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$\hat{\alpha g}$ Maximal Filters and $\hat{\alpha g}$ Convergence of $\hat{\alpha g}$ Filters

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Abstract

In this paper we introduce $\hat{\alpha g}$ maximal filters, study their existence and characterization. Then, we go on to study $\hat{\alpha g}$ convergence of $\hat{\alpha g}$ filters, define their $\hat{\alpha g}$ limit points and $\hat{\alpha g}$ limit points of a $\hat{\alpha g}$ filter base. We study their properties in detail. Finally, we define and study $\hat{\alpha g}$ cluster points of a $\hat{\alpha g}$ filter.

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1. Introduction

Throughout this paper (X, τ) denotes a topological space on which no separation axiom is assumed. (X, τ) will be simply denoted by X.N. Levine [3] introduced the concept of generalized closed sets and discussed the properties of sets. P.Bhattachariya and B.K. Lahiri [1] introduced the class of semi-generalized closed sets. O.Njastad [5] introduced α - open sets in general topology.

Senthilkumaran et al. [6] introduced $\hat{\alpha}$ generalized closed sets and established new type of filters and filter bases [7]. In this paper, we study about $\hat{\alpha}$g maximal filters and $\hat{\alpha}$g convergence of $\hat{\alpha}$g filters.

2. Preliminaries

Definition 2.1: A subset A of a topological space X is said to be
1)pre open if A$\subset$int cl A and pre closed if cl int A $\subset$A
2)regular open if A = int cl A and regular closed if A = cl int A
3)semi open if A $\subset$ cl int A and semi closed if int cl A$\subset$ A
4)α – open if A$\subset$int cl int A and α – closed if cl int cl A$\subset$A.

Definition 2.2: A subset A of a topological space X is called $\hat{\alpha}$ generalized closed set[6] ($\hat{\alpha}$g closed), if intclint A$\subset$ U, whenever A$\subset$ U and U is open in X.

The complement of $\hat{\alpha}$g closed set in X is $\hat{\alpha}$g open in X. The intersection of two $\hat{\alpha}$g open sets need not be $\hat{\alpha}$g open. In what follows, we assume finite intersection of $\hat{\alpha}$g open sets is $\hat{\alpha}$g open.

Definition 2.3[7]: Let X be a topological space A $\hat{\alpha}$g filter on X is a non empty family **F** of $\hat{\alpha}$g open subsets of X satisfying the following axioms:

1:$\phi\notin$**F**

2: If F$\in$**F** and H is $\hat{\alpha}$g open such that H$\supset$ F, then H$\in$**F**
3:If F$\in$**F** and H$\in$**F**, then F$\cap$ H$\in$**F.**

Definition 2.4[7]: Let X be a topological space. A $\hat{\alpha}$g filter base on X is a non empty family **B** of $\hat{\alpha}$g open subsets of X satisfying the following axioms:

1) $\phi\notin$**B**
2)If F,H$\in$**B**, then there exists G$\in$**B** such that G$\subset$ F$\cap$ H.

3. $\hat{\alpha}$g maximal filters

Definition 3.1: A $\hat{\alpha}$g filter **F** on a topological space X is said to be a $\hat{\alpha}$g maximal filter on X if and only if **F** is not properly contained in any other $\hat{\alpha}$g filter on X.

Remark 3.2: A $\hat{\alpha}$g maximal filter on X is a maximal element of the collection of all $\hat{\alpha}$g filters on X partially ordered by the inclusion relation $\subset$

Definition 3.3: A $\hat{\alpha}$g filter base on a topological space X is called a $\hat{\alpha}$g maximal filter base if and only if it is a base of a $\hat{\alpha}$g maximal filter.

4. Existence of $\hat{\alpha g}$ maximal filters

Theorem 4.1: Every $\hat{\alpha g}$ filter on a topological space X is contained in a $\hat{\alpha g}$ maximal filter.

