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On Convergence of Random Fixed Point SP Iterative Scheme with Errors Using Three Random Operators

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Abstract

The aim of this article is to establish the convergence and almost stability results of random SP fixed point iterative scheme with errors using three asymptotically quasi-nonexpansive type random operators in a real separable Banach space. The results presented in this paper generalize several well known results in Banach spaces.

Mathematics Subject Classification: 47H10, 54 H25

Keywords: Random SP iterative scheme with errors, random asymptotically quasi-nonexpansive type mapping, almost stability, measurable spaces

1 Introduction and Preliminaries

Approximation of fixed points was studied by several authors in deterministic fixed point theory[6-9,13,16,17,20,22-25,27,30,31]. A parallel development in random fixed point theory have attracted much attention during the last few years due to its increasing role in mathematics and applied sciences. Some of the prominent references are noted in [1-4,5,10-12,14,15,18,19,21,26,28,29]. Recently, several general iterative schemes have been successfully applied for solutions of operator equations. The development of random fixed point iterations was initiated by Choudhury in [10,11,12], where random Ishikawa iteration scheme was defined and its strong convergence to a random fixed point in Hilbert spaces was discussed. After that several authors [1,2,14,15,26] have worked on random fixed point iterations to obtain fixed points in deterministic operator theory. Suppose $(\Omega, \sum)$ denotes a measurable space consisting of a set Ω and sigma algebra $\sum$ of subsets of Ω, X stands for a separable Banach space and C is a nonempty subset of X. We denote the nth iterate $T(t,(T(t,..........,T(t,x))))$ of $T:\Omega\times X \to X$ by $T^n(t,x)$, the set of random fixed point of a random operator T is denoted by RF(T) and identity random operator by $I:\Omega\times X \to X$ defined by $I(t,x)=x$ and $T^0=I$. A function $f:\Omega \to X$ is said to be measurable if $f^{-1}(B)\in \sum$, for every Borel subset B of X. A single-valued operator $T:\Omega\times X \to X$ is called a random operator if for every $x\in X$, the function $T(.,x):\Omega \to X$ is measurable. A random operator $T:\Omega\times X \to X$ is continuous if for each $t\in\Omega$ the function $T(t,.):X\to X$ is continuous. A measurable function $p:\Omega\to X$ is said to be a random fixed point of the random operator $T:\Omega\times X\to X$ if $T(t,p(t))=p(t)$ for all $t\in\Omega$.

The following iterative schemes are now well known:

Random Mann iterative scheme [7]:

$$x_{n+1}(w)=(1-\alpha_n)x_n(w)+\alpha_n T(w,x_n(w)), \text{ for } n>0, \ w\in\Omega, \tag{1.1}$$

where $0\le\alpha_n\le 1$ and $x_0:\Omega\to F$ is an arbitrary measurable mapping.

Random Ishikawa iterative scheme [12]:

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)x_n(w)+\alpha_n T(w,y_n(w)),\\ y_n(w) &= (1-\beta_n)x_n(w)+\beta_n T(w,x_n(w)), \text{ for } n>0, \ w\in\Omega, \end{aligned} \tag{1.2}$$

where $0\le\alpha_n,\beta_n\le 1$ and $x_0:\Omega\to F$ is an arbitrary measurable mapping.

Random SP iterative scheme [12]:

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)y_n(w)+\alpha_n T(w,y_n(w)),\\ y_n(w) &= (1-\beta_n)z_n(w)+\beta_n T(w,z_n(w)),\\ z_n(w) &= (1-\gamma_n)x_n(w)+\gamma_n T(w,x_n(w)) \text{ for } n>0, \ w\in\Omega, \end{aligned} \tag{1.3}$$

where { α_n }, { β_n } and { γ_n } are sequences of positive numbers in [0,1] and $x_0 : \Omega \to F$ is an arbitrary measurable mapping.

Definition 1.1 Let C be a nonempty subset of a separable Banach space X and $T : \Omega \times C \to C$ be a random operator. Then T is said to be

(i) an asymptotically nonexpansive random operator if there exists a sequence of measurable functions $r_n : \Omega \to [1, \infty)$ with $\lim_{n\to\infty} r_n(t) = 1$ such that

$$\left\| T^n(t,x) - T^n(t,y) \right\| \le r_n(t) \left\| x - y \right\|$$

for all $x, y \in C, n \in N$ and for each $t \in \Omega$.

