International Mathematical Forum, Vol. 10, 2015, no. 1, 47 - 56
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/imf.2015.411188

Strongly Regular Congruences on E-inversive Semigroups

Hengwu Zheng[1,2]

School of Mathematical Sciences
Qufu Normal University
Qufu, Shandong 273165, P.R. China

Yunlong Yu

School of Mathematical Sciences
Qufu Normal University
Qufu, Shandong 273165, P.R. China

Abstract

It is shown that every strongly regular congruence on an E-inversive semigroup is uniquely determined by its kernel and hyper-trace. Furthermore, strongly orthodox (resp., strongly regular) congruences on an E-inversive (resp., E-inversive E-)semigroup S are described in terms of certain congruence pairs (ξ, K), where ξ is a certain normal congruence on the subsemigroup $\langle E(S)\rangle$ generated by $E(S)$ and K is a certain normal subsemigroup of S.

Mathematics Subject Classification: 20M10

Keywords: E-inversive semigroup, E-semigroup, strongly regular (strongly orthodox) congruence, kernel, hyper-trace, congruence pair

[1]mailing address: Office of Educational Administration (Rizhao Campus), Qufu Normal University, Rizhao, Shandong 276826, P.R. China

[2]Supported by the National Natural Science Foundation of China (11171180).

1. Introduction

A semigroup S with non-empty set of idempotents is called *E-inversive* if for every $a \in S$ there exists $x \in S$ such that ax is idempotent. This concept was introduced by Thierrin [17]. Hence a semigroup S is E-inversive if and only if $W(a) = \{a' \in S : a' = a'aa'\} \neq \emptyset$ for any $a \in S$ (see [13]).

The class of E-inversive semigroups is wide and contains many different types, for example, all regular, all eventually regular (its every element has power which is regular, see [2]) semigroups [12]. A semigroup is an *E-semigroup* if its idempotents form a subsemigroup.

Congruences on regular semigroups have been explored extensively. The kernel-trace approach is an effective tool for handling congruences on regular semigroups, which had been investigated in the previous literature, such as Crvenković and Dolinka [1], Feigenbaum [3], Gomes [4], [5], Imaoka [7], Pastijn and Petrich [14], Petrich [15], Scheiblich [16], Trotter [18], [19] and the author [21].

Gomes developed the kernel-trace approach and then described R-unipotent (resp., orthodox) congruences on regular semigroups by means of their kernels and hyper-traces in [4] (resp., [5]). Lou and Li applied this approach to eventually regular semigroups and described R-unipotent (resp., orthodox) congruences on eventually regular semigroups by means of their kernels and hyper-traces in [8] (resp., [9]). The purpose of this paper is to describe strongly orthodox congruences on E-inversive semigroups by means of their kernels and hyper-traces. It is shown that every strongly regular congruence on an E-inversive semigroup is uniquely determined by its kernel and hyper-trace. A description of strongly orthodox (resp., strongly regular) congruences on an E-inversive (resp., E-inversive E-)semigroup S is given in terms of certain congruence pairs (ξ, K) consisting of a certain normal congruence ξ on the subsemigroup $\langle E(S) \rangle$ generated by $E(S)$ and a certain normal subsemigroup K of S. These results generalize the corresponding results obtained by Lou and Li [9] for eventually regular semigroups.

In [11] Lou et al. described strongly regular congruences on an E-inversive semigroup S by means of their kernels and traces, where the view of Pastijn and Petrich [14] was followed. Their approach gave quite a neat expression, though not a very explicit one, for the description of a congruence $\rho_{(K,\tau)}$ on S defined by a certain congruence pair (K, τ) for S.

By looking again at the eventually regular case presented in [9] we came to be believe that it should possible to describe any strongly regular congruence on an E-inversive E-semigroup, by means of its kernel and trace, in a clearer manner than that obtained in [11]. We wish to remark that because of the type of technique used here is the one used in [9] we shall not need to repeat the proofs for the "tricks" used are almost always the same.

As in the eventually regular case, our results give no information when the semigroup is idempotent generated.

2. Preliminaries

For standard terminology and notation in semigroup theory see Howie [6]. The set of idempotents and regular elements in a semigroup S are denoted by $E(S)$ and $Reg(S)$, respectively. For any $a \in Reg(S)$, we customarily denote the set of inverses of a by $V(a)$. The lattice of congruences on S is denoted by $C(S)$.

