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Analogs of Gray Identities for the Riemannian Curvature Tensor of Generalized Kenmotsu Manifolds

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Abstract

In this paper we consider the contact analogs of Gray identities for generalized Kenmotsu manifolds, based on these three classes of generalized Kenmotsu manifolds and we get the local structure of the classes of manifolds. It is proved that generalized almost C(λ)-Kenmotsu manifold is a Kenmotsu manifold of constant curvature -1 and obtain its local characterization.

Keywords: Generalized Kenmotsu manifolds, generalized Kenmotsu manifolds of class $CR_i, i = 1,2,3$, GK-manifolds of constant Φ-holomorphic sectional curvature, closely cosymplectic structure, generalized almost C(λ)- Kenmotsu manifold

1. Introduction

There are a number of important relationships between the almost contact metric structures and almost Hermitian structures. For example, [7] naturally identifies in a certain sense a complete system, consisting of 16 classes of almost Hermitian structures. Classification of almost contact metric structures, which is so to say «contact» analog (classification Gray-Hervella, almost Hermitian structures) are proposed in [11]. The first classification of similar classification (Gray-Hervella) was proposed in [5].

There is another principle of classification of almost Hermitian structures – for differential-geometric invariants of the second order, i.e., on the symmetry properties of the Riemann curvature tensor *R*. It is based on the principle put forward by Gray and formed in a number of his works [6] and others, according to which the key to understanding the differential geometric properties of almost Hermitian manifolds are identities satisfied by their Riemann curvature tensor (see Gray [6]):

$$R_1 : \langle R(X,Y)Z,W\rangle = \langle R(JX,JY)Z,W\rangle;$$
$$R_2 : \langle R(X,Y)Z,W\rangle = \langle R(JX,JY)Z,W\rangle + \langle R(JX,Y)JZ,W\rangle + \langle R(JX,Y)Z,JW\rangle;$$
$$R_3 : \langle R(X,Y)Z,W\rangle = \langle R(JX,JY)JZ,JW\rangle; X,Y,Z \in \mathcal{X}(M).$$

In this paper we consider the contact analogy of Gray identities for the Riemann curvature tensor of generalized Kenmotsu manifolds, we have considered in [1].

Generalized Kenmotsu manifolds Kenmotsu were introduced in [16]. In [12, 15] this class is called the class of nearly Kenmotsu manifolds. We denote the class of manifolds following Umnova as a class *GK*-manifolds.

This article is organized as follows. In Section 2 we present the preliminary information required in the sequel, we give the definition of generalized Kenmotsu manifolds, give the components of the Riemann-Christoffel tensor in the space of the associated *G*-structure. In Section 3 we consider contact analogs of Gray identities for *GK*-manifolds, we consider three classes of *GK*-manifolds and give the local structure of selected classes of *GK*-manifolds. The main results of paragraph 3 are given in Theorems 3.2–3.5. Paragraph 4 provides definitions of almost C(λ)-manifolds and C(λ)-manifold. These definitions are considered for the *GK*-manifolds. It is proved that generalized almost C(λ)- Kenmotsu manifold is a Kenmotsu manifold of constant curvature -1 , which means generalized almost C(λ)- Kenmotsu manifold canonically concircularly manifold $\boldsymbol{C}^n \times \boldsymbol{R}$, equipped with cosymplectic structure. The main results of section 4 are given in Theorems 4.2–4.5.

2. Preliminaries

Let *M* – smooth manifold of dimension $2n+1$, $\mathcal{X}(M) - C^\infty$-module of smooth vector fields on *M*. In the future, all manifolds, tensor fields, etc. objects are assumed to be smooth of class C^∞.