Proof: Let **F** be any $\hat{\alpha g}$ filter on X and let **M** be the class of all $\hat{\alpha g}$ filters containing **F**. Then **M** is non empty since $\mathbf{F}\in\mathbf{M}$. Also **M** is partially ordered by the inclusion relation $\subset$. Now, let **H** be a linearly ordered subset of **M**. Then, by the definition of linear ordering for any two members $\mathbf{F_1}$,$\mathbf{F_2}$ of **H,** we have either $\mathbf{F_1}\subset\mathbf{F_2}$or $\mathbf{F_2}\subset\mathbf{F_1}$.
Let $\mathbf{G} = \{\mathbf{F_\lambda}:\mathbf{F_\lambda}\in\mathbf{H}\}$. We claim that **G** is a $\hat{\alpha g}$ filter on X. **G** is evidently non empty. Also, Since $\mathbf{F_\lambda}$ is a $\hat{\alpha g}$ filter, we have $\phi\notin\mathbf{F_\lambda}$, for every $\mathbf{F_\lambda}\in\mathbf{H}$ and so $\phi\notin\mathbf{G}$.
Let $A\in\mathbf{G}$ and B be a $\hat{\alpha g}$ open set such that $B\supset A$.
Then $A\in\mathbf{F_\lambda}$ for some $\mathbf{F_\lambda}\in$ H and since $\mathbf{F_\lambda}$ is a $\hat{\alpha g}$ filter, $B\in\mathbf{F_\lambda}$. Hence $B\in\mathbf{G}$. Let $A,B\in\mathbf{G}$. Then $A\in\mathbf{F_\lambda}$ and $B\in\mathbf{F_\mu}$, for some $\mathbf{F_\lambda}$,$\mathbf{F_\mu}\in\mathbf{H}$. Since **H** is linearly ordered, we have either $\mathbf{F_\lambda}\subset\mathbf{F_\mu}$ or $\mathbf{F_\mu}\subset\mathbf{F_\lambda}$. Hence both A and B belong to either $\mathbf{F_\lambda}$ or $\mathbf{F_\mu}$. Hence $A\cap B\in\mathbf{G}$. So **G** is a $\hat{\alpha g}$ filter on X. Also **G** is finer then every member of **H** and hence **G** is an upper bound of **H**. We have proved that **M** is a non empty partially ordered set in which every linearly ordered subset has an upper bound. Hence by Zorn's lemma, **M** contains a maximal element say **F'.** The maximal element **F'** is by definition, a $\hat{\alpha g}$ maximal filter on X containing **F**.

A more general form of the above theorem is the following.

Theorem 4.2: Every $\hat{\alpha g}$ filter base on a topological space X is contained in a $\hat{\alpha g}$ maximal filter base on X.

Proof: The proof follows from theorem 6.6 [7] and the above theorem
A still more general form is the following.

Theorem 4.3: Let **A** be a non empty collection of $\hat{\alpha g}$ open subsets of X such that **A** has the FIP. Then there exists a $\hat{\alpha g}$ maximal filter **F** containing **A**.

Proof: Let $\mathbf{M} = \{\mathbf{C}$: **C** is a collection of $\hat{\alpha g}$ open subsets of X with the FIP such that $\mathbf{C}\supset\mathbf{A}\}$. Then **M** is non empty as $\mathbf{A}\in\mathbf{M}$. Also **M** is partially ordered by the inclusion relation $\subset$. Further every linearly ordered subset of **M** has an upper bound. Hence by Zorn's lemma, there exists a maximal element **F** of **M**. Let us prove that **F** is a $\hat{\alpha g}$ filter on X. As $\mathbf{F}\supset\mathbf{A}$ and **A** is non empty, it follows **F** is non empty. Further, since **F** has the FIP, no member of **F** can be empty. Hence$\phi\notin\mathbf{F}$. Let $A\in\mathbf{F}$ and B be $\hat{\alpha g}$ open subset of X such that $B\supset A$. Since **F** has the FIP, it follows $\{B\}\cup\mathbf{F}$ also has FIP. More over $\{B\}\cup\mathbf{F}\supset\mathbf{A}$**.** Hence $\{B\}\cup\mathbf{F}\in\mathbf{M}$. As **F** is

maximal, B∈**F**. Let A,B∈**F**. Since **F** has FIP,{A∩B}∪**F** also has FIP. Further {A∩B}∪**F**⊃ **A.** Hence {A∩B}∪**F**∈**M**. Since **F** is maximal, A∩ B∈**F**.
This completes the proof.

5. Characterization of $\hat{\alpha g}$ maximal filters

Theorem 5.1: A $\hat{\alpha g}$ filter **F** on a topological space X is a $\hat{\alpha g}$ maximal filter if and only if **F** contains all those $\hat{\alpha g}$ open subsets of X which intersect every member of **F**.

Proof: Let **F** be a $\hat{\alpha g}$ filter on X satisfying the condition. Let **F**' be a $\hat{\alpha g}$ filter finer than **F**. Let F'∈**F'**. F' intersects every member of **F'**. Since **F**⊂**F'**,F' intersect every member of **F**. Hence F'∈**F**, thereby **F'**⊂**F**. So, **F** is a $\hat{\alpha g}$ maximal filter on X.