(ii) an asymptotically quasi-nonexpansive random operator if there exists a sequence of measurable functions $r_n : \Omega \to [0, \infty)$ with $\lim_{n\to\infty} r_n(t) = 0$ such that

$$\left\| T^n(t, \eta(t)) - p(t) \right\| \le (1 + r_n(t)) \left\| \eta(t) - p(t) \right\|$$

for each $t \in \Omega$, where $p : \Omega \to C$ is a random fixed point of the operator T and $\eta : \Omega \to C$ is any measurable map.

(iii) an asymptotically nonexpansive type random operator if for all $x, y \in C$ and for each $t \in \Omega$, $\limsup_{n\to\infty} \left\{ \sup_{x,y\in C} \left[\left\| T^n(t,x) - T^n(t,y) \right\| - \left\| x - y \right\| \right] \right\} \le 0, n \in N$.

(iv) an asymptotically quasi-nonexpansive type random operator if for all $x \in C$ and for each $t \in \Omega$, $\limsup_{n\to\infty} \left\{ \sup_{x\in C} \left[\left\| T^n(t,x) - p(t) \right\| - \left\| x - p(t) \right\| \right] \right\} \le 0, \quad n \in N$

where $p : \Omega \to C$ is a random fixed point of T .

Definition 1.2 Let $T_i : \Omega \times C \to C, i = 1,2,3$ be three random operators, where C is a nonempty closed convex subset of a real separable Banach space X. Let $\xi_0 : \Omega \to C$ be any measurable mapping. The sequence $\{\xi_{n+1}(t)\}$ of measurable mappings from Ω to C, for $n = 0,1,2.......$ generated by the certain random iterative procedure involving three random operators $T_i, i = 1,2,3$ is denoted by $\{T_1, T_2, T_3, \xi_n(t)\}$ for each $t \in \Omega$. Suppose that $\xi_{n+1}(t) \to \xi^*(t)$ as $n \to \infty$ for each $t \in \Omega$ where $\xi^* \in RF = \bigcap_{i=1}^{3} RF(T_i) \ne \phi$. Let $\{\eta_n\}$ be any arbitrary sequence of measurable mappings from Ω to C. Define the sequence of measurable mappings $k_n : \Omega \to R$ by $k_n(t) = d(\eta_n(t), \{T_1, T_2, T_3, \eta_n(t)\})$. If for each $t \in \Omega, k_n(t) \to 0$ as $n \to \infty$ implies $\eta_n(t) \to \xi^*(t)$ as $n \to \infty$ for each $t \in \Omega$, then the random iterative procedure is said to be stable with respect to the random operators T_1, T_2, T_3. If for each $t \in \Omega$, $\sum_{n=1}^{\infty} k_n(t) < \infty$, implies $\eta_n(t) \to \xi^*(t)$ as $n \to \infty$ for each $t \in \Omega$, then we say that the random iterative procedure is said to be almost stable with respect to the random operators T_1, T_2, T_3. It is easy to see that an stable random iterative process is almost stable, but the converse may not be true.

Lemma 1.3[5] Let $(\Omega, \sum)$ be a measurable space, X be a separable Banach space and $T : \Omega \times X \to X$ be a continuous random operator. Then for any measurable function $x : \Omega \to X$,the function $t \to T(t, x(t))$ is also measurable.

Lemma 1.4[30] Let $\{a_n\}$ and $\{b_n\}$ be two sequences satisfying $a_{n+1} \le a_n + b_n$ for all $n \ge n_0$, where $\sum_{n=1}^{\infty} b_n < \infty$ and n_0 is a positive integer. Then the limit $\lim_{n\to\infty} a_n$ exists.

Now, for three random operators $T_i : \Omega \times C \to C$, $i = 1,2,3$, we define the following SP iterative scheme with errors as follows:

$$\left.\begin{aligned} \xi_{n+1}(t) &= \alpha_n \eta_n(t) + \beta_n T_1^n(t, \eta_n(t)) + \gamma_n u_n(t) \\ \eta_n(t) &= \alpha_n^{'} \zeta_n(t) + \beta_n^{'} T_2^n(t, \zeta_n(t)) + \gamma_n^{'} v_n(t) \\ \zeta_n(t) &= \alpha_n^{''} \xi_n(t) + \beta_n^{''} T_3^n(t, \xi_n(t)) + \gamma_n^{''} w_n(t) \end{aligned}\right\}, \qquad (1.4)$$

where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}, \{\alpha_n^{'}\}, \{\beta_n^{'}\}, \{\gamma_n^{'}\}, \{\alpha_n^{''}\}, \{\beta_n^{''}\}, \{\gamma_n^{''}\}$ are sequences of real numbers in $[0,1]$ with $\alpha_n + \beta_n + \gamma_n = \alpha_n^{'} + \beta_n^{'} + \gamma_n^{'} = \alpha_n^{''} + \beta_n^{''} + \gamma_n^{''} = 1$ and $\{u_n\}, \{v_n\}, \{w_n\}$ are bounded sequences of measurable functions from Ω to C.

