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On Decomposable Almost Pseudo Projectively Symmetric Manifolds

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Abstract

In this paper we study the almost pseudo projectively symmetric Riemannian manifold of dim n & decomposable almost pseudo projectively symmetric Riemannian manifold of dim n (n ≥ 3).

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1. Introduction

Let (M^n, g) be a Riemannian manifold of dim-$n(n \geq 3)$. A non-flat Riemannian manifold $(M^n, g), (n \geq 2)$ is called an almost pseudo symmetric manifold whose curvature tensor R of type (0,4) satisfies condition [18].

$$
\begin{aligned}
(\nabla_X R)(Y,Z,U,V) = [A(X)+B(X)]R(Y,Z,U,V) \\
+A(Y)R(X,Z,U,V)+A(Z)R(Y,X,U) \\
+A(U)R(Y,Z,X,V)+A(V)R(Y,Z,U,X) \qquad (1.1)
\end{aligned}
$$

where A and B are nowhere vanishing $1-$forms such that

$$A(X) = g(X,\rho) \;\&\; B(X) = g(X,Q) \qquad \text{for all } X, \qquad (1.2)$$

ρ and Q are the vector fields associated with the $1-$forms A and B, respectively. An $n-$dimensional almost pseudo symmetric manifold has been denoted by $A(PS)_n$. If $A = B$ in (1.1), then the manifold reduces to a pseudo symmetric manifold $(PS)_n$ introduced by Chaki[12], whose notion is different from that of introduced by Deszcz [14]. Pseudo symmetric spaces, generalized symmetric spaces, were also studied by Mikes[6,7]. Also almost pseudo symmetric manifold is not a particular case of a weakly symmetric manifold $(WS)_n$ introduced by Tamassy and Binh[11].

The notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent such as conformally symmetric manifolds by Chaki and Gupta[12], recurrent manifolds introduced by Walker[2], conformally recurrent manifolds by Adati and Miyazawa[17], projective symmetric manifolds by Soos[5], projective-symmetric and projectively recurrent affinely connected spaces by Mikes[6], pseudo-conformally symmetric spaces by De and Biswas[19], almost pseudo conformally symmetric manifolds by De and Gazi [20], weakly conharmonically symmetric manifolds by Shaikh and Hui[1].

The projective curvature W of type (0,4) of a Riemannian manifold $(M^n, g)(n > 3)$ is given by

$$W(Y,Z,U,V) = R(Y,Z,U,V) - \frac{1}{(n-1)}[S(Z,U)g(Y,V) - S(Y,U)g(Z,V)] \tag{1.3}$$

where S is the Ricci tensor of the manifold of type (0,2). The present paper is concerned with a non-projective flat Riemannian manifold (M^n, g) $(n > 3)$ whose projective curvature tensor W satisfies the condition

$$\begin{aligned}(\nabla_X W)(Y,Z,U,V) = [A(X) + B(X)]W(Y,Z,U,V)& \\ +A(Y)W(X,Z,U,V) + A(Z)W(Y,X,U,V)& \\ +A(U)W(Y,Z,X,V) + A(V)W(Y,Z,U,X)&\end{aligned} \tag{1.4}$$

where A & B are same as defined in (1.2). A manifold of this type will be known as almost pseudo projectively symmetric manifold denoted by $A(PPS)_n$. As the conformal curvature tensor vanishes identically for $n = 3$, we assume the condition $n > 3$ throughout the paper. The paper is organized as follows

In Section 2, it deals with an $A(PPS)_n$. In Section 3, it is concerned with a decomposable $A(PPS)_n$ and exactly defined each decomposition of an $A(PPS)_n$. In Section 4, it is shown that the integral curves of the unit torse-forming vector field ρ in an Einstein $A(PPS)_n$ are geodesics. Hence it is also found that the vector field Q is the torse-forming vector field and its integral curves are geodesics.

2. $A(PPS)_n$

L denotes the symmetric endomorphism of the tangent space at each point of the manifold corresponding to the Ricci tensor S of type (0,2), that is

$$g(LX,Y) = S(X,Y) \tag{2.1}$$

Lete_i, $(1 \leq i \leq n)$be an orthonormal basis of the tangent space at any point of the manifold. From (1.3), we have

$$F(Z,U) = \sum_{i=1}^{n} W(Z,e_i,e_i,U) = \sum_{i=1}^{n} W(e_i,Z,U,e_i)$$
$$= -\frac{r}{n-1} g(Z,U)$$

$$\sum_{i=}^{n} K(e_i,e_i,Y,Z) = \sum_{i=1}^{n} K(Y,Z,e_i,e_i) = 0 \tag{2.2}$$

where$r$ is the scalar curvature of the manifold.