Conversely, let **F** be a $\hat{\alpha g}$ maximal filter on X. Let A be a $\hat{\alpha g}$ open subset of X intersecting every member of **F**.
Consider **F**'= {F': F'⊃A∩ F for some F ∈**F**}. We claim **F**' is a $\hat{\alpha g}$ filter on X containing **F**. For F∈**F,** F⊃ A∩ F, so F∈**F'**. Hence **F'**⊃**F**.
As A intersects every member of **F**, we have A∩ F ≠ ϕ , for every F∈**F** .For F'∈ **F',** F'⊃ A∩ F≠ ϕ , for some F∈**F**, we have F' ≠ ϕ. Henceϕ∉ **F'**. Let F'∈**F'** and G' be a $\hat{\alpha g}$ open subset of X such that G'⊃ F'. Then F'⊃A∩ F for some F∈**F**. Hence G'⊃A∩ F and so G'∈**F'**. Let F', G'∈**F'**. Then F'⊃A∩ F, G'⊃A∩G, for some F, G∈**F**. So, F'∩ G'⊃(A∩F)∩(A∩G)=A∩ (F∩G).As F∩G∈**F**, F'∩G'∈**F'**. Hence **F'** is a $\hat{\alpha g}$ filter containing **F**. As **F** is a $\hat{\alpha g}$ maximal filter, we have **F'**=**F**. X∈**F** and A∩ X =A, so that A⊃ A∩X, it follows A∈**F'** =**F**.

Theorem 5.2: A collection **F** of nonempty $\hat{\alpha g}$ open subsets of a topological space X is a $\hat{\alpha g}$ maximal filter on X if the following condition are satisfied.
i.**F** has the FIP.
ii.for every $\hat{\alpha g}$ open subset A of X, either A∈**F** or X- A∈**F**.

Proof: Let **F'** be any $\hat{\alpha g}$ filter on X containing **F** (such a $\hat{\alpha g}$ filter exists by theorem 5.1 [7]). Let F'∈**F'**. Then X-F'∉**F'**.
Hence X - F'∉**F** so, F'∈**F**, by (ii). So, **F'**⊂**F**. Therefore **F'**=**F** and **F** is $\hat{\alpha g}$ maximal filter on X.

Corollary 5.3: A $\hat{\alpha g}$ filter **F** on a topological space X is a $\hat{\alpha g}$ maximal filter if A or X- A belongs to **F** for all $\hat{\alpha g}$ open subset A of X.

Theorem 5.4: A $\hat{\alpha g}$ filter **F** on a topological space X is a $\hat{\alpha g}$ maximal filter if and only if for any two $\hat{\alpha g}$ open subsets A, B of X such that A∪B∈**F,** we have either A∈**F** or B∈**F**.

Proof: Let **F** be a $\hat{\alpha g}$ maximal filter on X and let $A\cup B\in$ **F**. If possible, let $A\notin$ **F** and $B\notin$ **F**. Then either A or B is non empty. Assume that $B\neq\phi$. Consider **G**={G : $G\cup A\in$ **F**}. As $B\in$ **G**, **G** is non empty. Let us prove **G** is a $\hat{\alpha g}$ filter on X strictly finer than **F**. Since $\phi\cup A = A\notin$ **F**, $\phi\notin$ **G.** Let $G\in$ **G** and H be a $\hat{\alpha g}$ open subset of X such that $H\supset G$. Then $G\cup A\in$ **F** and since **F** is a $\hat{\alpha g}$ filter, $H\cup A\supset G\cup A$ implies $H\cup A\in$ **F**. This shows $H\in G$. Let G, $H\in$ **G**. Then $G\cup A\in$ **F** and $H\cup A\in$ **F**.$(G\cup A)\cap(H\cup A)=(G\cap A)\cup A\in$ **F**. Hence $G\cap H\in G$. Thus **G** is a $\hat{\alpha g}$ filter. Let us prove **G**$\supset$**F**. Let $F\in$ **F**. As $F\cup A\supset F, F\cup A\in$ **F**, there by $F\in$ **G**. Hence **F**$\subset$ **G**. since $B\in$ **G** and $B\notin$ **F**, we have **F** is a proper subset of **G**. This contradicts **F** is a $\hat{\alpha g}$ maximal filter. This completes one half of the theorem.

