Estimation Parameter of $R = P(Y<X)$ for Lomax Distribution with Presence of Outliers

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Abstract

The Lomax (Pareto II) distribution has found wide application in a variety of fields. This paper deals with the estimation parameters of $P(Y<X)$ when $Y$ has Lomax distribution with parameters $\theta$ and $\lambda$ and $X$ has Lomax distribution with presence of $k$ outliers with parameters $\alpha, \beta$ and $\lambda$ such that $X$ and $Y$ are independent. The maximum likelihood and moment estimator of Lomax distribution with presence of $k$ outliers and the maximum likelihood estimator of $P(Y<X)$ with presence of $k$ outliers are derived. Analysis of a simulated data set has also been presented for illustrative purposes.

Keywords: Lomax distributions; Maximum likelihood estimator; moment estimators; Outlier

1. Introduction

Surles and Padget (1998) considered the estimation of $P[Y<X]$, where $X$ and $Y$ are Burr type X random variable. The maximum likelihood estimator (MLE) of $P[Y<X]$, when $X$ and $Y$ are normally distributed, has been considered by Downtown (1973). Woodward and Kelley (1977) and Owen et al. (1977) considered the estimation of $P[Y<X]$ when $X$ and $Y$ are independent exponential random variables. Awad et al. (1981) considered the MLE of R, when $X$ and $Y$ have bivariate exponential distribution. In 1990, Gupta and Gupta considered the estimation of $P(aX > bY )$ in the multivariate normal case.

Recently, Nasiri et al. (2011) studied estimation of $P[Y<X]$ for generalized exponential distribution with presence of $K$ outliers.
The two-parameter Lomax distribution has the following p.d.f. and c.d.f. respectively:

\[ f(x; \theta, \lambda) = \frac{\theta}{\lambda} \left(1 + \frac{x}{\lambda}\right)^{-(1+\theta)}, \quad x > 0, \theta > 0, \lambda > 0 \]  
\[ (1-1) \]

\[ F(x; \theta, \lambda) = 1 - \left(1 + \frac{x}{\lambda}\right)^{-\theta} \]  
\[ (1-2) \]

Where \( \theta > 0 \) is the shape parameter, and \( \lambda > 0 \) is the scale parameter.

The main aim of this paper is to focus on the inference of \( R = P(Y < X) \), where \( Y \sim \text{lomax}(\theta, \lambda) \), with pdf denoted in equation (1-1) and \( X \), has Lomax Distribution with presence of \( k \) outliers.

2. Joint Distribution of \( (X_1, X_2, \ldots, X_n) \) with \( k \) Outliers

According to Dixit (1989) and Dixit and Nasiri (2001) model, we assume that the random variables \( (X_1, X_2, \ldots, X_n) \) are such that of \( k \) them are

\[ g(x, \beta, \lambda) = \frac{\beta}{\lambda} \left(1 + \frac{x}{\lambda}\right)^{-(1+\beta)}, \quad x > 0 \]  
\[ (2-1) \]

And remaining \((n-k)\) random variables are distribution as

\[ f(x, \alpha, \lambda) = \frac{\alpha}{\lambda} \left(1 + \frac{x}{\lambda}\right)^{-(1+\alpha)}, \quad x > 0 \]  
\[ (2-2) \]

The joint distribution of \( (X_1, X_2, \ldots, X_n) \) is given as:

\[ f(x_1, x_2, \ldots, x_n) = \frac{k!(n-k)!}{n!} \prod_{i=1}^{n} f(x_i, \alpha) \cdot \prod_{r=1}^{k} \frac{g(x_{A_r})}{f(x_{A_r})} \]  
\[ (2-3) \]

Where \( \sum^* = \sum_{A_1=1}^{n-k+1} \sum_{A_2=A_1+1}^{n-k+2} \cdots \sum_{A_k=A_{k-1}+1}^{n} \).

