Cohomology Associated to a Poisson Structure on Weil Bundles

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Abstract

Let M be a paracompact smooth manifold of dimension n , A a Weil algebra and M^A the Weil bundle associated. We define and describe the notion of $\tilde{d}$ -Poisson cohomology and of $\tilde{d}_A$ -Poisson cohomology on M^A .

Mathematics Subject Classification: 58A20, 58A32, 17D63, 53D17, 53D05

Keywords: Weil bundle, Weil algebra, Poisson manifold, cohomology

1 Introduction

A local algebra in the sens of André Weil or simply a Weil algebra is a real, unitary, commutative algebra of finite dimension with a unique maximal ideal of codimension 1 on $\mathbb{R}$ [10].

Let A be a Weil algebra and $\mathfrak{m}$ be its maximal ideal. We have

$$A = \mathbb{R} \oplus \mathfrak{m}.$$

The first projection

$$A = \mathbb{R} \oplus \mathfrak{m} \longrightarrow \mathbb{R}$$

is a homomorphism of algebra which is surjective, called augmentation and the unique none-zero integer $k \in \mathbb{N}$ such that $\mathfrak{m}^k \neq (0)$ and $\mathfrak{m}^{k+1} = (0)$ is the height of A .

If M is a smooth manifold, $C^\infty(M)$ the algebra of differentiable functions on M and A a Weil algebra of maximal ideal $\mathfrak{m}$, an infinitely near point to $x \in M$ of kind A is a homomorphism of algebras

$$\xi : C^\infty(M) \longrightarrow A$$

such that $[\xi(f) - f(x)] \in \mathfrak{m}$ for any $f \in C^\infty(M)$.

We denote M_x^A the set of all infinitely near points to $x \in M$ of kind A and

$$M^A = \bigcup_{x \in M} M_x^A.$$

The set M^A is a smooth manifold of dimension $\dim M \times \dim A$ called manifold of infinitely near points of kind A [6].

When both M and N are smooth manifolds and when

$$h : M \longrightarrow N$$

is a differentiable application, then the application

$$h^A : M^A \longrightarrow N^A, \xi \longmapsto h^A(\xi),$$

such that, for any $g \in C^\infty(N)$,

$$[h^A(\xi)](g) = \xi(g \circ h)$$

is also differentiable. When h is a diffeomorphism, it is the same for h^A .

Moreover, if $\varphi : A \longrightarrow B$ is a homomorphism of Weil algebras, for any smooth manifold M , the application

$$\varphi_M : M^A \longrightarrow M^B, \xi \longmapsto \varphi \circ \xi$$

is differentiable. In particular, the augmentation

$$A \longrightarrow \mathbb{R}$$

defines for any smooth manifold M , the projection

$$\pi_M : M^A \longrightarrow M,$$

which assigns every infinitely near point to $x \in M$ to its origin x . Thus (M^A, π_M, M) defines the bundle of infinitely near points or simply weil bundle [7],[4],[10].

If (U, φ) is a local chart of M with coordinate functions $(x_1, x_2, \dots, x_n)$, the application

$$U^A \longrightarrow A^n, \xi \longmapsto (\xi(x_1), \xi(x_2), \dots, \xi(x_n)),$$

is a bijection from U^A into an open of A^n . The manifold M^A is a smooth manifold modeled over A^n , that is to say an A -manifold of dimension n [1],[9].

The set, $C^\infty(M^A, A)$ of differentiable functions on M^A with values in A is a commutative, unitary algebra over A . When one identifies $\mathbb{R}^A$ with A , for $f \in C^\infty(M)$, the application

$$f^A : M^A \longrightarrow A, \xi \longmapsto \xi(f)$$

is differentiable. Moreover the application

$$C^\infty(M) \longrightarrow C^\infty(M^A, A), f \longmapsto f^A,$$

is an injective homomorphism of algebras and we have:

$$(f + g)^A = f^A + g^A; (\lambda \cdot f)^A = \lambda \cdot f^A; (f \cdot g)^A = f^A \cdot g^A$$

for $\lambda \in \mathbb{R}$, f and g belonging to $C^\infty(M)$.