From (1.4) and (2.2), we have

$$(\nabla_X F)(Z,U) = -\frac{r}{n-1}\{[A(X)+B(X)]\}g(Z,U)$$
$$+A(R(X,Z)U) - \frac{1}{n-1}\{S(Z,U)A(X) - S(X,U)A(Z)\}$$
$$+A(R(X,U)Z) - \frac{1}{n-1}\{S(U,Z)A(X) - S(X,Z)A(U) \tag{2.3}$$

Putting $Z = U = e_i$ in (2.3) and taking summation over i, $(1 \leq i \leq n)$, it follows from (2.2) that

$$\nabla_X r = r\left\{\left(\frac{n+2}{n}\right)A(X) + B(X)\right\} \tag{2.4}$$

Thus we can state it as a theorem as follows

Theorem−1 The scalar curvature tensor r of an$A(PPS)_n$ satisfies the following relation

$$\nabla_X r = r\left\{\left(\frac{n+2}{n}\right)A(X) + B(X)\right\} \qquad \textit{for all } X. \tag{2.5}$$

Let us consider $A(PPS)_n$ of constant scalar curvature.Thus from (2.5) and r not equal to 0, we get

$$\left(\frac{n+2}{n}\right)A(X) + B(X) = 0 \qquad \textit{for all } X \tag{2.6}$$

this implies the following theorem

Theorem−2 The two associated $1-$forms in an $A(PPS)_n$ of non-zero constant scalar curvature are linearly dependent**.**

3. Decomposable $A(PPS)_n$

If a Riemannian manifold (M^n, g) can be expressed as the product, $M_1^p \times M_2^{n-p}$ for $(2 \le p \le n-2)$, namely, if coordinates can be found so that its metric takes the form [16]

$$dS^2 = g_{ij}dx^i dx^j = \bar{g}_{ab}dx^a dx^b + {g^*}_{\alpha\beta}dx^\alpha dx^\beta \tag{3.1}$$

then it is called decomposable.
where $\bar{g}_{ab}$ are functions of $x^1, x^2, \dots\dots\dots x^p$ & ${g^*}_{\alpha\beta}$ are functions of $x^{p+1}, x^{p+2}, \dots\dots\dots x^n$ only. $a, b, c, \dots\dots$are taken to have range from 1 to p&α, β, γ ... ...are taken to have range from $p+1$ to n.The two parts of (3.1) are the metrices of $M_1^p (p \ge 2)$ and $M_2^{n-p}(n-p \ge 2)$ which are called the decomposable spaces of M^n [9].
From eq(3.1), we have

$$g_{ab} = \bar{g}_{ab}, g_{\alpha\beta} = {g^*}_{\alpha\beta}, g^{ab} = \bar{g}^{ab}, g^{\alpha\beta} = g^{*\alpha\beta}, g_{a\beta} = g^{a\beta} = 0 \tag{3.2}$$

Let $(M^n, g) = (M_1^p, \bar{g}) \times (M_2^{n-p}, g^*) (2 \le p \le n-2)$ be a decomposable Riemannian manifold. Throughout this paper objects from M_1 are denoted by 'bar' and that from M_2 are denoted by 'star'.
Let $\bar{X}, \bar{Y}, \bar{Z}, \bar{U}, \bar{V} \in \chi(M_1)$ and $X^*, Y^*, Z^*, U^*, V^* \in \chi(M_2)$. In decomposable Riemannian manifold the following relations hold [9]

$$\begin{gathered}
R(X^*, \bar{Y}, \bar{Z}, \bar{U}) = 0 = R(\bar{X}, Y^*, \bar{Z}, U^*) = R(\bar{X}, Y^*, Z^*, U^*) \\
(\nabla_{X^*}R)(\bar{Y}, \bar{Z}, \bar{U}, \bar{V}) = 0 = (\nabla_{\bar{X}}R)(\bar{Y}, Z^*, \bar{U}, V^*) = (\nabla_{X^*}R)(\bar{Y}, Z^*, \bar{U}, V^*) \\
R(\bar{X}, \bar{Y}, \bar{Z}, \bar{U}) = \bar{R}(\bar{X}, \bar{Y}, \bar{Z}, \bar{U}); \\
R(X^*, Y^*, Z^*, U^*) = R^*(X^*, Y^*, Z^*, U^*) \\
S(\bar{X}, \bar{Y}) = \bar{S}(\bar{X}, \bar{Y}) \\
S(X^*, Y^*) = S^*(X^*, Y^*) \\
(\nabla_{\bar{X}}S)(\bar{Y}, \bar{Z}) = (\bar{\nabla}_{\bar{X}}S)(\bar{Y}, \bar{Z}); \\
(\nabla_{X^*}S)(Y^*, Z^*) = ({\nabla^*}_{X^*}S)(Y^*, Z^*) \\
r = \bar{r} + r^*
\end{gathered} \tag{3.3}$$