The SP iterative scheme [13] is independent of Ishikawa [16] and Noor iterative schemes [24] and has better convergence rate as compared to other iterative schemes. This is the main reason for considering Random SP iterative scheme with errors in this paper.

Remark 1.5 Putting $T_1 = T_2 = T_3 = T$ and $\gamma_n = \gamma_n^{'} = \gamma_n^{''} = \beta_n^{'} = \beta_n^{''} = 0$, (1.4) reduces to the random Mann iterative scheme (1.1). Also, putting $T_1 = T_2 = T_3 = T$ and $\gamma_n = \gamma_n^{'} = \gamma_n^{''}$, (1.4) reduces to the random SP iterative (1.3).

2. Main results

Theorem 2.1 Let X be a real separable Banach space and let $T_i : \Omega \times X \to X$, $i = 1,2,3$ be three asymptotically quasi-nonexpansive type random operators. Suppose that RF=

$\bigcap_{i=1}^{3} RF(T_i) \neq \phi$. Then the random SP iterative scheme with errors defined by (1.4) with $\sum_{n=0}^{\infty}\beta_n < \infty, \sum_{n=0}^{\infty}\gamma_n < \infty, \sum_{n=0}^{\infty}\beta_n^{'} < \infty, \sum_{n=0}^{\infty}\gamma_n^{'} < \infty, \sum_{n=0}^{\infty}\beta_n^{''} < \infty, \sum_{n=0}^{\infty}\gamma_n^{''} < \infty$, converges strongly to a common random fixed point of the random operators $\{T_i, i = 1,2,3\}$ if and only if for all $t \in \Omega$, $\liminf_{n\to\infty} d(\xi_n(t), RF) = 0$, where $d(\xi_n(t), RF) = \inf\{\|\xi_n(t) - \xi(t)\| : \xi \in RF\}$.

Proof. The necessary part is obvious. To prove the sufficiency part, let $p \in RF$. As $\{u_n\},\{v_n\},\{w_n\}$ are bounded sequences of measurable functions from Ω to X, we can put for each $t \in \Omega$,

$$M(t) = \sup_{n\geq 1}\|u_n(t) - p(t)\| \vee \sup_{n\geq 1}\|v_n(t) - p(t)\| \vee \sup_{n\geq 1}\|w_n(t) - p(t)\|.$$

Obviously, $M(t) < \infty$, for each $t \in \Omega$. As $T_i : \Omega \times X \to X, i = 1,2,3$ are three asymptotically quasi-nonexpansive type random operators, for any given $\varepsilon > 0$, there exists a positive integer n_1 such that for any $n \geq n_1$, we have

$$\sup_{x\in X}\{\|T_1^n(t,x) - p(t)\| - \|x - p(t)\|\} < \varepsilon .$$

Since $\{\eta_n(t)\} \subset X$ for each $t \in \Omega$, it follows from above that for all $n \geq n_1$ and for each $t \in \Omega$,

$$\|T_1^n(t,\eta_n(t)) - p(t)\| - \|\eta_n(t) - p(t)\| < \varepsilon \qquad (2.1)$$

Similarly, we get that there exists a positive integer n_2 such that for any $n \geq n_2$ and for each $t \in \Omega$, we have $\|T_2^n(t,\zeta_n(t)) - p(t)\| - \|\zeta_n(t) - p(t)\| < \varepsilon \qquad (2.2)$

and there exists a positive integer n_3 such that for any $n \geq n_3$ and for each $t \in \Omega$, we have

$$\|T_3^n(t,\xi_n(t)) - p(t)\| - \|\xi_n(t) - p(t)\| < \varepsilon \qquad (2.3)$$

Let $n_4 = \max\{n_1, n_2, n_3\}$. Then using (2.3), for any $n \geq n_4$ and for each $t \in \Omega$, we have

$$\begin{aligned}\|\zeta_n(t) - p(t)\| &\leq \alpha_n^{''}\|\xi_n(t) - p(t)\| + \beta_n^{''}\|T_3^n(t,\xi_n(t)) - p(t)\| + \gamma_n^{''}\|w_n(t) - p(t)\| \\ &\leq \alpha_n^{''}\|\xi_n(t) - p(t)\| + \beta_n^{''}\left[\varepsilon + \|\xi_n(t) - p(t)\|\right] + \gamma_n^{''}M(t) \\ &\leq \|\xi_n(t) - p(t)\| + \beta_n^{''}\varepsilon + \gamma_n^{''}M(t) \qquad (2.4)\end{aligned}$$