We denote $\mathfrak{X}(M^A)$, the set of all vector fields on M^A . According to [1], We have the following equivalent assertions:

1. $X : C^\infty(M^A) \longrightarrow C^\infty(M^A)$ is a vector field on M^A ;
2. $X : C^\infty(M) \longrightarrow C^\infty(M^A, A)$ is a linear application which verifies

$$X(fg) = X(f) \cdot g^A + f^A \cdot X(g)$$

for any $f, g \in C^\infty(M)$ i.e is a derivation of $C^\infty(M)$ into $C^\infty(M^A, A)$ with respect to the module structure

$$C^\infty(M^A, A) \times C^\infty(M) \longrightarrow C^\infty(M^A, A), (\varphi, f) \longmapsto \varphi \cdot f^A.$$

Thus, the set $\mathfrak{X}(M^A)$ of all vector fields on M^A is a $C^\infty(M^A, A)$ -module.

When

$$\theta : C^\infty(M) \longrightarrow C^\infty(M)$$

is a vector field on M , the application

$$\theta^A : C^\infty(M) \longrightarrow C^\infty(M^A, A), f \longmapsto [\theta(f)]^A,$$

is a vector field on M^A . The vector field θ^A is the prolongation to M^A of the vector field θ .

Theorem 1 *If X is a vector field on M^A considered as a derivation of $C^\infty(M)$ into $C^\infty(M^A, A)$, then there exists, an unique derivation*

$$\tilde{X} : C^\infty(M^A, A) \longrightarrow C^\infty(M^A, A)$$

such that such that

1. $\tilde{X}$ is A -linear;
2. $\tilde{X} [C^\infty(M^A)] \subset C^\infty(M^A)$;
3. $\tilde{X}(f^A) = X(f)$ for any $f \in C^\infty(M)$.

Thus, the application

$$[,] : \mathfrak{X}(M^A) \times \mathfrak{X}(M^A) \longrightarrow \mathfrak{X}(M^A), (X, Y) \longmapsto \tilde{X} \circ Y - \tilde{Y} \circ X,$$

is A -bilinear and defines a structure of Lie algebra over A on $\mathfrak{X}(M^A)[1]$.

The goal of this paper is to define and describe the notion of $\tilde{d}$ -Poisson cohomology and of $\tilde{d}_A$ -Poisson cohomology.

2 Poisson structure on Weil bundles

In this section, M is a Poisson manifold i.e there exists a bracket $\{, \}$ on $C^\infty(M)$ such that the pair $(C^\infty(M), \{, \})$ is a real Lie algebra and for any $f \in C^\infty(M)$, the application

$$ad(f) : C^\infty(M) \longrightarrow C^\infty(M), g \longmapsto \{f, g\}$$

is a derivation of commutative algebra i.e

$$\{f, g \cdot h\} = \{f, g\} \cdot h + g \cdot \{f, h\}$$

for $f, g, h \in C^\infty(M)$ [5],[8].

We denote

$$C^\infty(M) \longrightarrow \text{Der}_{\mathbb{R}}[C^\infty(M)], f \longmapsto ad(f),$$

the adjoint representation and d the operator of cohomology associated to this representation. For any $p \in \mathbb{N}$,

$$\Lambda_{Pois}^p(M) = C^p[C^\infty(M), C^\infty(M)]$$

denotes the $C^\infty(M)$ -module of skew-symmetric multilinear forms of degree p from $C^\infty(M)$ into $C^\infty(M)$. We have

$$\Lambda_{Pois}^0(M) = C^\infty(M).$$

The A -algebra $C^\infty(M^A, A)$ is a Poisson algebra over A if there exists a bracket $\{, \}$ on $C^\infty(M^A, A)$ such that the pair $(C^\infty(M^A, A), \{, \})$ is a Lie algebra over A satisfying

$$\{\varphi_1 \cdot \varphi_2, \varphi_3\} = \{\varphi_1, \varphi_3\} \cdot \varphi_2 + \varphi_1 \cdot \{\varphi_2, \varphi_3\}$$

for any $\varphi_1, \varphi_2, \varphi_3 \in C^\infty(M^A, A)$ [3],[2].