we consider a decomposable $A(PPS)_n$, which is decomposable in M_1^p& $M_2^{n-p}(2 \le \mathrm{p} \le \mathrm{n}-2)$.
Then using (3.3), we have from (1.3)

$$W(X^*, \bar{Y}, \bar{Z}, \bar{U}) = W(\bar{X}, Y^*, Z^*, U^*) = 0$$

$$W(X^*, \bar{Y}, \bar{Z}, U^*) = \frac{1}{n-1}[S(\bar{Y}, \bar{Z})g(X^*, U^*)] \tag{3.4}$$

From (1.4) on the manifold M_1, we have

$$(\nabla_{\bar{X}} W)(\bar{Y}, \bar{Z}, \bar{U}, \bar{V}) = [A(\bar{X}) + B(\bar{X})]W(\bar{Y}, \bar{Z}, \bar{U}, \bar{V}) + A(\bar{Y})W(\bar{X}, \bar{Z}, \bar{U}, \bar{V}) + A(\bar{Z})W(\bar{Y}, \bar{X}, \bar{U}, \bar{V}) + A(\bar{U})W(\bar{Y}, \bar{Z}, \bar{X}, \bar{V}) + A(\bar{V})W(\bar{Y}, \bar{Z}, \bar{U}, \bar{X}) \tag{3.5}$$

Replacing $\bar{X}$ by X^* in (3.5) and using (3.4) & (3.3), we get

$$[A(X^*) + B(X^*)]W(\bar{Y}, \bar{Z}, \bar{U}, \bar{V}) = 0 \tag{3.6}$$

Similarly, replacing $\bar{Y}$by Y^*, we have

$$A(Y^*)W(\bar{X}, \bar{Z}, \bar{U}, \bar{V}) = 0 \tag{3.7}$$

Put $\bar{X} = X^*$&$\bar{U} = U^*$ in (3.5), we get

$$A(\bar{Y})W(X^*, \bar{Z}, U^*, \bar{V}) + A(\bar{Z})W(\bar{Y}, X^*, U^*, \bar{V}) + A(\bar{V})W(\bar{Y}, \bar{Z}, U^*, X^*) = 0 \tag{3.8}$$

Similarly putting $\bar{Y} = Y^*$&$\bar{V} = V^*$ in (3.5), we get

$$[A(\bar{X}) + B(\bar{X})]W(Y^*, \bar{Z}, \bar{U}, V^*) + A(\bar{Z})W(Y^*, \bar{X}, \bar{U}, V^*) + A(\bar{U})W(Y^*, \bar{Z}, \bar{X}, V^*) = 0 \tag{3.9}$$

Setting $\bar{X}$=X^*, $\bar{Y} = Y^*$&$\bar{V} = V^*$ in (3.5), we get

$$(\nabla_{X^*} W)(Y^*, \bar{Z}, \bar{U}, V^*) = [A(X^*) + B(X^*)]W(Y^*, \bar{Z}, \bar{U}, V^*) + A(Y^*)W(X^*, \bar{Z}, \bar{U}, V^*) + A(V^*)W(Y^*, \bar{Z}, \bar{U}, X^*) \tag{3.10}$$

Similarly replacing $\bar{Z}$=Z^*, $\bar{U} = U^*$&$\bar{V} = V^*$ in (3.5), we get

$$A(Z^*)W(\bar{Y}, \bar{X}, U^*, V^*) + A(U^*)W(\bar{Y}, Z^*, \bar{X}, V^*) + A(V^*)W(\bar{Y}, Z^*, U^*, \bar{X}) = 0 \tag{3.11}$$

