Fuzzy Regular Congruences on an E -Inversive E -Semigroup

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Abstract

In this paper, we introduce the definition of E -inversive E -semigroup with fuzzy ϕ -kernel normal systems. It is described that the fuzzy regular congruences on the E -inversive E -semigroup with fuzzy ϕ -kernel normal systems.

Keywords: E -inversive E -semigroup; fuzzy regular congruences; fuzzy ϕ -kernel normal systems

1 Introduction and preliminaries

A semigroup S is called E -inversive if for every $a \in S$, there exists $x \in S$ such that ax is idempotent; S is called E -semigroup if the set $E(S)$ of all idempotents of S forms a subsemigroup. An element $x \in S$ is called weak inverse of $a \in S$ if $xax = x$. Devote by $W(a)$ the set of all weak inverses of $a \in S$. M. Siripitukdet etc. [4] investigated band congruences and rectangular band on E -inversive E -semigroup. Some special congruences on E -inversive E -semigroup were given by Barbara Weipoltshammer [1]. Luo etc. [7] introduced and characterized regular congruences on E -inversive semigroup. The concepts of fuzzy sets and fuzzy congruences on inversive semigroups are introduced by Al-Thukair in [2]. Li etc. [8] introduced fuzzy kernel normal systems on an orthodox semigroup. Chen etc. [5] introduced fuzzy P -kernel normal system for weakly P -inversive semigroups. They have proved that there exists

a one-one correspondence between fuzzy strong P -congruences and fuzzy P -kernel normal systems on a weakly P -inversive semigroups. In this paper, we introduce the definition of E -inversive E -semigroup with fuzzy ϕ -kernel normal systems. Next, we define a fuzzy binary relation on E -inversive E -semigroup with fuzzy ϕ -kernel normal systems. Finally, it is showed that the fuzzy binary relation on E -inversive E -semigroup with fuzzy ϕ -kernel normal systems is a fuzzy regular congruences on the E -inversive E -semigroup with fuzzy ϕ -kernel normal systems.

Definition 1.1^[7] A congruence ρ on an E -inversive E -semigroup S is called to be regular if for any $a \in S$, there exists $a' \in W(a)$ such that $a\rho aa'a$.

Let X be a non-empty, a map $A : X \rightarrow [0, 1]$ is called a fuzzy subset of X . Devote the set of all fuzzy subsets on X by $F(X)$. Let $A \in F(X)$, $t \in [0, 1]$, define $A^t = \{a \in X | A(a) \geq t\}$, we call A^t a t -cut set. The mapping $\rho : X \times X \rightarrow [0, 1]$ is called a fuzzy relation on X , let ρ and σ be fuzzy relations on X , $\rho \leq \sigma$ means that $\rho(a, b) \leq \sigma(a, b)$ for all $a, b \in X$. Their composition denoted by $\rho \circ \sigma$ and defined as:

$$(\rho \circ \sigma)(a, c) = \bigvee \{ \{ \rho(a, b) \wedge \sigma(b, c) \}, b \in X \},$$

for any $a, c \in X$. We denote $\rho \circ \sigma$ by $\rho\sigma$ for the sake of simplicity.

Definition 1.2^[2] A fuzzy relation ρ is called a fuzzy equivalence relation on a semigroup S if

- (1) $\rho(a, a) = 1$ for all $a \in S$;
- (2) $\rho(a, b) = \rho(b, a)$ for all $a, b \in S$;
- (3) $\rho(a, c) \geq \{ \rho(a, b) \wedge \rho(b, c) \}$ for all $a, b, c \in S$.

A fuzzy equivalence relation ρ on a semigroup S is called a fuzzy congruence if $\rho(ab, cd) \geq \{ \rho(a, c) \wedge \rho(b, d) \}$ (or $\rho(ac, bc) \geq \rho(a, b)$ and $\rho(ca, cb) \geq \rho(a, b)$) for all $a, b, c, d \in S$.

Let ρ be a fuzzy equivalence relation on a semigroup S and $a \in S$. The fuzzy equivalence class determined by a and ρ , denoted by ρ_a is the fuzzy subset of S defined by $\rho_a(x) = \rho(a, x)$. And $\rho_a = \rho_b$ if and only if $\rho(a, b) = 1$.