With using the equation (2-1) and (2-2), the joint distribution is given by

\[ f(x_1, x_2, \ldots, x_n) = \frac{k!(n-k)!}{n!} \left(\frac{\alpha}{\lambda}\right)^n \prod_{i=1}^{n} (1 + \frac{x_i}{\lambda})^{-(1+\alpha)} \sum^* \prod_{r=1}^{k} \frac{\beta}{\lambda} \left(1 + \frac{x_{A_r}}{\lambda}\right)^{-(1+\beta)} \]

\[ = \frac{k!(n-k)!}{n!} \left(\frac{\alpha^{n-k} \beta^k}{\lambda^*}\right) \prod_{i=1}^{n} (1 + \frac{x_i}{\lambda})^{-(1+\alpha)} \sum^* \prod_{i=1}^{k} (1 + \frac{x_{A_r}}{\lambda})^{\alpha-\beta} \]  
\[ (2-4) \]
By using the equation (2-4) the marginal distribution of $X$ is given by
\[
f(x; \alpha, \beta, \lambda) = \frac{k}{n} \left(1 + \frac{x}{\lambda}\right)^{-(1+\beta)} + \frac{n-k}{n} \left(1 + \frac{x}{\lambda}\right)^{-(1+\alpha)} \cdot x > 0	ag{2-5}
\]

3. Moment estimators of Lomax Distribution with presence of $k$ outliers

The raw moments of $X$ may be determined from marginal distribution of $X$. For $r \in \mathbb{N}$, we find that
\[
E(X^r) = \int_0^\infty x^r \left(\frac{k}{n} \left(1 + \frac{x}{\lambda}\right)^{-(1+\beta)} + \frac{n-k}{n} \left(1 + \frac{x}{\lambda}\right)^{-(1+\alpha)} \right) dx
\]
\[
= \frac{k}{n} \frac{\beta}{\lambda} \int_0^\infty x^r \left(1 + \frac{x}{\lambda}\right)^{-(1+\beta)} dx + \frac{(n-k)\alpha}{n} \int_0^\infty x^r \left(1 + \frac{x}{\lambda}\right)^{-(1+\alpha)} dx
\]
\[
= \frac{k}{n} \sum_{i=0}^{\infty} \frac{r}{(r+i)\lambda^i} \left(\binom{-\beta}{i} + \frac{n-k}{n} \binom{-\alpha}{i}\right) \frac{1}{r+i}
\]
\[
= \frac{r}{n} \sum_{i=0}^{\infty} \frac{r}{(r+i)\lambda^i} \left[\binom{-\beta}{i} + \frac{n-k}{n} \binom{-\alpha}{i}\right] \tag{3-1}
\]

For $r = 1, 2, 3$, $E(X^r)$ is given by
\[
E(X) = \frac{1}{n} \sum_{i=0}^{\infty} \frac{1}{(1+i)\lambda^i} \left[\binom{-\beta}{i} + \frac{n-k}{n} \binom{-\alpha}{i}\right] 
\]
\[
E(X^2) = \frac{2}{n} \sum_{i=0}^{\infty} \frac{1}{(2+i)\lambda^i} \left[\binom{-\beta}{i} + \frac{n-k}{n} \binom{-\alpha}{i}\right] 
\]
\[
E(X^3) = \frac{3}{n} \sum_{i=0}^{\infty} \frac{1}{(3+i)\lambda^i} \left[\binom{-\beta}{i} + \frac{n-k}{n} \binom{-\alpha}{i}\right] 
\]

To estimation of all parameters, it can be consider the following normal equations,
\[
\hat{E}(X^r) = \frac{1}{n} \sum_{i=1}^{n} X_i^r \tag{3-5}
\]

4. Maximum Likelihood estimators of Lomax distribution with presence of $k$ outliers

For observations, $(X_1, X_2, ..., X_n)$ the likelihood function is given by
\[
f(x_1, x_2, ..., x_n) = \frac{k!(n-k)!}{n!} \frac{\alpha^{n-k} \beta^k}{\lambda^n} \prod_{i=1}^{n} \left(1 + \frac{x_i}{\lambda}\right)^{-(1+\alpha)} \sum \prod_{i=1}^{n} \left(1 + \frac{x_i}{\lambda}\right)^{-(1+\beta)} \tag{4-1}
\]
The log-likelihood function of the equation (4-1) is given by