Again for any $n \geq n_4$ and for each $t \in \Omega$, we have by using (2.2) and (2.4),

$$\|\eta_n(t)-p(t)\|=\|\alpha_n{}'\zeta_n(t)+\beta_n{}'T_2^n(t,\zeta_n(t))+\gamma_n{}'v_n(t)-p(t)\|$$

$$=\|\alpha_n{}'(\zeta_n(t)-p(t))+\beta_n{}'(T_2^n(t,\zeta_n(t))-p(t))+\gamma_n{}'(v_n(t)-p(t))\|$$

$$\le\alpha_n{}'\|\zeta_n(t)-p(t)\|+\beta_n{}'\|T_2^n(t,\zeta_n(t))-p(t)\|+\gamma_n{}'\|v_n(t)-p(t)\|$$

$$\le\alpha_n{}'\|\zeta_n(t)-p(t)\|+\beta_n{}'[\varepsilon+\|\zeta_n(t)-p(t)\|]+\gamma_n{}'M(t)$$

$$\le\alpha_n{}'[\|\xi_n(t)-p(t)\|+\beta_n{}''\varepsilon+\gamma_n{}''M(t)]$$

$$+\beta_n{}'[\varepsilon+\|\xi_n(t)-p(t)\|+\beta_n{}''\varepsilon+\gamma_n{}''M(t)]+\gamma_n{}'M(t)$$

$$=(\alpha_n{}'+\beta_n{}')\|\xi_n(t)-p(t)\|+\alpha_n{}'\beta_n{}''\varepsilon+\alpha_n{}'\gamma_n{}''M(t)+\beta_n{}'\varepsilon$$

$$+\beta_n{}'\beta_n{}''\varepsilon+\beta_n{}'\gamma_n{}''M(t)+\gamma_n{}'M(t)$$

$$=(\alpha_n{}'+\beta_n{}')\|\xi_n(t)-p(t)\|+[\alpha_n{}'\beta_n{}''+\beta_n{}'+\beta_n{}'\beta_n{}'']\varepsilon$$

$$+[\alpha_n{}'\gamma_n{}''+\beta_n{}'\gamma_n{}''+\gamma_n{}']M(t) \qquad (2.5)$$

Again for any $n\ge n_4$ and for each $t\in\Omega$, by using (2.1) and (2.5), we have

$$\|\xi_{n+1}(t)-p(t)\|=\|\alpha_n\eta_n(t)+\beta_nT_1^n(t,\eta_n(t)+\gamma_nu_n(t)-p(t))\|$$

$$=\|\alpha_n\eta_n(t)+\beta_nT_1^n(t,\eta_n(t))+\gamma_nu_n(t)-p(t)(\alpha_n+\beta_n+\gamma_n)\|$$

$$\le\alpha_n\|\eta_n(t)-p(t)\|+\beta_n\|T_1^n(t,\eta_n(t))-p(t)\|+\gamma_n\|u_n(t)-p(t)\|$$

$$\le\alpha_n\|\eta_n(t)-p(t)\|+\beta_n[\varepsilon+\|\eta_n(t)-p(t)\|]+\gamma_nM(t)$$

$$=(\alpha_n+\beta_n)\|\eta_n(t)-p(t)\|+\beta_n\varepsilon+\gamma_nM(t)$$

$$=(\alpha_n+\beta_n)\left[\begin{array}{c}(\alpha_n{}'+\beta_n{}')\|\xi_n(t)-p(t)\|+(\alpha_n{}'\beta_n{}''+\beta_n{}'+\beta_n{}'\beta_n{}'')\varepsilon\\ +(\alpha_n{}'\gamma_n{}''+\beta_n{}'\gamma_n{}''+\gamma_n{}')M(t)\end{array}\right]+\beta_n\varepsilon+\gamma_nM(t)$$

$$=(\alpha_n+\beta_n)(\alpha_n{}'+\beta_n{}')\|\xi_n(t)-p(t)\|+(\alpha_n+\beta_n)(\alpha_n{}'\beta_n{}''+\beta_n{}'+\beta_n{}'\beta_n{}'')\varepsilon$$