Conversely, let the given condition hold. Let A be any $\hat{\alpha g}$ open subset of X. As **F** is a $\hat{\alpha g}$ filter, $X\in$ **F**. But $X = (X-A)\cup A$. Hence either $A\in$ **F** or $X-A\in$ **F**. Hence by the preceding corollary, **F** is a maximal filter on X.

Corollary 5.5: If **F** is a $\hat{\alpha g}$ maximal filter on a topological space X, then for every $\hat{\alpha g}$ open subset A of X, we have either $A\in$ **F** or $X-A\in$ **F**.

Corollary 5.6: If **F** is a $\hat{\alpha g}$ maximal filter on a topological space X, and A_1, $A_2,\ldots A_n$ are $\hat{\alpha g}$ open subsets of X then $A_1\cup A_2\cup A_3\cup\ldots\cup A_n\in$ **F** implies at least one $A_i\in$ **F**.

Corollary 5.7: If **F** is a $\hat{\alpha g}$ maximal filter on a topological space X and A_1, A_2, $\ldots A_n$ are $\hat{\alpha g}$ open subsets of X such that they cover X, than at least one $A_i\in$ **F**.
The corollary 5.3 and corollary 5.5 can be combined to give the following theorem.

Theorem 5.8: A $\hat{\alpha g}$ filter on a topological space X is a $\hat{\alpha g}$ maximal filter if and only if A or X - A belongs to **F** for all $\hat{\alpha g}$ open subsets A of X.

Theorem 5.9: Every $\hat{\alpha g}$ filter on a topological space X is the intersection of all the $\hat{\alpha g}$ maximal filters finer than **F**.

Proof: Let **H** $=\cap$ {**G**: **G** is a $\hat{\alpha g}$ maximal filter containing **F**}
Let us prove **H** = **F**. As **F**$\subset$**G**, for every **G**, **F**$\subset$**H**.
Conversely, let $A\in$ **H**. Then $A\in$ **G**, for every **G**. If possible, let $A\notin$ **F**. Then A contains no member of **F**. Hence every $F\in$ **F** intersects X – A. Then by, theorem 5.5[7], there exists a $\hat{\alpha g}$ filter **F'** finer than **F** and containing X-A. Again by theorem 4.1 there exists $\hat{\alpha g}$ maximal filter, say $\mathbf{G_0}$ finer than **F'**. Now $X-A\in$ **F'** implies $A\notin$ **F'** and since $\mathbf{G_0}\supset$ **F'.** we have $A\notin G_0$. This contradicts $A\in$ **G**, for every **G**. Hence $A\in$ **F**. This completes the proof.

Theorem 5.10: Let A be a $\hat{\alpha g}$ open subset of a topological space X which has a non empty intersection with every member of a $\hat{\alpha g}$ maximal filter **F**, then A must belong to **F**.

Proof: Let **A** = {A}∪**F**. Evidently, the intersection of any finite number of members of **A** is non empty.
Hence **A** generates a $\hat{\alpha g}$ filter **G** containing all the sets in **A** and so, in particular A. As **F** is a $\hat{\alpha g}$ maximal filter, we have **F** = **G** Hence A ∈**F**.

Theorem 5.11: The collection **F** (p) of all $\hat{\alpha g}$ open subsets of a topological space X , which contains a given element p∈X is a $\hat{\alpha g}$ maximal filter on X.

Proof: **F**(p) = {F:F is $\hat{\alpha g}$ open and p∈F}. Let A be any $\hat{\alpha g}$ closed subset of X. Then p∈A or p∈ X-A. Hence either A∈ **F** (p) or X-A∈**F**(p). Clearly **F**(p) has the FIP. Hence by theorem 5.2 **F**(p) is a $\hat{\alpha g}$ maximal filter on X.

Corollary 5.12: In a discrete topological space, every $\hat{\alpha g}$ open neighborhood filter is a $\hat{\alpha g}$ maximal filter.

Proof: In a discrete space X, the $\hat{\alpha g}$ open neighborhood filter of a point x∈X is the collection of all subsets of X which contain x. Hence by the preceding theorem, every $\hat{\alpha g}$ open neighborhood of X is a $\hat{\alpha g}$ maximal filter. It is clear that no other $\hat{\alpha g}$ filter can be maximal filter.

Theorem 5.13: Let **F** be a $\hat{\alpha g}$ maximal filter on a topological space X, in which every singleton is $\hat{\alpha g}$ closed. Then ∩ {F:F∈**F**} is either empty or singleton subset of X.