Let S be a semigroup, we called a map $\mu : S \rightarrow [0, 1]$ is a fuzzy subsemigroup on semigroup S if $\mu(xy) \geq \mu(x) \wedge \mu(y)$, for any $x, y \in S$. μ is a fuzzy subsemigroup on semigroup S if and only if $\mu^t = \{a \in S | \mu(a) \geq t\}$, for any $t \in [0, 1]$, is a subsemigroup on S .

Definition 1.3^[5] Let S be an E -inversive E -semigroup. A fuzzy congruence ρ is called a fuzzy regular congruence on S if exists $a' \in W(a)$ such that $\rho(a, aa'a) = 1$ for any $a \in S$.

2 Fuzzy regular congruences

Definition 2.1 Let S be an E -inversive E -semigroup and ϕ be a fuzzy congruence on S . Let set $\Phi = \{\phi_e : e \in E(S)\}$. The S is called an E -inversive E -semigroup with fuzzy ϕ -kernel normal systems, if the following conditions are satisfied, for $a, b \in S, e, f \in E(S)$,

- (K_1) $\phi_e^t \cap \phi_f^t \neq \emptyset \Leftrightarrow \phi_e^t = \phi_f^t$, for any $t \in [0, 1]$;
- (K_2) $E(S) \subseteq \bigcup_{e \in E(S)} \phi_e^1$;
- (K_3) $\phi_e(a') \geq \phi_e(a)$, for any $a' \in W(a)$;
- (K_4) $\phi_e(aa'afa') = \phi_e(afaf')$ and $\phi_e(a'faa'a) = \phi_e(a'fa)$, for any $a' \in W(a)$;
- (K_5) $\phi_e(b) \geq \phi_e(a) \wedge \phi_e(ba) \wedge \phi_e(b'b)$, for any $b' \in W(b)$;
- (K_6) $\phi_e(xby) \geq \phi_f(a) \wedge \phi_f(b) \wedge \phi_e(xay)$, for any $x, y \in S$.

Let S be an E -inversive E -semigroup with fuzzy ϕ -kernel normal systems, define a fuzzy binary relation ρ_Φ on S by: for any $a, b \in S$,

$$\rho_\Phi(a, b) = \bigvee_{a' \in W(a), b' \in W(b)} (\phi_{aa'}(bb') \wedge \phi_{aa'}(ab') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(a'b)).$$

Lemma 2.2 Let S be an E -inversive E -semigroup with fuzzy ϕ -kernel normal systems, $e, f \in E(S)$, $s \in S$, then we have

- (1) $\phi_e(f) = \phi_f(e)$;
- (2) $\phi_e(s) \geq \phi_e(f) \wedge \phi_f(s)$.

Proof. (1) Let $\phi_e(f) = t \in [0, 1]$, then $f \in \phi_e^t$. And $f \in \phi_f^t$. By (K_1) we have $\phi_e^t = \phi_f^t$. So $e \in \phi_e^t = \phi_f^t$, then $\phi_f(e) \geq t = \phi_e(f)$. Similarly, $\phi_e(f) \geq \phi_f(e)$. Thus $\phi_e(f) = \phi_f(e)$.

(2) Let $\phi_e(f) = t$, $\phi_f(s) = t'$, $t, t' \in [0, 1]$, then $\phi_e(f) = t \geq t \wedge t'$, so $f \in \phi_e^{t \wedge t'}$. And $f \in \phi_f^{t \wedge t'}$, by (K_1) we have $\phi_e^{t \wedge t'} = \phi_f^{t \wedge t'}$. By $\phi_f(s) = t' \geq t \wedge t'$, so $s \in \phi_f^{t \wedge t'} = \phi_e^{t \wedge t'}$. Thus $\phi_e(s) \geq t \wedge t' = \phi_e(f) \wedge \phi_f(s)$. $\square$

Lemma 2.3 Let S be an E -inversive E -semigroup with fuzzy ϕ -kernel normal systems, $a, b \in S$, $a' \in W(a)$, $b' \in W(b)$, then $\phi_{aa'}(ba') \wedge \phi_{a'a}(b'a) \geq \phi_{aa'}(bb') \wedge \phi_{aa'}(ab') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(a'b)$.