A congruence ρ on an E-inversive semigroup S is called *strongly regular* if it satisfies

$$(\forall a \in S)\ (\exists a' \in W(a))\ a \,\rho\, aa'a.$$

In [11] strongly regular congruences on an E-inversive semigroup are called regular congruences.

An orthodox congruence on an E-inversive semigroup is generally not strongly regular. For example, let $S = (\mathbb{N}, \cdot)$ be the multiplicative semigroup of all non-negative integers and ρ be the congruence determined by the partition (see [11]):

$$\{\{0\}, \{1\}, \{m \in \mathbb{N} : m \geq 2\}\}.$$

Then S is E-inversive, and $S/\rho = \{\overline{0}, \overline{1}, \overline{2}\}$ is a semilattice. Hence ρ is an orthodox congruence on S. But ρ is not strongly regular. Conversely, a strongly regular congruence on an E-inversive semigroup is generally not orthodox. This follows from that a regular semigroup is generally not orthodox. An orthodox congruence ρ on an E-inversive semigroup is called *strongly orthodox* if ρ is strongly regular. We have seen that a congruence on an eventually regular semigroup is regular if and only if it is strongly regular (see [9] or Lemma 5.4 in [20]). Hence a congruence on an eventually regular semigroup is orthodox if and only if it is strongly orthodox.

We give a example of a strongly orthodox congruence on an E-inversive semigroup as follows.

Example 2.1. Let S be the semigroup with Cayley table (Example 5.1 in [20]):

S	a	b	c	d	e	f	g
a	e	g	a	e	e	g	e
b	c	d	e	d	e	e	f
c	e	f	c	e	e	f	e
d	e	d	e	d	e	e	e
e	e	e	e	e	e	e	e
f	c	e	e	e	e	e	f
g	a	e	e	e	e	e	g.

Then S is E-inversive, since S is eventually regular. Let ρ be the congruence on S determined by the partition $\{\{a,c,e,f,g\},\{b,d\}\}$. Then $S/\rho=\{\bar{a},\bar{b}\}$ is a semilattice, that is, ρ is a semilattice congruence on S. Hence ρ is a strongly orthodox congruence on S.

Let S be a semigroup and $e,f\in E(S)$ and let

$$M(e,f)=\{g\in E(S):ge=g=fg\}$$

and

$$S(e,f)=\{g\in E(S):ge=g=fg,egf=ef\}.$$

$S(e,f)$ is called the sandwich set of e and f. It is known that $M(e,f)\neq\emptyset$ (resp., $S(e,f)\neq\emptyset$) for all $e,f\in E(S)$ in an E-inversive (resp., a regular) semigroup S ([6], [11]).

Throughout this paper, S is always an E-inversive semigroup, unless otherwise stared.

We list some known results which will be used in the sequel.

Lemma 2.2. [11] *Let $a,b\in S, a'\in W(a), b'\in W(b)$. If $g\in M(a'a,bb')$, then $b'ga'\in W(ab)\cap V(agb)$.*

Lemma 2.3. [11] *Let ρ be a strongly regular congruence on S. If $a\rho$ is an idempotent of S/ρ, then an idempotent e can be found in $a\rho$ such that $H_e\leq H_a$.*

Let ρ be a congruence on a semigroup S. The subset $\{a\in S: a\rho a^2\}$ of S is called the kernel of ρ and is denoted by $\ker\rho$. The restriction of ρ to the subset $E(S)$ of S is called the trace of ρ and is denoted by $\mathrm{tr}\rho$. The restriction of ρ to the subsemigroup $\langle E(S)\rangle$ of S generated by $E(S)$ is called the hyper-trace of ρ and is denoted by $\mathrm{htr}\rho$. By Lemma 2.3, if ρ is a strongly regular congruence on S, then

$$\ker\rho=\{a\in S: a\;\rho\;e \text{ for some } e\in E(S)\}.$$

Lemma 2.4. [11] *Let ρ be a strongly regular congruence on S. If $a,b\in S$ such that $b\rho\in W(a\rho)$, then there exists $a'\in W(a)$ such that $a'\rho b$ and $H_{a'}\leq H_b$.*

Lemma 2.5. [11] *Let ρ be a strongly regular congruence on S. If $e,f\in E(S)$ such that $e\rho f$, then there exists $g\in E(S)$ such that $e\rho g\rho f$ and $g\in M(e,f)$.*