$$+(\alpha_n+\beta_n)(\alpha_n{}'\gamma_n{}''+\beta_n{}'\gamma_n{}''+\gamma_n{}')M(t)+\beta_n\varepsilon+\gamma_nM(t)$$

$$= (\alpha_n + \beta_n)(\alpha_n' + \beta_n')\|\xi_n(t) - p(t)\| + \begin{bmatrix} \alpha_n\alpha_n'\beta_n'' + \alpha_n\beta_n' + \alpha_n\beta_n'\beta_n'' \\ +\alpha_n'\beta_n\beta_n'' + \beta_n\beta_n' + \beta_n\beta_n'\beta_n'' \end{bmatrix}\varepsilon$$

$$+\left[\alpha_n\alpha_n'\gamma_n'' + \alpha_n\beta_n'\gamma_n'' + \alpha_n\gamma_n' + \alpha_n'\beta_n\gamma_n'' + \beta_n\beta_n'\gamma_n'' + \beta_n\gamma_n'\right]M(t) + \beta_n\varepsilon + \gamma_n M(t)$$

$$\leq \|\xi_n(t) - p(t)\| + \left[\alpha_n\alpha_n'\beta_n'' + \alpha_n\beta_n' + \alpha_n\beta_n'\beta_n'' + \alpha_n'\beta_n\beta_n'' + \beta_n\beta_n' + \beta_n\beta_n'\beta_n'' + \beta_n\right]\varepsilon$$

$$+\left[\alpha_n\alpha_n'\gamma_n'' + \alpha_n\beta_n'\gamma_n'' + \alpha_n\gamma_n' + \alpha_n'\beta_n\gamma_n'' + \beta_n\beta_n'\gamma_n'' + \beta_n\gamma_n' + \gamma_n\right]M(t)$$

$$= \|\xi_n(t) - p(t)\| + \sigma_n(t), \qquad (2.6)$$

where $\sigma_n(t) = \left[\alpha_n\alpha_n'\beta_n'' + \alpha_n\beta_n' + \alpha_n\beta_n'\beta_n'' + \alpha_n'\beta_n\beta_n'' + \beta_n\beta_n' + \beta_n\beta_n'\beta_n'' + \beta_n\right]\varepsilon$

$$+\left[\alpha_n\alpha_n'\gamma_n'' + \alpha_n\beta_n'\gamma_n'' + \alpha_n\gamma_n' + \alpha_n'\beta_n\gamma_n'' + \beta_n\beta_n'\gamma_n'' + \beta_n\gamma_n' + \gamma_n\right]M(t).$$

(2.6) further yields that for all $n \geq n_4$,

$$d(\xi_{n+1}(t), RF) \leq d(\xi_n(t), RF) + \sigma_n(t) \qquad (2.7)$$

Using given conditions of the theorem it is easy to see that $\sum_{n=0}^{\infty}\sigma_n(t) < \infty$ for each $t \in \Omega$.

Hence Lemma 1.4 together with (2.7), yields that $\lim_{n\to\infty} d(\xi_n(t), RF)$ exists for each $t \in \Omega$.

Therefore using the conditions of the theorem we have for all $t \in \Omega$,

$$\lim_{n\to\infty} d(\xi_n(t), RF) = 0 \qquad (2.8)$$

Also, from (2.6) it follows that for each $t \in \Omega$ and for any natural numbers m and for all $n \geq n_4$,

$$\|\xi_{n+m}(t) - p(t)\| \leq \|\xi_{n+m-1}(t) - p(t)\| + \sigma_{n+m-1}(t)$$

$$\leq \|\xi_{n+m-2}(t) - p(t)\| + \sigma_{n+m-2}(t) + \sigma_{n+m-1}(t)$$

$$\leq .... \leq \|\xi_n(t) - p(t)\| + \sum_{k=n}^{n+m-1}\sigma_k(t). \qquad (2.9)$$

Therefore for any $p \in RF$ we have for all $t \in \Omega$,

$$\|\xi_{n+m}(t) - \xi_n(t)\| \leq \|\xi_n(t) - p(t)\| + \sum_{k=n}^{n+m-1}\sigma_k(t) + \|\xi_n(t) - p(t)\|$$

$$= 2\|\xi_n(t) - p(t)\| + \sum_{k=n}^{n+m-1}\sigma_k(t) \qquad (2.10)$$

As $\sum_{n=0}^{\infty}\sigma_n < \infty$ and $\lim_{n\to\infty} d\left(\xi_n(t), RF\right) = 0$, so there exists $n_5(\geq n_4)\in N$ such that for all $n \geq n_5$,

we have $d\left(\xi_n(t), RF\right) < \dfrac{\varepsilon}{4}$ *and* $\sum_{k=n}^{\infty}\sigma_k(t) < \dfrac{\varepsilon}{2}$