Proof. By (K_6) we have $\phi_{aa'}(ba'ab') \geq \phi_{aa'}(bb'bb') \wedge \phi_{a'a}(a'a) \wedge \phi_{a'a}(b'b) = \phi_{aa'}(bb') \wedge \phi_{a'a}(b'b)$, and $\phi_{aa'}(aa'ab'ba') = \phi_{aa'}(ab'ba') \geq \phi_{aa'}(aa'aa') \wedge \phi_{a'a}(a'a) \wedge \phi_{a'a}(b'b) = \phi_{a'a}(b'b)$.

By (K_5) we have

$$\begin{aligned} \phi_{aa'}(ba') &\geq \phi_{aa'}(ab') \wedge \phi_{aa'}(ba'ab') \wedge \phi_{aa'}(aa'ab'ba'ab'ba') \\ &\geq \phi_{aa'}(ab') \wedge \phi_{aa'}(bb') \wedge \phi_{a'a}(b'b) \wedge \phi_{aa'}(aa'ab'ba') \\ &\geq \phi_{aa'}(ab') \wedge \phi_{aa'}(bb') \wedge \phi_{a'a}(b'b). \end{aligned}$$

Similarly, $\phi_{a'a}(b'a) \geq \phi_{a'a}(a'b) \wedge \phi_{aa'}(bb') \wedge \phi_{a'a}(b'b)$. Thus

$$\phi_{aa'}(ba') \wedge \phi_{a'a}(b'a) \geq \phi_{aa'}(bb') \wedge \phi_{aa'}(ab') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(a'b).$$

□

Lemma 2.4 Let S be an E -inversive E -semigroup with fuzzy ϕ -kernel normal systems, then exists $a' \in W(a), b' \in W(b)$ such that $\phi_{aa'}(bb') \wedge \phi_{aa'}(ab') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(a'b) = \rho_{\Phi}(a, b)$, for any $a, b \in S$. □

Lemma 2.5 Let S be an E -inversive E -semigroup with fuzzy ϕ -kernel normal systems, then ρ_{Φ} is a fuzzy equivalence relation on S .

Proof. (1) It is clear that $\rho_{\Phi}(a, a) = 1$, for any $a \in S$.

(2) For any $a, b \in S$, by lemma 2.4, exists $a' \in W(a), b' \in W(b)$, such that

$$\phi_{aa'}(bb') \wedge \phi_{aa'}(ab') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(a'b) = \rho_{\Phi}(a, b).$$

And by lemma 2.3 we have

$$\phi_{aa'}(ba') \wedge \phi_{a'a}(b'a) \geq \phi_{aa'}(bb') \wedge \phi_{aa'}(ab') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(a'b) = \rho_{\Phi}(a, b).$$

And by lemma 2.2 we have

$$\begin{aligned} \phi_{bb'}(aa') \wedge \phi_{bb'}(ba') \wedge \phi_{b'b}(a'a) \wedge \phi_{b'b}(b'a) &\geq \phi_{aa'}(bb') \wedge \phi_{bb'}(aa') \\ &\wedge \phi_{aa'}(ba') \wedge \phi_{a'a}(b'b) \wedge \phi_{b'b}(a'a) \wedge \phi_{a'a}(b'a) \\ &= \phi_{aa'}(bb') \wedge \phi_{aa'}(ba') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(b'a) \\ &= \rho_{\Phi}(a, b). \end{aligned}$$

Thus,

$$\begin{aligned} \rho_{\Phi}(b, a) &= \bigvee_{b' \in W(b), a' \in W(a)} (\phi_{bb'}(aa') \wedge \phi_{bb'}(ba') \wedge \phi_{b'b}(a'a) \wedge \phi_{b'b}(b'a)) \\ &\geq \phi_{bb'}(aa') \wedge \phi_{bb'}(ba') \wedge \phi_{b'b}(a'a) \wedge \phi_{b'b}(b'a) \\ &\geq \phi_{aa'}(bb') \wedge \phi_{aa'}(ba') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(b'a) \\ &= \rho_{\Phi}(a, b). \end{aligned}$$

Similarly, $\rho_{\Phi}(a, b) \geq \rho_{\Phi}(b, a)$. Thus $\rho_{\Phi}(a, b) = \rho_{\Phi}(b, a)$.