Proof: Let G = ∩{F:F∈**F**} and let G≠ϕ. Then, there exist at least one x∈G. If possible, let there be another different element y∈G. Since **F** is a $\hat{\alpha g}$ maximal filter, either {x}∈**F** or X- {x}∈**F**. If {x}∈**F**, then, as y ≠x, y∉ {x}and hence y∉G, which is a contradiction. Similarly, if X- {x}∈**F**, then x ∉G, which is again a contradiction. Hence G = {x}. This completes the proof.

6. $\hat{\alpha g}$ Convergence of $\hat{\alpha g}$ filters

Definition 6.1: Let **F** be a $\hat{\alpha g}$ filter on a topological space X and let A be a $\hat{\alpha g}$ open subset of X. Then **F** is said to be $\hat{\alpha g}$ eventually in A if and only if A∈**F**.

Definition 6.2: A $\hat{\alpha g}$ filter on a topological space X is said to be $\hat{\alpha g}$ frequently in a $\hat{\alpha g}$ open subset A of X if and only if A intersects every member of **F**.

Remark 6.3: **F** is α̂g frequently in A if **F** is α̂g eventually in A.
The converse of the above remark need not be true as can be seen in the following example.

Example 6.4: Let X = {a,b,c}, τ = {ϕ,{a},{a,b},X},
α̂g open sets = {ϕ,{a},{b},{a,b},{a,c},X}. Let **F** = {{b,c},X}, A = {b}
F is α̂g frequently in A but not α̂g eventually in A.

Theorem 6.5: A α̂g filter **F** on a topological space X, is α̂g frequently on a α̂g open subset A of X if and only if there exists a α̂g filter finer than **F** and containing A.

Proof: Let **F** be frequently on A. Let **G** = **F** ∪ {A}. **G** satisfies FIP. Hence by theorem 5.1 [7], there exists a α̂g filter **F'** containing **G**. **F'** is finer than **F**.
Conversely let there exist a α̂g filter **F'** finer than **F** and containing **A**. Since A∈**F'**, A∩ F≠ϕ,
for every F ∈**F**.

Definition 6.6: Let (X,τ) be a topological space and **F** be a α̂g filter on X. Then **F** is said to be τ-α̂g converge or simply α̂g converge to a point x ∈X if and only if **F** is α̂g eventually in each α̂g open neighbourhood of x, that is, if and only if every α̂g open neighbourhood of x is a member of **F** and we say that x is a α̂g limit point (or simply a α̂g limit) of **F** and we write **F**→x.

7. α̂g limit points of a α̂g filter base

Definition 7.1: A α̂g filter base **B** on a topological space X is said to be α̂g converge to a point x∈X if and only if the α̂g filter whose base is **B** (that is, the α̂g filter generated by **B**) α̂g converges to x and we say that x is a α̂g limit point of **B**. The set of all α̂g limit points of a α̂g filter shall be denoted by α̂g limit (**F**). Similarly α̂g lim (**B**) will denote the set of all α̂g limit points of a α̂g filter base **B**.

Theorem 7.2: Let X be a topological space and let **F** be a α̂g filter on X. Then the following statements are equivalent.
(a)**F**, α̂g converges to a point x∈X.
(b) Each α̂g open neighborhood of x belongs to **F**.
(c)**F** is finer than the α̂g open neighborhood filter **N**(x) on X.
(d) for every α̂g open neighborhood N of x, there exists F∈**F** such that F⊂N .

Proof:
(a)⇔(b) follows from definition 6.1and 7.1.
(b) ⇔(c) is an immediate consequence of definition 4.1 [7].
(b)⇔(d) follows from α̂g filter definition.

Remark 7.3: The $\alpha\hat{g}$ open neighborhood filter **N**(x) of a point x in a topological space X, $\alpha\hat{g}$ converges to x.

Theorem 7.4: In a discrete topological space X, the only $\alpha\hat{g}$ convergent filters are the $\alpha\hat{g}$ open neighborhood filters

Proof: By Corollary 5.12 that the $\alpha\hat{g}$ open neighborhood filters are the only $\alpha\hat{g}$ maximal filters on X and so they are the only $\alpha\hat{g}$ convergent filters.

Remarks 7.5: If a $\alpha\hat{g}$ filter **F** on a topological space X $\alpha\hat{g}$ converges to a point x $\in$X, then every $\alpha\hat{g}$ filter **F'** finer than **F** also $\alpha\hat{g}$ converges to x.