Definition 2.1[8]. Almost contact structure on a manifold *M* is a triple (η, ξ, Φ) tensor fields on the manifold, where η – a differential 1-form called the contact form of structure, ξ – vector field is called the characteristic,Φ – endomorphism of $\mathcal{X}(M)$ called the structure endomorphism. Here

1) $\eta(\xi) = 1$; 2) $\eta \circ \Phi = 0$; 3) $\Phi(\xi) = 0$; 4) $\Phi^2 = -id + \eta \otimes \xi$. (1)

Moreover if on M is fixed Riemannian structure $g = <\cdot,\cdot>$, such that
$\langle \Phi X, \Phi Y \rangle = \langle X, Y \rangle - \eta(X)\eta(Y),\ X, Y \in \mathcal{X}(M)$, (2)

Four $(\eta, \xi, \Phi, g = <\cdot,\cdot>)$ is called an almost contact metric (shorter, *AC*-) structure. Manifold with a fixed almost contact (metric) structure, called an almost contact (metric (in short, *AC*-)) manifold.

Skew-symmetric tensor $\Omega(X, Y) = < X, \Phi Y >,\ X, Y \in \mathcal{X}(M)$ is called the fundamental form of the AC-structures (see [11]).

Let $(M^{2n+1}, \Phi, \xi, \eta, g = \langle\cdot,\cdot\rangle)$–almost contact metric manifold.

In [9] Kenmotsu introduced a new class of almost contact metric structures, characterized by the identity
$\nabla_X(\Phi)Y = -\eta(Y)\Phi X - \langle X, \Phi Y\rangle \xi; X, Y \in \mathcal{X}(M)$. (3)

Put in this identity $Y = X$, we obtain
$\nabla_X(\Phi)X = -\eta(X)\Phi X; X \in \mathcal{X}(M)$. (4)

In the resulting identity will make a replacement $X \to X + Y$ (polarization along X), we obtain:
$\nabla_X(\Phi)Y + \nabla_Y(\Phi)X = -\eta(Y)\Phi X - \eta(X)\Phi Y; X, Y \in \mathcal{X}(M)$. (5)

Definition 2.2[16]. Class of almost contact metric manifolds, characterized by identity (5) is called generalized Kenmotsu manifolds (shorter, *GK*-) manifolds.

Recall the following theorem.

Theorem 2.1[1]. Complete structure equations *GK*-manifolds in the space of the associated *G*-structure has the form:
1) $d\omega = F_{ab}\omega^a \wedge \omega^b + F^{ab}\omega_a \wedge \omega_b$;
2) $d\omega^a = -\theta^a_b \wedge \omega^b + C^{abc}\omega_b \wedge \omega_c - \frac{3}{2}F^{ab}\omega \wedge \omega_b + \delta^a_b \omega \wedge \omega^b$;
3) $d\omega_a = \theta^b_a \wedge \omega_b + C_{abc}\omega^b \wedge \omega^c - \frac{3}{2}F_{ab}\omega \wedge \omega^b + \delta^b_a \omega \wedge \omega_b$;
4) $d\theta^a_b = -\theta^a_c \wedge \theta^c_b + \left(A^{ad}_{bc} - 2C^{adh}C_{hbc} - \frac{3}{2}F^{ad}F_{bc}\right)\omega^c \wedge \omega_d + \left(-\frac{1}{3}\delta^a_b F_{cd} + \frac{2}{3}\delta^a_c F_{db} + \frac{2}{3}\delta^a_d F_{bc}\right)\omega^c \wedge \omega^d + \left(\frac{1}{3}\delta^a_b F^{cd} - \frac{2}{3}\delta^c_b F^{da} - \frac{2}{3}\delta^d_b F^{ac}\right)\omega_c \wedge \omega_d$;
5) $dC^{abc} + C^{dbc}\theta^a_d + C^{adc}\theta^b_d + C^{abd}\theta^c_d = C^{abcd}\omega_d - 2\delta^{[a}_d F^{bc]}\omega^d - C^{abc}\omega$;
6) $dC_{abc} - C_{dbc}\theta^d_a - C_{adc}\theta^d_b - C_{abd}\theta^d_c = C_{abcd}\omega^d - 2\delta^d_{[a}F_{bc]}\omega_d - C_{abc}\omega$;
7) $dF^{ab} + F^{cb}\theta^a_c + F^{ac}\theta^b_c = -2F^{ab}\omega$;
8) $dF_{ab} - F_{cb}\theta^c_a - F_{ac}\theta^c_b = -2F_{ab}\omega$. (6)