\[
\ln f(x_1, x_2, \ldots, x_n) = \ln \left( \frac{k!(n-k)!}{n!} \cdot \frac{\alpha^{a-k} \beta^k}{\lambda^n} \prod_{i=1}^{n} \left( 1 + \frac{x_i}{\lambda} \right)^{-(1+\alpha)} \sum \prod_{r=1}^{k} \left( 1 + \frac{x_{Ar}}{\lambda} \right)^{\alpha-\beta} \right)
\]

\[
= \ln \left( \frac{k!(n-k)!}{n!} \right) + (n-k) \ln(\alpha) - n \ln(\lambda) + k \ln(\beta) + \ln(\prod_{i=1}^{n} \left( 1 + \frac{x_i}{\lambda} \right)^{-(1+\alpha)}) + \ln(\sum \prod_{r=1}^{k} \left( 1 + \frac{x_{Ar}}{\lambda} \right)^{\alpha-\beta})
\]

\[
= \ln \left( \frac{k!(n-k)!}{n!} \right) + (n-k) \ln(\alpha) - n \ln(\lambda) + k \ln(\beta) + \sum_{i=1}^{n} \ln\left( 1 + \frac{x_i}{\lambda} \right)^{-(1+\alpha)} + \ln(\sum \prod_{r=1}^{k} \left( 1 + \frac{x_{Ar}}{\lambda} \right)^{\alpha-\beta})
\]

(4-2)

Taking the derivative with respect to \(\alpha, \beta, \lambda\) and equating to 0, we can obtain the normal equations as:

\[
\frac{\partial \ln f(x_1, x_2, \ldots, x_n)}{\partial \alpha} = \frac{n-k}{\alpha} - \sum_{i=1}^{n} \ln \left( 1 + \frac{x_i}{\lambda} \right) + \frac{\partial}{\partial \alpha} \cdot \frac{\prod_{r=1}^{k} \left[ 1 + \frac{x_{Ar}}{\lambda} \right]^{\alpha-\beta}}{\sum_{r=1}^{k} \left[ 1 + \frac{x_{Ar}}{\lambda} \right]^{\alpha-\beta}} = 0
\]

\[
\frac{\partial \ln f(x_1, x_2, \ldots, x_n)}{\partial \beta} = \frac{k}{\beta} + \frac{\partial}{\partial \beta} \cdot \frac{\prod_{r=1}^{k} \left[ 1 + \frac{x_{Ar}}{\lambda} \right]^{\alpha-\beta}}{\sum_{r=1}^{k} \left[ 1 + \frac{x_{Ar}}{\lambda} \right]^{\alpha-\beta}} = 0
\]

\[
\frac{\partial \ln f(x_1, x_2, \ldots, x_n)}{\partial \lambda} = \frac{k}{\lambda} + \frac{\partial}{\partial \lambda} \left[ -(1+\alpha) \sum_{i=1}^{n} \ln \left( 1 + \frac{x_i}{\lambda} \right) + \sum_{r=1}^{k} \ln \left( 1 + \frac{x_{Ar}}{\lambda} \right) \right] = 0
\]

Here, we need to use either the scoring algorithm or the Newton Raphson method to solve the non-linear equation. Hence solution of the equation is

Newton - Raphson method is \(\alpha_{i+1} = \alpha_i - \frac{g(\alpha_i)}{g'(\alpha_i)}\), that \(\alpha_i\), would be the initial guess.

Where

\[
g(\alpha_i) = \frac{n-k}{\alpha} - \sum_{i=1}^{n} \ln \left( 1 + \frac{x_i}{\lambda} \right) + \frac{\prod_{r=1}^{k} \left[ 1 + \frac{x_{Ar}}{\lambda} \right]^{\alpha-\beta}}{\sum_{r=1}^{k} \left[ 1 + \frac{x_{Ar}}{\lambda} \right]^{\alpha-\beta}}\]
Estimation parameter of $R = P(Y < X)$

$g(\alpha_i)' = \frac{-n}{\alpha^2} + \sum^k_{r=1} \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}} \left( \ln \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}} + \frac{\ln \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}}}{\lambda} \right)^2$

$g(\beta_i)' = \frac{k}{\beta^2} - \frac{n}{\beta^2} + \sum^k_{r=1} \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}} \left( \ln \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}} + \frac{\ln \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}}}{\lambda} \right)^2$