Hence there exists $q \in RF$ such that

$$\left\|\xi_n(t) - q(t)\right\| < \frac{\varepsilon}{4} \text{ for all } n \geq n_5.$$

Therefore from (2.10), we have that for all $t \in \Omega$, for all $n \geq n_5$ and for any positive integer m,

$$\left\|\xi_{n+m}(t) - \xi_n(t)\right\| \leq 2\left\|\xi_n(t) - q(t)\right\| + \sum_{k=n}^{n+m-1}\sigma_k(t) < 2\frac{\varepsilon}{4} + \frac{\varepsilon}{2} = \varepsilon,$$

from which it follows that $\{\xi_n(t)\}$ is a Cauchy sequence for each $t \in \Omega$. So, $\xi_n(t) \to \xi(t)$ as $n \to \infty$ for each $t \in \Omega$, where $\xi : \Omega \to X$, being the limit of the sequence of measurable functions is also measurable.

Now we prove that $\xi \in RF$. Since for each $t \in \Omega, \xi_n(t) \to \xi(t)$ as $n \to \infty$, there exists $n_6 \in N$ such that

$$\left\|\xi_n(t) - \xi(t)\right\| < \frac{\varepsilon}{4} \text{ for all } n \geq n_6.$$

Also, $\lim_{n\to\infty} d\left(\xi_n(t), RF\right) = 0$ for each $t \in \Omega$,implies that there exists $n_7 \in N$ such that

$$d\left(\xi_n(t), RF\right) < \frac{\varepsilon}{4} \text{ for all } n \geq n_7.$$

Hence for each $t \in \Omega$, there exists $\xi^* \in RF$ such that

$$\left\|\xi_n(t) - \xi^*(t)\right\| \leq \frac{\varepsilon}{4} \text{ for all } n \geq n_7.$$

Let $n_8 = \max\{n_6, n_7, n_1\}$. Then for all $t \in \Omega$, we have

$$\begin{aligned}
\left\|T_1(t, \xi(t)) - \xi(t)\right\| &\leq \left\|T_1(t, \xi(t)) - \xi^*(t)\right\| + \left\|\xi^*(t) - \xi(t)\right\| \\
&= \left\|T_1(t, \xi(t)) - \xi^*(t)\right\| - \left\|\xi^*(t) - \xi(t)\right\| + 2\left\|\xi^*(t) - \xi(t)\right\| \\
&\leq \varepsilon + 2\left\|\xi^*(t) - \xi(t)\right\| \\
&\leq \varepsilon + 2\left[\left\|\xi^*(t) - \xi_n(t)\right\| + \left\|\xi_n(t) - \xi(t)\right\|\right] \\
&< \varepsilon + 2\left(\frac{\varepsilon}{4} + \frac{\varepsilon}{4}\right) = 2\varepsilon \qquad (2.11)
\end{aligned}$$

which yields $T_1\left(t,\xi(t)\right)=\xi(t)$ for each $t\in\Omega$. Again ξ is measurable, so $\xi\in RF\left(T_1\right)$. In a similar manner we can show that $\xi\in RF\left(T_2\right)$ and $\xi\in RF\left(T_3\right)$. Hence we have $\xi\in RF$. Thus $\{\xi_n\}$ converges strongly to a common random fixed point of T_1,T_2,T_3.

Remark 2.2 (i) As asymptotically quasi-nonexpansive type random operators are more general than asymptotically quasi-nonexpansive random operators, result similar to Theorem 2.1 holds for asymptotically quasi-nonexpansive random operators.

Now, we prove the almost stability of the random iterative procedure (1.4).