where

$$C^{[abc]} = C^{abc};\ C_{[abc]} = C_{abc};\ \overline{C^{abc}} = C_{abc};$$
$$F^{ab} + F^{ba} = 0;\ F_{ab} + F_{ba} = 0;\ \overline{F^{ab}} = F_{ab};\ \ A^{ad}_{[bc]} = A^{[ad]}_{bc} = 0;$$
$$C^{a[bcd]} = \frac{3}{2} F^{a[b}F^{cd]}, C_{a[bcd]} = \frac{3}{2} F_{a[b}F_{cd]},\ F_{ad}C^{dbc} = 0. \qquad (7)$$

Theorem 2.2[1]. Nonzero essential components of the Riemann-Christoffel tensor on the space of the associated *G*-structure are of the form:
1) $R^a_{00b} = F^{ac}F_{cb} + \delta^a_b;$
2) $R^a_{bcd} = \frac{1}{3}(-2\delta^a_b F_{cd} + \delta^a_c F_{db} + \delta^a_d F_{bc});$
3) $R^a_{bc\hat{d}} = A^{ad}_{bc} - C^{adh}C_{hbc} - \frac{1}{2}F^{ad}F_{bc} - \delta^a_c\delta^d_b;$
4) $R^a_{\hat{b}cd} = 2C^{abh}C_{hcd} + F^{ab}F_{cd} - 2\delta^a_{[c}\delta^b_{d]};$
5) $R^a_{\hat{b}\hat{c}\hat{d}} = -2C^{ab[cd]} - \frac{3}{2}F^{ab}F^{cd}.$ (8)
Plus ratio obtained based on the classical symmetry properties of the tensor *R*. The remaining components of this tensor - zero.

3. Contact analogs of the Gray identities and Gray classes for generalized Kenmotsu manifolds

A. Gray in a number of papers [6, etc.] allocated identity that meets their Riemann curvature tensor (see Gray [6]):

$$R_1: \langle R(X,Y)Z,W\rangle = \langle R(JX,JY)Z,W\rangle;$$
$$R_2: \langle R(X,Y)Z,W\rangle = \langle R(JX,JY)Z,W\rangle + \langle R(JX,Y)JZ,W\rangle + \langle R(JX,Y)Z,JW\rangle;$$
$$R_3: \langle R(X,Y)Z,W\rangle = \langle R(JX,JY)JZ,JW\rangle; X,Y,Z \in \mathcal{X}(M).$$

Contact analogues identities A. Gray R_1, R_2 and R_3 curvature of almost Hermitian manifolds for the Riemannian curvature tensor are of curvature identities CR_1, CR_2 and CR_3 for almost contact metric manifolds:

$$CR_1: \langle R(\Phi X,\Phi Y)\Phi Z,\Phi W\rangle = \langle R(\Phi^2 X,\Phi^2 Y)\Phi Z,\Phi W\rangle;$$
$$CR_2: \langle R(\Phi X,\Phi Y)\Phi Z,\Phi W\rangle = \langle R(\Phi^2 X,\Phi^2 Y)\Phi Z,\Phi W\rangle + \langle R(\Phi^2 X,\Phi Y)\Phi^2 Z,\Phi W\rangle + \langle R(\Phi^2 X,\Phi Y)\Phi Z,\Phi^2 W\rangle;$$
$$CR_3: \langle R(\Phi X,\Phi Y)\Phi Z,\Phi W\rangle = \langle R(\Phi^2 X,\Phi^2 Y)\Phi^2 Z,\Phi^2 W\rangle; X,Y,Z \in \mathcal{X}(M).$$

Let M^{2n+1} –*GK*-manifold. We call *GK*-manifold having identities CR_1, CR_2 and CR_3, respectively, CR_1, CR_2 and CR_3– manifold.

The following theorem holds.