$g(\lambda)' = \frac{(n+m)}{\lambda} + (1 + \theta) \sum^m_{i=1} \frac{y_i}{\lambda + y_i} + \alpha \sum^n_{i=1} \frac{x_i}{\lambda + x_i} - \sum^k_{r=1} (\alpha - \beta) \frac{x_{Ar}}{\lambda^2} \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}}^{\alpha - \beta - 1}$

$g(\lambda)' = \frac{(n+m)}{\lambda^2} - (1 + \theta) \sum^m_{i=1} \frac{(2\lambda + y_i)x_i}{(\lambda + y_i)^2} + \sum^k_{r=1} (\alpha - \beta) \frac{2x_{Ar}}{\lambda^3} \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}}^{\alpha - \beta - 1} + \sum^k_{r=1} (\alpha - \beta) \frac{2x_{Ar}}{\lambda^3} \frac{1 + \frac{x_{Ar}}{\lambda}}{1 + \frac{x_{Ar}}{\lambda}}^{\alpha - \beta - 1}$

5. Maximum likelihood estimator of R

Let $(Y_1, Y_2, ..., Y_m)$ be a random sample for Y with pdf

$$g(y, \theta, \lambda) = \frac{\theta}{\lambda} \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\theta)} y > 0$$

And $(X_1, X_2, ..., X_n)$ be a random sample for X with pdf
\[ f(x, \alpha, \beta, \lambda) = \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta)} + \frac{n-k \alpha}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\alpha)} \quad x > 0 \]

Such that of \( k \) them are

\[ g(x, \beta, \lambda) = \frac{\beta}{\lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta)} \]

And remaining \( (n-k) \) random variables are distribution as

\[ f(x, \alpha, \lambda) = \frac{\alpha}{\lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\alpha)}. \]

Then,

\[ R = P(Y < X) = \int_{0}^{\infty} \int_{0}^{x} g(y, \theta, \lambda) f(x, \alpha, \beta, \lambda) dy dx = \]

\[ \int_{0}^{\infty} \left[ \int_{0}^{x} \frac{\beta}{\lambda} \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\beta)} dy \left[ \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta)} + \frac{n-k \alpha}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\alpha)} \right] \right] dx \]

\[ = \int_{0}^{\infty} \left[ \int_{0}^{x} \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta)} \left[ \frac{\theta}{\lambda} \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\theta)} \right] \right] dy dx \]

\[ + \int_{0}^{\infty} \left[ \int_{0}^{x} \frac{n-k \alpha}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\alpha)} \left[ \frac{\theta}{\lambda} \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\theta)} \right] \right] dy dx \]

\[ I = \int_{0}^{\infty} \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta)} \int_{0}^{x} \frac{\beta}{\lambda} \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\theta)} dy dx \]

\[ \quad = \int_{0}^{\infty} \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta)} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\theta)} \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\theta)} \right]_0^x dx = \]

\[ \int_{0}^{\infty} \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta)} \left[ 1 - \left[ 1 + \frac{x}{\lambda} \right]^{-\theta} \right] \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\theta)} \right]_0^x dx = \]

\[ \int_{0}^{\infty} \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta)} \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\theta)} \right]_0^x dx - \int_{0}^{\infty} \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta+\theta)} \right]_0^x dx = \]

\[ \int_{0}^{\infty} \frac{k \beta}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\beta+\theta)} \right]_0^x dx = \frac{k}{n} \left[ 1 + \frac{x}{\lambda} \right]^{-\beta} - \frac{\beta}{\beta + \theta} \frac{k}{n} \left[ 1 + \frac{x}{\lambda} \right]^{-(\beta+\theta)} \right]_0^\infty = \]

\[ \frac{k}{n} - \frac{k \beta}{n \beta + \theta} = \frac{k}{n} \left( 1 - \frac{\beta}{\beta + \theta} \right) = \frac{k}{n} \frac{\theta}{\beta + \theta} \]