Theorem 2.3 Let X be a real separable Banach space and let $T_i:\Omega\times X\to X$, $i=1,2,3$ be three asymptotically quasi-nonexpansive type random operators. Suppose that $RF=\bigcap_{i=1}^{3}RF\left(T_i\right)\neq\phi$. Let $\{\xi_n\}$ be the random SP iterative sequence with errors defined by (1.4) satisfying $\sum_{n=0}^{\infty}\beta_n<\infty,\sum_{n=0}^{\infty}\gamma_n<\infty,\sum_{n=0}^{\infty}\beta_n^{'}<\infty,\sum_{n=0}^{\infty}\gamma_n^{'}<\infty,\sum_{n=0}^{\infty}\beta_n^{''}<\infty,\sum_{n=0}^{\infty}\gamma_n^{''}<\infty$. Let $\{x_n\}$ be any arbitrary sequence of measurable function from Ω to C. Define the seqence of measurable mappings $k_n:\Omega\to R$ by

$$\left.\begin{aligned}
k_n(t)&=\left\|x_{n+1}(t)-\alpha_n y_n(t)-\beta_n T_1^n\left(t,y_n(t)\right)-\gamma_n f_n(t)\right\|\\
y_n(t)&=\alpha_n^{'}z_n(t)+\beta_n^{'}T_2^n\left(t,z_n(t)\right)+\gamma_n^{'}g_n(t)\\
z_n(t)&=\alpha_n^{''}x_n(t)+\beta_n^{''}T_3^n\left(t,x_n(t)\right)+\gamma_n^{''}h_n(t)\quad,\ n\geq 0,\forall t\in\Omega
\end{aligned}\right\}\qquad(2.12)$$

where $\{f_n\},\{g_n\},\{h_n\}$ are bounded sequences of measurable functions from Ω to C. Then

(i) the random iterative process is almost stable with respect to the random operators T_1,T_2,T_3, provided for all $t\in\Omega,\liminf_{n\to\infty}d\left(x_n(t),RF\right)=0$.

(ii) If $\{x_n\}$ converges to a common random fixed point of T_1,T_2,T_3 then , $\lim_{n\to\infty}k_n(t)=0$ for each $t\in\Omega$.

Proof. Let $p\in RF$. Since $\{f_n\},\{g_n\},\{h_n\}$ are bounded sequences of measurable functions from Ω to X, we can put for each $t\in\Omega$,

$$M^{'}(t)=\sup_{n\geq1}\left\|f_n(t)-p(t)\right\|\vee\sup_{n\geq1}\left\|g_n(t)-p(t)\right\|\vee\sup_{n\geq1}\left\|h_n(t)-p(t)\right\|.$$

Obviously $M^{'}(t)<\infty$ for each $t\in\Omega$. Similar to the proof of Theorem 2.1 for all $n\geq n_4$ and for all $t\in\Omega$, we have

$$\left\|x_{n+1}(t)-p(t)\right\| = \left\| \begin{matrix} x_{n+1}(t)-p(t)+\left[-\alpha_n y_n(t)-\beta_n T_1^n(t,y_n(t))-\gamma_n f_n(t)\right] \\ +\left[\alpha_n y_n(t)+\beta_n T_1^n(t,y_n(t))+\gamma_n f_n(t)\right] \end{matrix} \right\|$$

$$\le \left\|x_{n+1}(t)-\alpha_n y_n(t)-\beta_n T_1^n(t,y_n(t))-\gamma_n f_n(t)\right\|$$

$$+\left\|\alpha_n y_n(t)+\beta_n T_1^n(t,y_n(t))+\gamma_n f_n(t)-p(t)\right\|$$

$$= k_n(t)+\left\|\alpha_n y_n(t)+\beta_n T_1^n(t,y_n(t))+\gamma_n f_n(t)-p(t)\right\|$$

$$= k_n(t)+\left\|\alpha_n y_n(t)+\beta_n T_1^n(t,y_n(t))+\gamma_n f_n(t)-p(t)(\alpha_n+\beta_n+\gamma_n)\right\|$$

$$\le k_n(t)+\alpha_n\left\|y_n(t)-p(t)\right\|+\beta_n\left\|T_1^n(t,y_n(t))-p(t)\right\|+\gamma_n\left\|f_n(t)-p(t)\right\|$$

$$= k_n(t)+\alpha_n\left\|y_n(t)-p(t)\right\|+\beta_n\left[\varepsilon+\left\|y_n(t)-p(t)\right\|\right]+\gamma_n\left\|f_n(t)-p(t)\right\|$$

$$\le k_n(t)+\alpha_n\left\|y_n(t)-p(t)\right\|+\beta_n\left[\varepsilon+\left\|y_n(t)-p(t)\right\|\right]+\gamma_n M'(t)$$

$$= k_n(t)+(\alpha_n+\beta_n)\left\|y_n(t)-p(t)\right\|+\beta_n\varepsilon+\gamma_n M'(t)$$

$$\le k_n(t)+\left\|y_n(t)-p(t)\right\|+\beta_n\varepsilon+\gamma_n M'(t)$$

$$= \left\|y_n(t)-p(t)\right\|+k_n(t)+\beta_n\varepsilon+\gamma_n M'(t)$$

$$= \left\|y_n(t)-p(t)\right\|+\sigma_n'(t) \ \textit{where} \ \sigma_n'(t)=k_n(t)+\beta_n\varepsilon+\gamma_n M'(t) \qquad (2.13)$$

Clearly $\sum_{n=0}^{\infty} k_n(t)<\infty$ implies that $\sum_{n=0}^{\infty}\sigma_n'(t)<\infty$ So by Lemma 1.4, $\lim_{n\to\infty}\left\|y_n(t)-p(t)\right\|$ exists.