Theorem 3.1. Let $S = (\xi, \eta, \Phi, g = <\cdot,\cdot>)$–*AC*-structure. Then:
(1) $S = (\xi, \eta, \Phi, g = <\cdot,\cdot>)$ is structure of class CR_1 if and only if the space of the associated *G*-structure $R_{abcd} = R_{\hat{a}bcd} = R_{\hat{a}\hat{b}cd} = 0;$
(2) $S = (\xi, \eta, \Phi, g = <\cdot,\cdot>)$ is structure of class CR_2 if and only if the space of the associated *G*-structure $R_{abcd} = R_{\hat{a}bcd} = 0;$

(3) $S = (\xi, \eta, \Phi, g = <\cdot,\cdot>)$is structure of class CR_3 if and only if the space of the associated *G*-structure $R_{\hat{a}bcd} = 0$.

The proof of this theorem is similar to that of the corresponding theorem for Riemannian curvature tensor of almost Hermitian manifolds [11], and we omit it.

From this theorem follow inclusion $CR_1 \subset CR_2 \subset CR_3$.

Theorem 3.2. *GK*-manifold is a manifold of class CR_3 if and only if the manifold is *SGK*-manifold of type II.

Proof. Let M^{2n+1} is a *GK*-manifold of class CR_3. Then, according to Theorems2.2 and3.1, we have $R^a_{\hat{b}cd} = \frac{1}{3}(-2\delta^a_b F_{cd} + \delta^a_c F_{db} + \delta^a_d F_{bc}) = 0$. The last equality we turn to the indices a and c, then $F_{bd} + F_{bd} = 0$, i.e. $(n+1)F_{bd} = 0$. Since $n \geq 1$, then $F_{bd} = 0$, i.e. manifold is a special generalized manifold Kenmotsu (shorter, *SGK*-) of type II (see [1];[16]). Since for the *SGK*-manifolds of type II takes place$F_{bd} = 0$, i.e.$R^a_{\hat{b}cd} = 0$, i.e. manifold is a manifold of class CR_3.

Given the local structure of *SGK*-manifolds of type II [16], Theorem 4 can be formulated as follows.

Theorem 3.3. *GK*-manifold is a manifold of class CR_3 if and only if the manifold obtained from the nearly cosymplectic manifold canonical concircular transformation nearly cosymplectic structure.

Now let M^{2n+1} is a *GK*-manifold of class CR_2. Then, according to Theorem 3.1, we have $R^{\hat{a}}_{bcd} = -2C_{ab[cd]} + F_{ab}F_{cd} - 2F_{a[c}F_{|b|d]} = 0$. Since $F_{bd} = 0$, then $C_{ab[cd]} = 0$. The last equality, together with equality $C_{a[bcd]} = \frac{3}{2}F_{a[b}F_{cd]} = 0$ gives$C_{abcd} = 0$.

Under the conditions $C_{abcd} = 0$ and $F_{bd} = 0$, we find that $R_{abcd} = R_{\hat{a}bcd} = 0$, i.e., *GK*-manifold is the *GK*-manifold of class CR_2. Thus, we have the following theorem.

Theorem 3.4. *GK*-manifold is a manifold of class CR_2 if and only if on the space of the associated *G*-structure have $C_{abcd} = C^{abcd} = 0$ and $F^{ab} = F_{bd} = 0$.

Last theorem can be reformulated as follows:

Theorem 3.5. *GK*-manifold is a manifold of class CR_2 if and only if it is *SGK*- manifold of type II for which $C_{abcd} = C^{abcd} = 0$.