Also,

\[ II = \int_{0}^{\infty} \int_{0}^{x} \left[ \frac{n-k \alpha}{n \lambda} \left[ 1 + \frac{x}{\lambda} \right]^{-(1+\alpha)} \left[ \frac{\theta}{\lambda} \left[ 1 + \frac{y}{\lambda} \right]^{-(1+\theta)} \right] \right] dy dx = \]
\[
\int_{0}^{\infty} \frac{n - k}{n} \frac{\alpha}{\lambda} \left[1 + \frac{x}{\lambda}\right]^{-(1+\alpha)} \int_{0}^{\frac{x}{\lambda}} \frac{\theta}{1 + \frac{y}{\lambda}}^{-(1+\theta)} \, dy \, dx = \\
\int_{0}^{\infty} \frac{n - k}{n} \frac{\alpha}{\lambda} \left[1 + \frac{x}{\lambda}\right]^{-(1+\alpha)} \left[- \left[1 + \frac{y}{\lambda}\right]^{-\theta}\right] \, dx \\
= \int_{0}^{\infty} \frac{n - k}{n} \frac{\alpha}{\lambda} \left[1 + \frac{x}{\lambda}\right]^{-(1+\alpha)} \left[1 - \left[1 + \frac{x}{\lambda}\right]^{-\theta}\right] \, dx = \\
\int_{0}^{\infty} \frac{n - k}{n} \frac{\alpha}{\lambda} \left[1 + \frac{x}{\lambda}\right]^{-(1+\alpha)} \left[- \left[1 + \frac{y}{\lambda}\right]^{-\theta}\right] \, dx \\
= \frac{n - k}{n} \frac{\alpha}{\lambda} \left[1 + \frac{x}{\lambda}\right]^{-(1+\alpha+\theta)} \left[0 - \frac{\alpha}{\lambda} \frac{n - k}{n} \frac{\alpha}{\lambda} \left[- \left[1 + \frac{y}{\lambda}\right]^{-(\alpha+\theta)}\right] \right] \int_{0}^{\infty} \\
n - k - n - k \frac{\alpha}{\lambda + \theta} = n - k \left(1 - \frac{\alpha}{\lambda + \theta}\right) = \frac{n - k}{n} \frac{\theta}{\lambda + \theta}
\]

\[1 + \frac{\alpha}{\lambda + \theta} \Rightarrow R = P(Y < X) = \frac{k}{\beta + \hat{\theta}} + \frac{n - k}{\hat{\theta} + \beta} \frac{\lambda}{\alpha + \theta} \tag{5-1}\]

Therefore, the MLE of \( R \) becomes

\[\hat{R} = \frac{k}{\hat{\theta}} + \frac{n - k}{\hat{\theta} + \beta} \frac{\lambda}{\alpha + \theta}\]

Where \( \hat{\alpha}, \hat{\beta} \) he MLE of \( \alpha, \beta \) and the MLE of \( \theta \) can be obtained as

\[
\frac{\partial L(\theta, \alpha, \beta, \lambda)}{\partial \theta} = \frac{m}{\theta} - \sum_{i=1}^{m} \ln \left[1 + \frac{y_i}{\lambda}\right] = 0 \Rightarrow \frac{m}{\theta} = \sum_{i=1}^{m} \ln \left[1 + \frac{y_i}{\lambda}\right] \Rightarrow \hat{\theta} \]

\[
= \frac{\sum_{i=1}^{m} \ln \left[1 + \frac{y_i}{\lambda}\right]}{m} \]

6. Numerical experiments and discussions

In order to have some idea about Mean Square Error (MSE) of MLE, we perform sampling experiment using a MATLAB. We consider two cases separately to draw inference on R, namely when (i) \( \lambda \) is unknown and (ii) \( \lambda \) is known.

In both cases we consider the following small sample size: \((n, m) = (10, 10), (10, 15), (10, 20), (10, 25), (15, 10), (15, 15), (15, 20), (15, 25), (20, 10), (20, 15), (20, 20), (20, 25),

(25, 10), (25, 15), (25, 20), (25, 25).\)

In both cases we take \( \theta = 1.5, \alpha = 2.75, \beta = 2.5 \) and \( k = 1. \)

Without loss of generality, we take \( \lambda = 1. \)

In all cases considered, all the results are based on 1000 replications.

The data has been truncated after four decimal places and it has been presented below.