Theorem 7.6: Let **M** be the collection of all those $\alpha\hat{g}$ filters on a topological space X which $\alpha\hat{g}$ converge to the same point x $\in$X. Then the intersection **F** of all the $\alpha\hat{g}$ filters in **M** also $\alpha\hat{g}$ converges to x. Also **F** =**N**(x).

Proof: **F** is a actually a $\alpha\hat{g}$ filter on X by theorem 4.3 [7]. Since all the $\alpha\hat{g}$ filters in **M** $\alpha\hat{g}$ converge to x, the $\alpha\hat{g}$ neighborhood filter **N**(x) is coarser than each $\alpha\hat{g}$ filter in **M** and consequently coarser than **F**. Since **N**(x) $\alpha\hat{g}$ converges to x, **F** also $\alpha\hat{g}$ converges to x. Let us prove **F** = **N**(x). We have shown that **N**(x)$\subset$**F**. **N**(x), $\alpha\hat{g}$ converges to x. Hence **N**(x) is a member of **M**. So **F**$\subset$**N**(x).

Theorem 7.7: A $\alpha\hat{g}$ filter **F** on a topological space X $\alpha\hat{g}$ converges to a point x$\in$X if and only if every $\alpha\hat{g}$ maximal filter containing **F**, $\alpha\hat{g}$ converges to x.

Proof: If **F**, $\alpha\hat{g}$ converges to x, then every $\alpha\hat{g}$ maximal filter containing **F** also $\alpha\hat{g}$ converges to x by remark 8.5.
Conversely, let every $\alpha\hat{g}$ maximal filter containing x, $\alpha\hat{g}$ converges to x. By theorem 5.9 , **F** is the intersection of all $\alpha\hat{g}$ maximal filters on X finer than **F**. So, **F**, $\alpha\hat{g}$ converge to x by the above theorem.

Theorem 7.8: Let X be a topological space and **B** be a $\alpha\hat{g}$ filter base on X. Then **B**, $\alpha\hat{g}$ converges to a point x$\in$X if and only if every member of the $\alpha\hat{g}$ open local base $\mathbf{B}^*$(x) at x contains a member of **B**.

Proof: Let **B**, $\alpha\hat{g}$ converge to x. Then the $\alpha\hat{g}$ filter **F** generated by **B** also $\alpha\hat{g}$ converges to x. Also by definition 6.9[7], every member of **F** contains a member of **B**. Since **F**, $\alpha\hat{g}$ converges to x, every member of $\mathbf{B}^*$(x) belongs to **F** and so, every member of $\mathbf{B}^*$(x) must contain a member of **B**.
Conversely, let every member of $\mathbf{B}^*$(x) contain a member of **B** and let **F** be the $\alpha\hat{g}$ filter generated by **B**. Then by definition 6.9[7], **F** consists of all those $\alpha\hat{g}$ open subsets of X, which contain a member of **B**. Hence, every member of $\mathbf{B}^*$(x) belongs to **F**, by the definition of $\alpha\hat{g}$ filter. Let N be any $\alpha\hat{g}$ open neighbourhood of x. Then by the definition of $\alpha\hat{g}$ open local base, there exist **B** $\in\mathbf{B}^*$(x) such that B$\subset$N. Hence N$\in$**F**. So **F**, $\alpha\hat{g}$ converges to x. This completes the proof.

Corollary 7.9: **B**,$\alpha\hat{g}$ converges to x if and only if every $\alpha\hat{g}$ open neighborhood of x contains a member of **B**.

Proof: Let **B**,$\alpha\hat{g}$ converge to x and let N be any $\alpha\hat{g}$ open neighborhood of x. Then, there exists B∈**B***(x) such that B⊂ N. Hence by the above theorem, N must contain a member of **B**.
Conversely, let every $\alpha\hat{g}$ open neighborhood of x contain a member of **B**. Then every member of **B***(x) must contain a member of **B**. Hence by the above theorem, **B**, $\alpha\hat{g}$ converges to x.

Theorem 7.10: Let X be a topological space and A be a subspace of X. Then a point x∈X belongs to αg cl A if and only if there exists a $\alpha\hat{g}$ filter base on A $\alpha\hat{g}$ converging to x.