Now let M^{2n+1} is a *GK*-manifold of class CR_1. Then, according to Theorem 3.1, we have $R^a_{\hat{b}cd} = 2C^{abh}C_{hcd} + F^{ab}F_{cd} - 2\delta^a_{[c}\delta^b_{d]} = 0$, i.e. $C^{abh}C_{hcd} = \delta^a_{[c}\delta^b_{d]}$. In view of the last equality, we have $R^a_{bc\hat{d}} = A^{ad}_{bc} - \frac{1}{2}\delta^a_c\delta^d_b - \frac{1}{2}\delta^a_b\delta^d_c$.Symmetrize resulting equation over the indices *a*, *d* and *b*, *c*, we obtain

$R^{(a\ \ d)}_{(bc)} = -\delta^{(a}_{(c}\delta^{d)}_{b)}$, i.e., *GK*-manifold of class CR_1 is *GK*-manifold of constant Φ-holomorphic sectional curvature $c = -1$. Conversely, for *GK*-manifolds of constant Φ-holomorphic sectional curvature $c = -1$ we have $A^{ad}_{bc} = 0$ and$F^{ab} = 0$. So$C^{abh}C_{hcd} = \delta^{a}_{[c}\delta^{b}_{d]}$, i.e. $R_{abcd} = R_{\hat{a}bcd} = R_{\hat{a}\hat{b}cd} = 0$.And *GK*-manifold is the *GK*-manifold of class CR_1.

Thus, we can formulate the following theorem.

Theorem 3.6. *GK*-manifold is the *GK*-manifold of class CR_1if and only if it is a manifold of globally constant Φ-holomorphic sectional curvature $c = -1$.

According to Theorem 3. 7 in [1] we can formulate the following theorem.

Theorem 3.7. *GK*-manifold is the *GK*-manifold of class CR_1 if and only if the manifold canonical concircularly $\boldsymbol{C}^n \times \boldsymbol{R}$, endowed with the canonical nearly cosymplectic structure.

4. Generalized almost C(λ)-Kenmotsu manifold

The notion of almost C(λ)-manifolds, where λ - the real number was introduced and started their investigation in [8]. Subsequently, these varieties in their works accessed [14], [10], [3], [4], [13].

Definition 4.1[8, 14]. Almost contact thermal diversity called almost C(λ)-manifold if its Riemann curvature tensor satisfies

$$\langle R(Z,W)Y,X\rangle = \langle R(\Phi Z,\Phi W)Y,X\rangle - \lambda\{g(X,W)g(Y,Z) - g(X,Z)g(Y,W) - g(X,\Phi W)g(Y,\Phi Z) + g(X,\Phi Z)g(Y,\Phi W)\}, \quad (9)$$

where$X,Y,Z,W \in \mathcal{X}(M)$, and λ - real number.

Definition 4.2[8, 14]. The normal almost C(λ)-manifold is called the C(λ)-manifold.

In [8] it is shown that a symplectic manifold, Sasaki its diversity and variety Kenmotsu manifolds are respectively C(0)-, C(1)- and C(–1)-manifolds. The following theorem holds.

Theorem 4.1[10]. *AC*-manifold is almost C(λ)-manifold if and only if the components of the Riemann curvature tensor it in the space of the associated *G*-structure satisfy the relations: $R_{\hat{a}\hat{b}cd} = \lambda\delta^{ab}_{cd}$; $R_{\hat{a}0b0} = \lambda\delta^{a}_{b}$; $R_{\hat{a}bc\hat{d}}$– any, in view of the identity Ricci satisfying identity $R_{\hat{a}bc\hat{d}} - R_{\hat{a}cb\hat{d}} = -\lambda\delta^{ad}_{bc}$, where λ – the real number, $\delta^{ab}_{cd} = \delta^{a}_{c}\delta^{b}_{d} - \delta^{a}_{d}\delta^{b}_{c}$, while the other components are zero.

Knowing the expressions for the components of the Riemann curvature tensor in the space of the associated *G*-structure, according to the formula $S_{ij} = -R^{k}_{ijk}$ we obtain expressions for the components of the Ricci tensor almost C(λ)-manifold on the space of the associated *G*-structure:

$$S_{00} = 2\lambda n;\ S_{a\hat{b}} = S_{\hat{b}a} = R^{b}_{ca\hat{c}} + \lambda n\delta^{b}_{a}, \tag{10}$$

the other components are zero.