Lemma 2.6. [11] *Let ρ be a strongly regular congruence on S. If $g\rho\in M(e\rho,f\rho)$ with $e,f,g\in E(S)$, then for each $x\in e\rho\cap E(S), y\in f\rho\cap E(S)$ there exists $z\in g\rho\cap E(S)$ such that $z\in M(x,y)$.*

Lemma 2.7. [20] *Let S be an E-semigroup. Then*

(1) $(\forall a\in S, a'\in W(a), e,f\in E(S))\ ea', a'f, ea'f\in W(a)$;

(2) $(\forall a\in S, a'\in W(a), e\in E(S))\ aea', a'ea\in E(S)$;

(3) $(\forall e\in E(S))\ W(e)\subseteq E(S)$.

Lemma 2.8. [13] *Let S be an E-inversive semigroup. Then $W(E^n(S)) \subseteq E^{n+1}(S)$ for $n = 1, 2, \ldots$. Thus, the subsemigroup $\langle E(S)\rangle$ of S is again E-inversive.*

3. Strongly regular congruences on E-inversive semigroups

In this section it is shown that every strongly regular congruence on S is uniquely determined by its kernel and hyper-trace, and a description of strongly orthodox (resp., strongly regular) congruences on an E-inversive (resp., E-inversive E-) semigroup S is given in terms of strongly orthodox (resp., strongly regular) congruence pairs for S.

We begin by some definitions.

Definition 3.1. A subsemigroup K of S is called normal if

(i) $Reg(K) = Reg(S) \cap K$;

(ii) $E(S) \subseteq K$, i.e., K is full;

(iii) for all $a \in S$ and $a' \in W(a)$, $aKa', a'Ka \subseteq K$, i.e., K is weakly self-conjugate.

Let K be a full subsemigroup of S. As Lemma 3.1 in [10] we have that $Reg(K) = Reg(S) \cap K$ if and only if for all $a \in K$ and $a' \in W(a)$, $a' \in K$. So the condition (i) in Definition 3.1 can be replaced by the following condition:

(i′) for all $a \in K$ and $a' \in W(a)$, $a' \in K$.

Definition 3.2. A congruence ξ on the subsemigroup $\langle E(S)\rangle$ of S is called normal if for all $x, y \in \langle E(S)\rangle, a \in S$ and $a' \in W(a)$,

$$x\,\xi\,y \Rightarrow axa'\,\xi\,aya',\ a'xa\,\xi\,a'ya$$

whenever $axa', aya', a'xa, a'ya \in \langle E(S)\rangle$.

Definition 3.3. The pair (ξ, K) consisting of a normal congruence ξ on $\langle E(S)\rangle$ such that ξ is a strong band congruence on $\langle E(S)\rangle$ and a normal subsemigroup K of S is called a strongly orthodox congruence pair for S if, for all $a, b \in S, x \in \langle E(S)\rangle$,

(O1) $(\exists a^+ \in W(a))\ (\forall a' \in W(a))\ (aa^+aa'\,\xi\,aa', a'aa^+a\,\xi\,a'a)$;

(O2) $xa \in K, x\,\xi\,aa^+ \Rightarrow a \in K$;

(O3) $ab \in K, b' \in W(b), a^+a\,\xi\,a^+abb' \Rightarrow axb \in K$;

(O4) $a \in K \Rightarrow$

$$aa^+xaa^+\,\xi\,aa^+a^+xaaa^+ \text{ and } a^+axa^+a\,\xi\,a^+aaxa^+a^+a$$

whenever $a^+xa, axa^+ \in \langle E(S)\rangle$.

Given such a pair (ξ, K), we define a binary relation $\rho_{(\xi,K)}$ on S by

$$a\ \rho_{(\xi,K)}\ b \Leftrightarrow \begin{array}{c}(\forall a' \in W(a))(\exists b' \in W(b))\ (a'b \in K, aa'\,\xi\,bb', a'a\,\xi\,b'b)\ \&\\ (\forall b' \in W(b))(\exists a' \in W(a))\ (b'a \in K, aa'\,\xi\,bb', a'a\,\xi\,b'b).\end{array}$$

Notation. We represent the set of all strongly orthodox congruences on S by $SOC(S)$ and the set of all strongly orthodox congruence pairs for S by $SOCP(S)$.