Proof: Let **B** be a $\alpha\hat{g}$ filter base on A which $\alpha\hat{g}$ converges to x. Then by the preceding corollary, for each $\alpha\hat{g}$ open neighbourhood of x, there exists B∈**B** such that B⊂ N. As B is non empty $\alpha\hat{g}$ open subset of A, N contains at least one point of A. Hence x ∈αgcl A.
Conversely, let x ∈αg clA. Then every member of $\alpha\hat{g}$ open neighbourhood filter **N**(x) must intersect A. Consider the collection.
B={A∩N : N∈**N**(x)}. A∩N ≠ϕ, for every N ∈**N**(x). Let A∩N, A∩M ∈**B**. (A∩N) ∩(A∩M) = A∩(N∩M) ∈ **B**.
Hence **B** is a filter base on A. The $\alpha\hat{g}$ filter generated by **B** is evidently the $\alpha\hat{g}$ open neighbourhood filter **N**(x). As **N**(x), $\alpha\hat{g}$ converges to x, the result follows.

Theorem 7.11: Let X be a topological space and A be a subspace of X. If A is $\alpha\hat{g}$ open, then A belongs to every $\alpha\hat{g}$ filter which $\alpha\hat{g}$ converges to a point of A.

Proof: Let A be $\alpha\hat{g}$ open and let **F** be a $\alpha\hat{g}$ filter which $\alpha\hat{g}$ converges to a point x ∈A (There exists one such $\alpha\hat{g}$ filter, namely,$\alpha\hat{g}$ open neighbourhood filter of x). Then **F** contains every $\alpha\hat{g}$ open neighbourhood of x, by theorem 8.2(b). As A is $\alpha\hat{g}$ open, it is a $\alpha\hat{g}$ open neighbourhood of x. Hence A∈ **F**.

Theorem 7.12: If a topological space X is $\alpha\hat{g}$ Hausdorff, then every $\alpha\hat{g}$ convergent filter has a unique limit.

Proof: Let X be $\alpha\hat{g}$ Hausdorff and let **F** be a $\alpha\hat{g}$ filter on X, $\alpha\hat{g}$ converging to two distinct point x and y. Then **F**⊃**N**(x) and **F**⊃**N**(y). As X is $\alpha\hat{g}$ Hausdorff, there exist $\alpha\hat{g}$ open neighbourhoods N and M of x and y respectively, such that M∈**F** and N ∈**F** and N∩M =ϕ , a contradiction. This completes the proof.
The converse of the above theorem need not be true can be seen from the following example.

Example 7.13: Let X = {a,b,c}, τ= {ϕ,{b},{a,b},{b,c},X}.
α̂g open sets of X = {ϕ,{b},{a,b},{b,c},X}. The α̂g filters are
$\mathbf{F}_1$= {{b},{b,c},X}, $\mathbf{F}_2$ = {{a,b},X}, $\mathbf{F}_3$= {{b,c},X}, $\mathbf{F}_4$ = {X}
Evidently $\mathbf{F}_1$,α̂g converges to c and no other point can be a α̂g limit point of $\mathbf{F}_1$
Evidently $\mathbf{F}_2$,α̂g converges to a and no other point can be a α̂g limit point of $\mathbf{F}_2$
Evidently $\mathbf{F}_3$,α̂g converges to c and no other point can be a α̂g limit point of $\mathbf{F}_3$
$\mathbf{F}_1$, **F2, F3** have unique α̂g limits. But the space X is not α̂g Hausdorff.

8. α̂g cluster points of a α̂g filter

Definition 8.1: Let X be a topological space and let **F** be a α̂g filter on X. Then a point x ∈X is said to be a α̂g cluster point or α̂g adherent point of **F** if and only if F∩ N≠ϕ for every F∈**F** and every α̂g open neighbourhood N of x.

Definition 8.2 : A point x in a topological space X is said to be a α̂g cluster point or α̂g adherent point of a α̂g filter base **B** if and only if B∩ N ≠ϕfor every B∈**B** and every α̂g open neighbourhood N of x.
The set of all α̂g adherent points of **F** is called the α̂g adherence of **F** and is denoted by α̂gadh(**F**).

Theorem 8.3: If x is a α̂g cluster point of a α̂g filter base **B** and **B'** is any other α̂g filter base equivalent to **B**, then X is also a α̂g cluster point of **B'**.

Proof: x is a α̂g cluster point of **B**⇒B∩ N≠ϕ , for every B∈**B**&N∈**N**(x)

Let B'∈**B’**. There exists B∈**B** such that B’⊃ B. Hence B'∩N≠ϕ. So, x is a α̂g cluster point of **B'**.