We calculate the scalar curvature χ almost C(λ)-manifold on the space of the associated *G*-structure the formula $\chi = g^{ij}S_{ij}$, where g^{ij} –contravariant components of the metric tensor. Using (2) and the matrix of the metric tensor g, we obtain

$$\chi = 2\lambda n + 2R^{b}_{ab\hat{a}} + 2\lambda n^{2}. \tag{11}$$

Let us – almost C(λ)-manifold is a generalized Kenmotsu manifold. From Theorems 2.2 and 4.1 we have:

1) $R^{a}_{00b} = F^{ac}F_{cb} + \delta^{a}_{b} = -\lambda\delta^{a}_{b}$;

2) $R^{a}_{bcd} = \frac{1}{3}(-2\delta^{a}_{b}F_{cd} + \delta^{a}_{c}F_{db} + \delta^{a}_{d}F_{bc}) = 0$;

3) $R^{a}_{bc\hat{d}} - R^{a}_{cb\hat{d}} = -2C^{adh}C_{hbc} - F^{ad}F_{bc} - \delta^{a}_{c}\delta^{d}_{b} - \delta^{a}_{b}\delta^{d}_{c} = -\lambda(\delta^{a}_{b}\delta^{d}_{c} - \delta^{a}_{c}\delta^{d}_{b})$;

4) $R^{a}_{\hat{b}cd} = 2C^{abh}C_{hcd} + F^{ab}F_{cd} - 2\delta^{a}_{[c}\delta^{b}_{d]} = \lambda(\delta^{a}_{c}\delta^{b}_{d} - \delta^{a}_{d}\delta^{b}_{c})$;

5) $R^{a}_{\hat{b}\hat{c}\hat{d}} = C^{acdb} - \frac{3}{2}F^{ab}F^{cd} = 0.$ (12)

Let's turn the equation (12: 2) to the indices a and b , we obtain $(n+1)F_{bc} = 0$. Since $n \geq 1$, the $F_{bc} = 0$; generalized almost C(λ)-Kenmotsu manifold is a special generalized Kenmotsu manifold type II. Then (12:1) takes the form $-\lambda\delta^{a}_{b} = \delta^{a}_{b}$. Let's turn the resulting equation over the indices *a* and *b*, we obtain that $\lambda = -1$, i.e., generalized almost C(λ)- Kenmotsu manifold is a special generalized almost C(–1)- Kenmotsu manifold of type II. Thus, we have proved

Theorem 4.2. A generalized almost C(λ)- Kenmotsu manifold is a special generalized almost C(–1)- Kenmotsu manifold of type II.

Since $F_{bc} = 0$ and $\lambda = -1$, then (12:4) takes the form: $C^{abh}C_{hcd} = 0$, i.e. $\sum_{abc}|C_{abc}|^{2} = 0$, i.e. $C_{abc} = 0$. In view of the preceding theorem we obtain the following theorem.

Theorem 4.3. A generalized almost C(λ)-Kenmotsu manifold is a Kenmotsu manifold.

In view of the resulting theorem 4.3, the relation (12) in the space of the associated *G*-structure relations take the form:

1) $R_{0a0\hat{b}} = \delta^{b}_{a}$; 2) $R_{ab\hat{c}\hat{d}} = 0$; 3) $R_{a\hat{b}c\hat{d}} = -A^{bd}_{ac} = \delta^{d}_{a}\delta^{b}_{c}$; 4) $R_{abcd} = 0$; 5) $R_{\hat{a}bcd} = 0$. (13)

Thus manifold is a manifold of constant curvature -1. The previous theorem can be stated as follows.

Theorem 4.4. A generalized almost C(λ)-Kenmotsu manifold is a Kenmotsu manifold of constant curvature -1.

Then, by Theorem 5.2 of [11], the previous theorem can be stated as:

Theorem 4.5. Generalized almost C(λ)-Kenmotsu manifold is canonically concircularly of manifold $\boldsymbol{C}^n \times \boldsymbol{R}$, provided with cosymplectic structure.

Note. From (12:2) and (12:5) it follows that generalized almost C(λ)-Kenmotsu manifold is the manifold of the class CR_2.

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