The following theorem is our main result in this section.

Theorem 3.4. *If $(\xi, K) \in SOCP(S)$, then $\rho_{(\xi,K)}$ is the unique strongly orthodox congruence on S such that $htr\rho_{(\xi,K)} = \xi$ and $ker\rho_{(\xi,K)} = K$.*

Conversely, if $\rho \in SOC(S)$, then $(htr\rho, ker\rho) \in SOCP(S)$ and $\rho = \rho_{(htr\rho, ker\rho)}$.

We give the proof of Theorem 3.4 by a series of lemmas.

As Lemma 3.6 in [9], we may obtain the following

Lemma 3.5. *Let $(\xi, K) \in SOCP(S)$. Then*

(A) $a \in K \Rightarrow (aa^+)\xi \in V((a^+a)\xi)$;

(B) $a \in K, a' \in W(a) \Rightarrow (aa')\xi, (a'a)\xi \in W((aa^+a^+a)\xi)$;

(C) $a \in K, a' \in W(a) \Rightarrow (aa')\xi \in V((a'a)\xi)$;

(D) $a \in K, a' \in W(a), x \in \langle E(S)\rangle \Rightarrow$

$$aa'xaa' \,\xi\, aa'a'xaaa' \quad \text{and} \quad a'axa'a \,\xi\, a'aaxa'a'a$$

whenever $a'xa, axa' \in \langle E(S)\rangle$.

Lemma 3.6. *Let $(\xi, K) \in SOCP(S)$. Then $\rho_{(\xi,K)} \in C(S)$.*

Proof. Notice that ξ is a strongly regular congruence on the E-inversive semigroup $\langle E(S)\rangle$. So by Lemma 2.5 for any $e, f \in E(S)$ with $e \,\xi\, f$, there exists $g \in M(e, f)$ such that $e \,\xi\, g \,\xi\, f$. As Lemma 3.8 in [9], we may deduce that $\rho_{(\xi,K)} \in C(S)$. □

Lemma 3.7. *If $(\xi, K) \in SOCP(S)$, then $\rho_{(\xi,K)} \in SOC(S)$ such that $htr\rho_{(\xi,K)} = \xi$ and $ker\rho_{(\xi,K)} = K$.*

Proof. To simplify the notation, for $(\xi, K) \in SOCP(S)$, let $\rho = \rho_{(\xi,K)}$. We first show that ρ is strongly regular. Notice that ξ is a strongly regular congruence on the E-inversive semigroup $\langle E(S)\rangle$. So by Lemma 2.4 for any $x, y \in \langle E(S)\rangle$ with $y\xi \in W(x\xi)$, there exists $x' \in W(x)$ such that $x' \,\xi\, y$ and $H_{x'} \leq H_y$. Moreover, if $y \in E(S)$, then $x'y = x'$. As Lemma 3.9 in [9], we thus have $a \,\rho\, aa^+a$, hence ρ is strongly regular.

Next we show that $\ker\rho = K$. Let $a \in \ker\rho$. Since ρ is strongly regular, there exists $e \in E(S)$ such that $a \,\rho\, e$. As Lemma 3.9 in [9] we may obtain that $a \in K$.

Conversely, suppose that $a \in K$. We show that $a \,\rho\, a^2$. Now $\langle E(S)\rangle/\xi$ is a band, hence is a regular semigroup. For any $a' \in W(a)$, take $x\xi \in S((a'a)\xi, (aa')\xi)$, where $x \in \langle E(S)\rangle$. Since ξ is a strongly regular congruence on $\langle E(S)\rangle$, there exists $f \in E(S)$ such that $x \,\xi\, f$. Then by Lemma 2.6 there exists $e \in E(S)$ such that $x \,\xi\, f \,\xi\, e$ and $e \in M(a'a, aa')$. As Lemma 3.9 in [9] we may show that $a\rho a^2$.

Notice that ξ is a strongly regular congruence on $\langle E(S)\rangle$, as Lemma 3.9 in [9] we have that $\mathrm{htr}\rho = \xi$.