Now, (2.13) yields, for all $n \ge n_4$ and $t\in\Omega$,

$$d\left(x_{n+1}(t),RF\right)\le d\left(y_n(t),RF\right)+\sigma_n'(t)$$

Hence using Lemma 1.4, $\lim_{n\to\infty} d\left(y_n(t),RF\right)$ exists. As $\liminf_{n\to\infty} d\left(y_n(t),RF\right)=0$, so we have $\lim_{n\to\infty} d\left(y_n(t),RF\right)=0$.

Similarly as in the proof of Theorem 2.1, we have that $\{x_n\}$ converges strongly to a common random fixed point of $\{T_i, i\in\{1,2,3\}\}$. This completes the proof of (i).

To prove (ii), let $p\in RF$. Then for all $t\in\Omega$, we have

$$k_n(t) = \left\|x_{n+1}(t) - \alpha_n y_n(t) - \beta_n T_1^n(t, y_n(t)) - \gamma_n f_n(t)\right\|$$
$$= \left\|x_{n+1}(t) - p(t) + p(t) - \alpha_n y_n(t) - \beta_n T_1^n(t, y_n(t)) - \gamma_n f_n(t)\right\|$$
$$= \left\|x_{n+1}(t) - p(t) + p(t)(\alpha_n + \beta_n + \gamma_n) - \alpha_n y_n(t) - \beta_n T_1^n(t, y_n(t)) - \gamma_n f_n(t)\right\|$$
$$= \left\|x_{n+1}(t) - p(t)\right\| + \alpha_n \left\|y_n(t) - p(t)\right\| + \beta_n \left\|T_1^n(t, y_n(t)) - p(t)\right\| + \gamma_n \left\|f_n(t) - p(t)\right\| \quad (2.14)$$

Then as in the calculation in (2.6), we have that for all $n \geq n_4$, for all $p \in RF$ and $t \in \Omega$,

$$k_n(t) \leq \left\|x_{n+1}(t) - p(t)\right\| + \alpha_n \left\|y_n(t) - p(t)\right\| + \beta_n \left\|T_1^n(t, y_n(t)) - p(t)\right\| + \gamma_n \left\|f_n(t) - p(t)\right\|$$
$$\leq \left\|x_{n+1}(t) - p(t)\right\| + \alpha_n \left\|y_n(t) - p(t)\right\| + \beta_n \left[\varepsilon + \left\|y_n(t) - p(t)\right\|\right] + \gamma_n M'(t)$$
$$= \left\|x_{n+1}(t) - p(t)\right\| + \alpha_n \left\|y_n(t) - p(t)\right\| + \beta_n \varepsilon + \beta_n \left\|y_n(t) - p(t)\right\| + \gamma_n M'(t)$$
$$= \left\|x_{n+1}(t) - p(t)\right\| + (\alpha_n + \beta_n)\left\|y_n(t) - p(t)\right\| + \beta_n \varepsilon + \gamma_n M'(t)$$
$$\leq \left\|x_{n+1}(t) - p(t)\right\| + \left\|y_n(t) - p(t)\right\| + \beta_n \varepsilon + \gamma_n M'(t)$$
$$= \left\|x_{n+1}(t) - p(t)\right\| + \left\|y_n(t) - p(t)\right\| + \lambda_n'(t) \text{ where } \lambda_n'(t) = \beta_n \varepsilon + \gamma_n M'(t) \quad (2.15)$$

Using given conditions we have $\sum_{n=0}^{\infty} \lambda_n'(t) < \infty$. Let $\{x_n\}$ converges to a common random fixed point ξ (say) of T_1, T_2, T_3 then (2.15) yields

$$k_n(t) \leq \left\|x_{n+1}(t) - \xi(t)\right\| + \left\|x_n(t) - \xi(t)\right\| + \lambda_n'(t)$$

which implies that $\lim_{n \to \infty} k_n(t) = 0$ for each $t \in \Omega$.

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