Theorem 8.4: Let X be a topological space and let **F** be a α̂g filter on X. Then the following statements are equivalent.
(a)x∈X is a α̂g cluster point of **F**.
(b)Every α̂g open neighbourhood of x intersects every member of **F**.
(c)x∈ α̂g cl F for all F∈**F**.
(d) **N**(x) ∪**F** has the FIP
(e) There exists a α̂g filter finer than both **N**(x) and **F**.
(f)There exists a α̂g filter than **F** and α̂g converging to x.

Proof:
(a) ⇔(b) follows from definition
(b)⇔(c) follows from the properties of α̂g cl F
(b)⇔ (d) obvious
(d) ⇔(e) By theorem 5.1[7]
(e) ⇔(f) Let **G** be the α̂g filter which is finer than both **F** and **N**(x). Then **G**,α̂g converges to x, since it contains **N**(x).

Conversely, let **G** be a α̂g filter which is finer than **F** and α̂g converges to x. Then **G** is finer than **N**(x).

The above theorem remains true if we replace the word α̂g filter by the word α̂g filter base.

Theorem 8.5: If xis a α̂g limit point of a α̂g filter **F** on a topological space X, then x is also a α̂g cluster point of **F**.

Proof: x is a α̂g limit point of **F** implies every α̂g open neighbourhood of x belongs to **F.** This implies every α̂g open neighbourhood of x intersects every member of **F**.

The converse of the above theorem need not be true can be seen from the following example.

Example 8.6: Refer example 7.13.
b is a α̂g cluster point of $\mathbf{F}_1$,but b is not a α̂g limit point of $\mathbf{F}_1$, as the α̂g open neighbourhood {a,b}∉$\mathbf{F}_1$.

Theorem 8.7: Let X be a topological space and let **B** be a α̂g filter base on X. Then a point x∈X is a α̂g cluster point if and only if every member of the α̂g open local base $\mathbf{B}^*$(x) at x intersects every member of **B**.

Proof: x is aα̂g cluster point of **B**⇔every α̂g open neighbourhood of x intersects every member of **B**⇔every member of $\mathbf{B}^*$(x) intersects every member of **B**.

Theorem 8.8: Let **F** be a maximal filter on a topological space X and let x∈X. Then **F**, α̂g converges to x if and only if x is a α̂g cluster point of **F**.

Proof: Let **F**,α̂g converge to x. Then every α̂g open neighbourhood of x belongs to **F** and so every α̂g open neighbourhood of x intersects **F**.
Conversely, let x be a α̂g cluster point of **F**.By theorem 9.4(f), there exists a α̂g filter **F'** finer than **F** and α̂g converging to x. As **F** is a α̂g maximal filter, **F'** = **F**.

Theorem 8.9: Let **B** be a α̂g filter base on a topological space X. Then α̂gadh (**B**) = { α̂gcl B : B∈**B**}.

Proof: **x**∈α̂gadh (**B**)⇔ xis a α̂g cluster point of **B**⇔each α̂g open neighbourhood of x intersects each member of **B**⇔ x∈ cl α̂g B, for every B∈**B**⇔α̂gadh (**B**) = {α̂gcl B: B∈**B**}

Corollary 8.10: If **F** is a $\hat{\alpha}$g filter on X, then $\hat{\alpha}$gadh (**F**) = {$\hat{\alpha}$gcl F: F∈**F**}

Theorem 8.11: Let x be a $\hat{\alpha}$g cluster point of a $\hat{\alpha}$g filter **F** on a topological space X and **F'** be a $\hat{\alpha}$g filter on X coarser than **F**. Then x is also a $\hat{\alpha}$g cluster point of **F'**.

Proof: x is a $\hat{\alpha}$g cluster point of **F**⇒ x∈$\hat{\alpha}$g cl F, for every F∈**F**⇒x ∈$\hat{\alpha}$gcl G, for every G∈**F'**, as **F'**⊂**F**.⇒ x is a cluster point of **F'**.

The converse of the above theorem need not be true can be seen from the following example.

Example 8.12: Let X = {a,b,c}, τ= {ϕ,{a},{a,b},X}
$\hat{\alpha}$g open sets = {ϕ,{a},{b},{a,b},{a,c},X}. $\mathbf{F}_1$ = {{a},{b},{a,b},{a,c},X}
$\mathbf{F}_2$ = {{a,b},X} $\mathbf{F}_2 \subset \mathbf{F}_1$. b is a $\hat{\alpha}$g cluster point of $\mathbf{F}_2$ but not a $\hat{\alpha}$g cluster of $\mathbf{F}_1$, as {b} ∩{a} =ϕ.

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