Finally, to show that ρ is an orthodox congruence on S, let $a\rho, b\rho \in E(S/\rho)$. Then by Lemma 2.3 there exist $e, f \in E(S)$ such that $a\,\rho\, e, b\,\rho\, f$. Since ξ is a band congruence on $\langle E(S)\rangle$, we have that $(ef)^2 \,\xi\, ef$. Now $(ef)^2, ef \in \langle E(S)\rangle$, and $\mathrm{htr}\rho = \xi$, we obtain $(ef)^2 \,\rho\, ef$, that is, $a\rho b\rho \cdot a\rho b\rho = a\rho b\rho$. Hence S/ρ is an orthodox semigroup. Therefore, $\rho \in SOC(S)$. $\square$

That $\mathrm{htr}\rho$ is a strong band congruence on $\langle E(S)\rangle$ follows from that $\rho \in SOC(S)$ and $\langle E(S)\rangle$ is an E-inversive subsemigroup of S, as Lemma 3.10 in [9] we may obtain the following

Lemma 3.8. *If $\rho \in SOC(S)$, then $ker\rho$ is a normal subsemigroup of S and $htr\rho$ is a normal congruence on $\langle E(S)\rangle$ such that $htr\rho$ is a strong band congruence on $\langle E(S)\rangle$.*

As the proofs of the corresponding results in [9] we may obtain the following three results.

Lemma 3.9. *If $\rho \in SOC(S)$, then $(htr\rho, ker\rho) \in SOCP(S)$.*

Lemma 3.10. *Let ρ be a strongly regular congruence on S and $a, b \in S$. Then*

$$a\,\rho\, b \Leftrightarrow \begin{array}{c} (\forall a' \in W(a))(\exists b' \in W(b))\ (a'b \in ker\rho, aa'\, htr\rho\, bb', a'a\, htr\rho\, b'b)\ \& \\ (\forall b' \in W(b))(\exists a' \in W(a))\ (b'a \in ker\rho, aa'\, htr\rho\, bb', a'a\, htr\rho\, b'b). \end{array}$$

Corollary 3.11. *Every strongly regular congruence on S is uniquely determined by its kernel and hyper-trace.*

Up to now, we have completed the proof of Theorem 3.4.

Define a relation $\leq$ on $SOCP(S)$ by

$$(\xi, K) \leq (\xi', K') \Leftrightarrow \xi \subseteq \xi' \text{ and } K \subseteq K'.$$

It is clear that $\leq$ is a partial order on $SOCP(S)$.

Now the next result follows immediately.

Corollary 3.12. *The mappings*

$$\varphi : SOC(S) \to SOCP(S),\ \rho \mapsto (htr\rho, ker\rho)$$

and

$$\psi : SOCP(S) \to SOC(S),\ (\xi, K) \mapsto \rho_{(\xi,K)}$$

are mutually inverse order isomorphisms of partial order sets.

Remark 1. In Lemma 3.10 we can substitute "htr" by "tr", since elements $aa', bb', a'a, b'b \in E(S)$.

Corollary 3.13. *Every strongly regular congruence on S is uniquely determined by its kernel and trace.*

Remark 2. Corollary 3.13 has been established already by Lou et al. [11].

Remark 3. Theorem 3.4 is a generalization of Theorem 3.7 obtained by Lou et al. [9].

By Lemma 2.3, we have that every strongly regular congruence on an E-inversive E-semigroup is orthodox. We give a description of strongly regular congruences on an E-inversive E-semigroup which is a consequence of Theorem 3.4.

Definition 3.14. The pair (ξ, K) consisting of a normal congruence ξ on the subsemigroup $E(S)$ of an E-inversive E-semigroup S such that the band congruence ξ is a strong band congruence on $E(S)$ and a normal subsemigroup K of S is called a strongly regular congruence pair for S if, for all $a, b \in S, e \in E(S)$,

(O1) as in Definition 3.3;
(R2) $ea \in K, e\,\xi\, aa^+ \Rightarrow a \in K$;
(R3) $a \in K \Rightarrow a^+ea\,\xi\, a^+a^+eaa$ and $aea^+\,\xi\, aaea^+a^+$.

Given such a pair (ξ, K), we define a binary relation $\rho_{(\xi,K)}$ on S as in Definition 3.3.

As Theorem 4.3 in [9] we may obtain the following

Theorem 3.15. *If (ξ, K) is a strongly regular congruence pair for an E-inversive E-semigroup S, then $\rho_{(\xi,K)}$ is the unique strongly regular congruence on S such that $tr\rho_{(\xi,K)} = \xi$ and $ker\rho_{(\xi,K)} = K$.*

Conversely, if ρ is a strongly regular congruence on an E-inversive E-semigroup S, then $(tr\rho, ker\rho)$ is a strongly regular congruence pair for S and $\rho = \rho_{(tr\rho, ker\rho)}$.