Setting $\bar{X} = X^*$, $\bar{Z}$=Z^*, $\bar{U} = U^*$&$\bar{V} = V^*$ in (3.5), we get

$$A(\bar{Y})W(X^*, Z^*, U^*, V^*) = 0 \tag{3.12}$$

Similarly putting $\bar{Y} = Y^*$, $\bar{Z}$=Z^*, $\bar{U} = U^*$&$\bar{V} = V^*$ in (3.5), we get

$$[A(\bar{X}) + B(\bar{X})]W(Y^*, Z^*, U^*, V^*) = 0 \tag{3.13}$$

Now setting$\bar{X} = X^*\bar{Y} = Y^*$, $\bar{Z}$=Z^*, $\bar{U} = U^*$&$\bar{V} = V^*$ in (3.5), we get

$$(\nabla_{X^*} W)(Y^*, Z^*, U^*, V^*) = [A(X^*) + B(X^*)]W(Y^*, Z^*, U^*, V^*) + A(Y^*)W(X^*, Z^*, U^*, V^*) + A(Z^*)W(Y^*, X^*, U^*, V^*) + A(U^*)W(Y^*, Z^*, X^*, V^*) + A(V^*)W(Y^*, Z^*, U^*, X^*) \tag{3.14}$$

Thus from (3.6), (3.7), (3.12) & (3.13) we have following theorem

Theorem−3 Let an $A(PPS)n$ be a decomposable space such that $M^n = M_1^p \times M_2^{n-p}$ for $(2 \leq \text{p} \leq \text{n} - 2)$, then one of the decomposition is projectively flat and on the other $A = B = 0$.

Let us now deal with each decomposition individually. Let one of the decompositions be projectively flat. Then, we get

$$W(\bar{Y},\bar{Z},\bar{U},\bar{V}) = 0 \; for \; \bar{Y},\bar{Z},\bar{U},\bar{V} \in \chi(M_1)$$

From (1.3), we have

$$R(\bar{Y},\bar{Z},\bar{U},\bar{V}) = \frac{1}{n-1}[S(\bar{Z},\bar{U})g(\bar{Y},\bar{V}) - S(\bar{Y},\bar{U})g(\bar{Z},\bar{V})] \tag{3.15}$$

Contracting over $\bar{Y}$&$\bar{V}$, we get

$$S(\bar{Z},\bar{U}) = 0 \tag{3.16}$$

Hence manifold M_1 is of vanishing curvature.

Let us consider the other decomposition, that is, $A = B = 0 \; on \; M_2$. Then from (3.10), we get

$(\nabla_{X^*}W)(Y^*,\bar{Z},\bar{U},V^*) = 0$

Which implies that

$$(\nabla_{X^*}S)(Y^*,V^*) = 0 \tag{3.17}$$

Hence we see that the manifold M_1 is Ricci-symmetric.

By virtue of equation(3.14), we have

$$(\nabla_{X^*}W)(Y^*,Z^*,U^*,V^*) = 0 \tag{3.18}$$

Therefore we can state the following

Theorem−4 Let $A(PPS)n$ be decomposable Riemannian manifold $M^n = M_1^p \times M_2^{n-p}$ for $(2 \leq p \leq n-2)$. Then the following holds

(1) If one of the decompositions is projectively flat, then it is of vanishing curvature.

(2) If 1-forms A & B vanish on the other then this decomposition is Ricci-symmetric and is projectively symmetric.

Now contracting in (3.8) over $X^*$and U^*, we get

$$A(\bar{Y})\{(n-p)S(\bar{Z},\bar{V}) + r^*g(\bar{Z},\bar{V})\} - A(\bar{Z})\{(n-p)S(\bar{Y},\bar{V}) + r^*g(\bar{Y},\bar{V})\} = 0 \tag{3.19}$$

Again contracting over $\bar{Z}$and $\bar{V}$, we obtain

$$A(L\bar{Y}) = \left\{\frac{(p-1)}{(n-p)}r^* + \bar{r}\right\}A(\bar{Y}) \tag{3.20}$$

Repeating similar operation for (3.9), we have

$$\begin{aligned} 0 = {} & [A(\bar{X}) + B(\bar{X})]\{(n-p)S(\bar{Z},\bar{U}) + r^*g(\bar{Z},\bar{U})\} \\ & + A(\bar{Z})\{(n-p)S(\bar{X},\bar{U}) + r^*g(\bar{X},\bar{U})\} \\ & + A(\bar{U})\{(n-p)S(\bar{Z},\bar{X}) + r^*g(\bar{Z},\bar{X})\} \end{aligned} \tag{3.21}$$

and

$$A(\bar{X})\{(n-p)\bar{r}+(p+2)r^*\}+B(\bar{X})\{(n-p)\bar{r}+pr^*\}+2(n-p)A(L\bar{X})=0 \tag{3.22}$$