(3) For any $a, b \in S$, by lemma 2.4, exists $a' \in W(a), b', b^+ \in W(b)$, and $c' \in W(c)$ such that

$$\phi_{aa'}(bb') \wedge \phi_{aa'}(ab') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(a'b) = \rho_{\Phi}(a, b);$$

and

$$\phi_{bb^+}(cc') \wedge \phi_{bb^+}(bc') \wedge \phi_{b+b}(c'c) \wedge \phi_{b+b}(b^+c) = \rho_{\Phi}(b, c).$$

So we have $\phi_{aa'}(bb') \geq \rho_\Phi(a, b)$, $\phi_{a'a}(b'b) \geq \rho_\Phi(a, b)$, $\phi_{bb^+}(cc') \geq \rho_\Phi(b, c)$, $\phi_{b+b}(c'c) \geq \rho_\Phi(b, c)$. By lemma 2.2 and (K_3) we have

$$\begin{aligned}\phi_{bb'}(bb^+) &\geq \phi_{bb'}(bb^+bb'bb^+) \wedge \phi_{bb^+bb'bb^+}(bb^+) \\ &= \phi_{bb'}(bb^+bb'bb^+) \wedge \phi_{bb^+}(bb^+bb'bb^+) \\ &\geq \phi_{bb'}(bb') \wedge \phi_{bb^+}(bb^+) \\ &= 1.\end{aligned}$$

Thus, $\phi_{bb'}(bb^+) = 1$. Similarly, $\phi_{b'b}(b^+b) = 1$.

And by lemma 2.2 we have

$$\phi_{aa'}(cc') \geq \phi_{aa'}(bb') \wedge \phi_{bb'}(bb^+) \wedge \phi_{bb^+}(cc') \geq \rho_\Phi(a, b) \wedge \rho_\Phi(b, c)$$

and

$$\phi_{a'a}(c'c) \geq \phi_{a'a}(b'b) \wedge \phi_{b'b}(b^+b) \wedge \phi_{b+b}(c'c) \geq \rho_\Phi(a, b) \wedge \rho_\Phi(b, c).$$

By lemma 2.3 and lemma 2.4 we have $\phi_{aa'}(ba') \geq \rho_\Phi(a, b)$ and $\phi_{b'b}(c'b) \geq \rho_\Phi(b, c)$. By (K_6) we have

$$\begin{aligned}\phi_{aa'}(ac'ba') &\geq \phi_{aa'}(ab'ba') \wedge \phi_{b'b}(b'b) \wedge \phi_{b'b}(c'b) \\ &\geq \phi_{aa'}(aa'aa') \wedge \phi_{a'a}(b'b) \wedge \phi_{b'b}(c'b) \\ &= \phi_{a'a}(b'b) \wedge \phi_{b'b}(c'b) \\ &\geq \rho_\Phi(a, b) \wedge \rho_\Phi(b, c).\end{aligned}$$

By (K_4) and (K_6) we have

$$\begin{aligned}\phi_{aa'}(ca'ac'ca'ac') &= \phi_{aa'}(cc'ca'ac'ca'ac') \\ &= \phi_{aa'}(cc'ca'ac') \\ &\geq \phi_{aa'}(cc'cc'cc') \wedge \phi_{a'a}(a'a) \wedge \phi_{a'a}(c'c) \\ &= \phi_{aa'}(cc') \wedge \phi_{a'a}(c'c) \\ &\geq \rho_\Phi(a, b) \wedge \rho_\Phi(b, c).\end{aligned}$$