(i) \( \lambda \) is unknown and \( \theta = 1.5, \alpha = 2.75, \beta = 2.5, k = 1, n = m = 25 \)
The $Y$ values:

\[
\begin{array}{cccccccccccc}
0.5856 & 3.2988 & 0.1688 & 0.0460 & 0.1093 & 2.4655 & 4.3285 & 0.3632 & 3.7382 & 8.6357 \\
1.1215 & 0.3395 & 0.4308 & 0.3032 & 0.1675 & 0.9054 & 4.4180 & 0.7440 & 0.1747 & 1.2076 \\
0.2289 & 3.6355 & 2.2237 & 0.0533 & 0.1717 & & & & & \\
\end{array}
\]

And the corresponding $X$ values are:

\[
\begin{array}{cccccccccccc}
0.8104 & 0.0706 & 0.5474 & 0.2670 & 0.1873 & 6.0553 & 0.0973 & 0.7607 & 0.7869 & 0.0875 \\
0.6132 & 0.0700 & 0.0010 & 0.4033 & 0.0394 & 0.2601 & 1.4897 & 0.3744 & 3.2598 & 0.0386 \\
0.4307 & 4.0534 & 0.2869 & 0.3458 & 0.0600 & & & & & \\
\end{array}
\]

Then $\hat{\theta} = 1.3845$, $\hat{\alpha} = 2.0524$, $\hat{\beta} = 3.2771$, $\hat{\lambda} = 2.0365$ and $\hat{R} = 0.3986$

(ii) $\lambda$ is known and $\theta = 1.5, \alpha = 2.75, \beta = 2.5, k = 1, n = m = 25$

The $Y$ values:

\[
\begin{array}{cccccccccccc}
6.8681 & 1.2438 & 1.2122 & 0.5148 & 3.1058 & 0.1650 & 0.5820 & 1.6796 & 0.1311 & 3.2672 \\
0.0005 & 0.2026 & 0.5495 & 0.6389 & 1.3791 & 0.0253 & 1.1399 & 0.6955 & 0.5208 & 0.1768 \\
0.8051 & 0.0651 & 3.0956 & 0.0952 & 0.3308 & & & & & \\
\end{array}
\]

And the corresponding $X$ values are:

\[
\begin{array}{cccccccccccc}
0.2866 & 0.2424 & 0.1147 & 0.6077 & 0.4151 & 0.1088 & 0.2942 & 0.1605 & 0.0384 & 0.4919 \\
0.5605 & 0.1920 & 0.8610 & 0.7614 & 0.0818 & 1.2485 & 0.1529 & 0.8169 & 0.4351 & 0.2525 \\
0.6562 & 0.1279 & 0.5139 & 2.3124 & 0.0791 & & & & & \\
\end{array}
\]

Then $\hat{\theta} = 1.6685$, $\hat{\alpha} = 2.8822$, $\hat{\beta} = 2.9120$, and $\hat{R} = 0.3665$

7. Conclusions

In this paper, we have addressed the problem of estimating parameters of $R=P(Y<X)$ for Lomax distribution with presence $k$ outliers, when they have the same scale parameter. We consider both the cases, namely the common scale parameter is unknown or known, and the results are given in tables 1 and 2. It is observed that the maximum likelihood estimator of $R$, when $\lambda$ is unknown and known works quit well. We report the average estimates and the MSEs based on 1000 replications.

In this case as expected when $m = n$ and $m, n$ increase then the average MLEs decrease. The result of MSE of the estimators are given in Tables 1 to 2, for $\theta = 1.5, \alpha = 2.75, \beta = 2.5, \lambda = 1$. 

\[\text{Parviz Nasiri}\]
Table 1
\[ \lambda \text{ is unknown } \theta = 1.5, \alpha = 2.75, \beta = 2.5, \lambda = 1 \]

<table>
<thead>
<tr>
<th>(n)</th>
<th>(m)</th>
<th>(\hat{\theta})</th>
<th>(\hat{\alpha})</th>
<th>(\hat{\beta})</th>
<th>(\hat{\lambda})</th>
<th>(\hat{R})</th>
<th>(MSE(\hat{R}))</th>
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