Using (3.20), we have

$$3A(\bar{X})+B(\bar{X})=0 \tag{3.23}$$

Thus we can state it as follows

Theorem−5 Let (M^n, g) be decomposable Riemannian manifold$M_1^p \times M_2^{n-p}$ for $(2 \leq \mathrm{p} \leq \mathrm{n}-2)$ If M is an $A(PPS)n$, then the following relations are satisfied

$$A(L\bar{X})=\left\{\frac{(p-1)}{(n-p)}r^*+\bar{r}\right\}A(\bar{X})$$

and

$$3A(\bar{X})+B(\bar{X})=0$$

on M_1.

Contracting in (3.12) over $\bar{Y}$, $\bar{X}$and Z^* ,V^*, respectively, we obtain

$$A(LU^*)=\left\{r^*+\frac{(p-1)}{p}\bar{r}\right\}A(U^*)$$

onM_2. Hence we can state the following

Theorem−6 Let (M^n, g) be decomposable Riemannian manifold$M_1^p \times M_2^{n-p}$ for $(2 \leq \mathrm{p} \leq \mathrm{n}-2)$

If M is an $A(PPS)n$, then the following relations are satisfied

$$A(LU^*)=\left\{r^*+\tfrac{(p-1)}{p}\bar{r}\right\}A(U^*)$$

on M_2.

4. The torse-forming vector field ρ

We consider an $A(PPS)n$ defined by (1.4) which is an Einstien manifold. Then its Ricci tensor S satiafies

$$S(Z,U)=\frac{r}{n}g(Z,U) \tag{4.1}$$

It follows that

$$dr(Z)=0 \ \ and \ \ (\nabla_X S)(Z,U)=0 \tag{4.2}$$

We suppose that ρ is a unit-torse forming vector [19] defined by

$$\nabla_X \rho=\lambda X+\omega(X)\rho \tag{4.3}$$

where λ is a non-zero scalar and ω is a non-zero $1-$form, called the scalar and $1-$form of the vector field ρ, respectively. Some properties of torse -forming vector fields in Riemannian spaces have been studied by Rachunek and Mikes [10] and various mathematicians.

Now by (4.2), we have

$$(\nabla_X S)(Z,\rho) = 0 \tag{4.4}$$

Since $(\nabla_X S)(Z,\rho) = \nabla_X S(Z,\rho) - S(\nabla_X Z,\rho) - S(Z,\nabla_X \rho)$ and using (1.2) and (4.1),we have

$$\frac{r}{n}(\nabla_X A)(Z) - S(Z,\nabla_X \rho) = 0 \tag{4.5}$$

Substituting (4.3) in (4.5) and using (4.1), we get

$$\frac{r}{n}(\nabla_X A)(Z) - \lambda S(Z,X) - \frac{r}{n}\omega(X)A(Z) = 0 \tag{4.6}$$

Putting $Z = \rho$ in (4.6) and remembering that ρ is a unit vector, thus equation(4.6) reduces to

$$(\nabla_X A)(\rho) = \lambda A(X) + \omega(X) \tag{4.7}$$

Since ρ is a unit vector, we get

$$(\nabla_X A)(\rho) = -A(\nabla_X \rho) \tag{4.8}$$

From (4.3) and (4.8), it follows that

$$(\nabla_X A)(\rho) = -\lambda A(X) - \omega(X) \tag{4.9}$$

From(4.7) and (4.9), we get

$$\omega(X) = -\lambda A(X)$$

It means that

$$\lambda = -\omega(\rho) \tag{4.10}$$

Using (4.10) in (4.3), we have

$$\nabla_X \rho = \omega(\rho)X + \omega(X)\rho \tag{4.11}$$

Hence it follows that $\nabla_\rho \rho = 0$.

Therefore we can state it as follows

Theorem−7 If in an Einstein $A(PPS)_n$ the vector field ρ is a unit torse-forming vector field, then the integral curves of the vector ρ are geodesics.

As we know that an Einstein manifold is of constant scalar curvature, so from Theorem-2, two associated $1-$forms in an Einstein $A(PPS)_n$ whose scalar curvature is non-zero are linearly dependent. Also, it follows from (1.2) and (2.6) that

$$\rho = -\left(\frac{n}{n+2}\right)Q \tag{4.12}$$

From equations (4.11) and (4.12), we can say that the vector field Q is also a torse-forming vector field. Thus from Theorem-7, the integral curves of the vector field Q are also geodesics.

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