Then by (K_5) we have

$$\begin{aligned}\phi_{aa'}(ac') &\geq \phi_{aa'}(ba') \wedge \phi_{aa'}(ac'ba') \wedge \phi_{aa'}(ca'ac'ca'ac') \\ &\geq \rho_\Phi(a, b) \wedge \rho_\Phi(b, c).\end{aligned}$$

Similarly, $\phi_{a'a}(a'c) \geq \rho_\Phi(a, b) \wedge \rho_\Phi(b, c)$. Thus

$$\begin{aligned}\rho_\Phi(a, c) &= \bigvee_{a' \in W(a), c' \in W(c)} (\phi_{aa'}(cc') \wedge \phi_{aa'}(ac') \wedge \phi_{a'a}(c'c) \wedge \phi_{a'a}(a'c)) \\ &\geq \phi_{aa'}(cc') \wedge \phi_{aa'}(ac') \wedge \phi_{a'a}(c'c) \wedge \phi_{a'a}(a'c) \\ &\geq \rho_\Phi(a, b) \wedge \rho_\Phi(b, c).\end{aligned}$$

□

Lemma 2.6 Let S be an E -inversive E -semigroup with fuzzy ϕ -kernel normal systems, then ρ_Φ is a fuzzy congruence on S .

Proof. For any $a, b \in S$, by lemma 2.4, exists $a' \in W(a), b' \in W(b)$, such that

$$\phi_{aa'}(bb') \wedge \phi_{aa'}(ab') \wedge \phi_{a'a}(b'b) \wedge \phi_{a'a}(a'b) = \rho_\Phi(a, b).$$

And by lemma 2.3, we have $\phi_{aa'}(ba') \geq \rho_\Phi(a, b), \phi_{a'a}(b'b) \geq \rho_\Phi(a, b)$. For any $c \in S, c' \in W(c)$, By (K_3) we have, $\phi_{aa'}(aa'c'cbb'c'caa') \geq \phi_{aa'}(aa') = 1$, and $\phi_{bb'}(bb'c'cbb') \geq \phi_{bb'}(bb') = 1$. Thus $\phi_{aa'}(aa'c'cbb'c'caa') = \phi_{bb'}(bb'c'cbb') = 1$. Let $x = (a'c'cbb'c')(ca), y = (a'c'cbb'c'caa'c'cbb'c')(cb)$. Then by (K_6) we have

$$\begin{aligned} \phi_{a'a}(x) &= \phi_{a'a}(a'c'cbb'c'ca) \\ &= \phi_{a'a}(a'aa'c'cbb'c'caa'a) \\ &\geq \phi_{a'a}(a'aa'a) \wedge \phi_{aa'}(aa') \wedge \phi_{aa'}(aa'c'cbb'c'caa') \\ &\geq \phi_{a'a}(a'aa'a) \wedge \phi_{aa'}(aa'c'cbb'c'caa') \\ &= 1 \end{aligned}$$

and

$$\begin{aligned} \phi_{a'a}(yx) &= \phi_{a'a}(a'c'cbb'c'caa'c'cbb'c'cba'c'cbb'c'ca) \\ &= \phi_{a'a}(a'aa'c'cbb'c'caa'c'cbb'c'cba'c'cbb'c'caa'a) \\ &\geq \phi_{a'a}(a'aa'c'cbb'c'caa'c'cbb'c'caa'c'cbb'c'caa'a) \wedge \phi_{aa'}(ba') \wedge \phi_{aa'}(aa') \\ &\geq \phi_{a'a}(a'aa'c'cbb'c'caa'a) \wedge \phi_{aa'}(ba') \\ &\geq \phi_{a'a}(a'aa'a) \wedge \phi_{aa'}(aa'c'cbb'c'caa') \wedge \phi_{aa'}(ba') \\ &= \phi_{aa'}(ba') \\ &\geq \rho_\Phi(a, b). \end{aligned}$$