Remark 4. Theorem 3.15 is a generalization of Theorem 4.3 obtained by Lou et al. [9].

References

[1] S. Crvenković and I. Dolinka, Congruences on *-regular semigroups, Periodica Math. Hungarica, 45 (2002), 1-13.
http://dx.doi.org/10.1023/a:1022355610931

[2] P. M. Edwards, Eventually regular semigroups, Bull. Austral. Math. Soc., 28 (1983), 23-38.

http://dx.doi.org/10.1017/s0004972700026095

[3] R. Feigenbaum, Regular semigroup congruences, Semigroup Forum, 17 (1979), 373-377.
http://dx.doi.org/10.1007/bf02194336

[4] G. M. S. Gomes, R-unipotent congruences on regular semigroups, Semigroup Forum, 31 (1985), 265-280.
http://dx.doi.org/10.1007/bf02572656

[5] G. M. S. Gomes, Orthodox congruences on regular semigroups, Semigroup Forum, 37 (1988), 149-166.
http://dx.doi.org/10.1007/bf02573131

[6] J. M. Howie, Fundamentals of Semigroup Theory, Clarendon Press, Oxford, (1995).

[7] T. Imaoka, Representation of *-congruences on regular *-semigroups, Proc. of 1984 Marquette Conf. on Semigroups, Marquette Univ. 65-72 (1984).

[8] Y. F. Luo and X. L. Li, R-unipotent congruences on an eventually regular semigroup, Algebra Colloquium, 14 (2007), 37-52.
http://dx.doi.org/10.1142/s1005386707000053

[9] Y. F. Luo and X. L. Li, Orthodox congruences on an eventually regular semigroup, Semigroup Forum, 74 (2007), 319-336.
http://dx.doi.org/10.1007/s00233-006-0670-x

[10] Y. F. Luo and X. L. Li, The maximum idempotent-separating congruences on an eventually regular semigroup, Semigroup Forum, 74 (2007), 306-318.
http://dx.doi.org/10.1007/s00233-006-0646-x

[11] Y. F. Luo, X. K. Fan and X. L. Li, Regular congruences on an E-inversive semigroup, Semigroup Forum, 76 (2008), 107-123.
http://dx.doi.org/10.1007/s00233-007-9015-7

[12] H. Mitsch, Subdirect products of E-inversive semigroups, J. Austral. Math. Soc., 48 (1990), 66-78.
http://dx.doi.org/10.1017/s1446788700035199

[13] H. Mitsch and M. Petrich, Basic properties of E-inversive semigroups, Commun. Algebra, 28 (2000), 5169-5182.
http://dx.doi.org/10.1080/00927870008827148

[14] F. Pastijn and M. Petrich, Congruences on regular semigroups, Trans. Am. Math. Soc., 295 (1986), 607-633.
http://dx.doi.org/10.1090/s0002-9947-1986-0833699-3

[15] M. Petrich, Inverse Semigroups, Wiley, New York, (1984).

[16] H. E. Scheiblich, Kernels of inverse semigroup homomorphisms, J. Austral. Math. Soc., 18 (1974), 289-292.
http://dx.doi.org/10.1017/s1446788700022862

[17] G. Thierrin, Sur les demi-groups inverses, C. R. Acad. Sci. Paris, 234 (1952), 1336-1338.

[18] P. G. Trotter, Normal partitions of idempotents of regular semigroups, J. Austral. Math. Soc., 26 (1978), 110-114.
http://dx.doi.org/10.1017/s1446788700011587

[19] P. G. Trotter, Congruences on regular and completely regular semigroups, J. Austral. Math. Soc., 32 (1982), 388-398.
http://dx.doi.org/10.1017/s1446788700024940

[20] B. Weipoltshammer, Certain congruences on E-inversive E-semigroups, Semigroup Forum, 65 (2002), 233-248.
http://dx.doi.org/10.1007/s002330010131

[21] H. W. Zheng, Strong $\mathcal{P}$-congruences on $\mathcal{P}$-regular semigroups, Semigroup Forum, 51 (1995), 217-223.
http://dx.doi.org/10.1007/bf02573630

Received: November 22, 2014; Published: January 12, 2015