By (K_3) we have $\phi_{bb'}(bb'c'caa'c'cbb'c'cbb') \geq \phi_{bb'}(bb') = 1$. Thus, $\phi_{bb'}(bb'c'caa'c'cbb'c'cbb') = 1$. Let $y^+ = b'c'caa'c'cbb'c'ca$, so $y^+ \in W(y)$. Therefore,

$$\begin{aligned} \phi_{a'a}(y^+y) &\geq \phi_{a'a}(b'b) \wedge \phi_{b'b}(y^+y) \\ &= \phi_{a'a}(b'b) \wedge \phi_{bb'}(b'c'caa'c'cbb'c'caa'c'cbb'c'caa'c'cbb'c'cb) \\ &= \phi_{a'a}(b'b) \wedge \phi_{b'b}(b'bb'c'caa'c'cbb'c'cbb'b) \\ &\geq \phi_{a'a}(b'b) \wedge \phi_{b'b}(b'bb'b) \wedge \phi_{bb'}(bb'c'caa'c'cbb'c'cbb') \\ &= \phi_{a'a}(b'b) \\ &\geq \rho_\Phi(a, b). \end{aligned}$$

By (K_5) we have, $\phi_{a'a}(y) \geq \phi_{a'a}(x) \wedge \phi_{a'a}(yx) \wedge \phi_{a'a}(y^+y) \geq \rho_\Phi(a, b)$.
Therefore,

$$\begin{aligned} \phi_{(ca)'(ca)}(ca)'(cb) &\geq \phi_{(ca)'(ca)}(x) \wedge \phi_x(y) \\ &\geq \phi_x(ca)'(ca) \wedge \phi_x(a'a) \wedge \phi_{a'a}(y) \\ &\geq \phi_x(a'aa'c'cbb'c'caa'c'cbb'c'caa'a) \wedge \phi_x(a'a) \wedge \phi_{a'a}(y) \\ &= \phi_x(x) \wedge \phi_{a'a}(y) \\ &\geq \rho_\Phi(a, b). \end{aligned}$$

Similarly, $\phi_{(ca)(ca)'}(ca)(cb)' \geq \rho_\Phi(a, b)$. And,

$$\begin{aligned} \phi_{(ca)'(ca)}(cb)'(cb) &\geq \phi_{(ca)'(ca)}(x) \wedge \phi_x(cb)'(cb) \\ &= \phi_x(cb)'(cb) \\ &\geq \phi_x(a'a) \wedge \phi_{a'a}(b'b) \wedge \phi_{b'b}(cb)'(cb) \\ &= \phi_{a'a}(b'b) \wedge \phi_{b'b}(b'c'cbb'c'cb) \\ &\geq \phi_{a'a}(b'b) \wedge \phi_{b'b}(b'c'cbb'c'cbb'b) \\ &= \phi_{a'a}(b'b) \wedge \phi_{b'b}(b'bb'c'cbb'b) \\ &\geq \phi_{a'a}(b'b) \wedge \phi_{b'b}(b'bb'b) \wedge \phi_{bb'}(bb'c'cbb') \\ &= \phi_{a'a}(b'b) \\ &\geq \rho_\Phi(a, b). \end{aligned}$$

Similarly, $\phi_{(ca)(ca)'}(cb)(cb)' \geq \rho_\Phi(a, b)$. Therefore,

$$\begin{aligned} \rho_\Phi(ca, cb) &\geq \phi_{(ca)(ca)'}(cb)(cb)' \wedge \phi_{(ca)(ca)'}(ca)(cb)' \\ &\quad \wedge \phi_{(ca)'(ca)}(cb)'(cb) \wedge \phi_{(ca)'(ca)}(ca)'(cb) \\ &\geq \rho_\Phi(a, b). \end{aligned}$$

Thus, $\rho_\Phi(ca, cb) \geq \rho_\Phi(a, b)$. Similarly, $\rho_\Phi(ac, bc) \geq \rho_\Phi(a, b)$. Thus, ρ_Φ is fuzzy congruence on S . $\square$

Theorem 2.7 Let S be an E -inversive E -semigroup with fuzzy ϕ -kernel normal systems, then ρ_Φ is a fuzzy regular congruence on S .

Proof. By Lemma 2.6 and definition of ρ_Φ we have $\rho_\Phi(a, aa'a) = 1$, for any $a' \in W(a)$. Therefore, ρ_Φ is a fuzzy regular congruence on E -inversive E -semigroup S . $\square$

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