When M is a Poisson manifold with bracket $\{, \}$, for any $f, g \in C^\infty(M)$,

$$ad(fg) = ad(f) \cdot g + f \cdot ad(g).$$

For any $f \in C^\infty(M)$, let

$$[ad(f)]^A : C^\infty(M) \longrightarrow C^\infty(M^A, A), g \longmapsto \{f, g\}^A,$$

be the prolongation of the vector field $ad(f)$ and let

$$[\widetilde{ad(f)}]^A : C^\infty(M^A, A) \longrightarrow C^\infty(M^A, A)$$

be the unique A -linear derivation such that

$$[\widetilde{ad(f)}]^A(g^A) = [ad(f)]^A(g) = \{f, g\}^A$$

for any $g \in C^\infty(M)$.

Theorem 2 [3] *For $\varphi \in C^\infty(M^A, A)$, the application*

$$\tau_\varphi : C^\infty(M) \longrightarrow C^\infty(M^A, A), f \longmapsto -[\widetilde{ad(f)}]^A(\varphi)$$

is a vector field on M^A .

We denote

$$\widetilde{\tau}_\varphi : C^\infty(M^A, A) \longrightarrow C^\infty(M^A, A)$$

the unique A -linear derivation such that

$$\widetilde{\tau}_\varphi(f^A) = \tau_\varphi(f)$$

for any $f \in C^\infty(M)$. We have for $f \in C^\infty(M)$,

$$\widetilde{\tau}_{f^A} = [\widetilde{ad(f)}]^A,$$

and for $\varphi, \psi \in C^\infty(M^A, A)$ and for $a \in A$,

$$\begin{aligned}\widetilde{\tau}_{\varphi+\psi} &= \widetilde{\tau}_\varphi + \widetilde{\tau}_\psi; \\ \widetilde{\tau}_{a \cdot \varphi} &= a \cdot \widetilde{\tau}_\varphi; \\ \widetilde{\tau}_{\varphi \cdot \psi} &= \varphi \cdot \widetilde{\tau}_\psi + \psi \cdot \widetilde{\tau}_\varphi.\end{aligned}$$

For any $\varphi, \psi \in C^\infty(M^A, A)$, we let

$$\{\varphi, \psi\}_A = \widetilde{\tau}_\varphi(\psi).$$

In [3] we show that this bracket defines a structure of A -Poisson algebra on $C^\infty(M^A, A)$.

Theorem 3 *If M is a Poisson manifold with bracket $\{\cdot, \cdot\}$, then $\{\cdot, \cdot\}_A$ is the prolongation on M^A of the structure of Poisson on M defined by $\{\cdot, \cdot\}$.*

3 $\widetilde{d}$ -Poisson cohomology

Proposition 4 *When M is a Poisson manifold with bracket $\{\cdot, \cdot\}$, the map*

$$C^\infty(M) \longrightarrow \text{Der}_A [C^\infty(M^A, A)], f \longmapsto -[\widetilde{ad(f)}]^A$$

is a representation of $C^\infty(M)$ into $C^\infty(M^A, A)$.

We denote $\widetilde{d}$ the operator of cohomology associated to this representation. For any $p \in \mathbb{N}$,

$$\Lambda_{Pois}^p(M^A, \sim) = \mathcal{C}^p[C^\infty(M), C^\infty(M^A, A)]$$

denotes the $C^\infty(M^A, A)$ -module of skew-symmetric multilinear forms of degree p from $C^\infty(M)$ into $C^\infty(M^A, A)$. We have

$$\Lambda_{Pois}^0(M^A, \sim) = C^\infty(M^A, A).$$

We denote

$$\Lambda_{Pois}(M^A, \sim) = \bigoplus_{p=0}^n \Lambda_{Pois}^p(M^A, \sim).$$

Thus, for $\Omega \in \Lambda_{Pois}^p(M^A, \sim)$ and $f_1, \dots, f_{p+1} \in C^\infty(M)$, we have

$$\begin{aligned} \tilde{d}\Omega(f_1, \dots, f_{p+1}) &= \sum_{i=1}^{p+1} (-1)^i [\widetilde{ad(f_i)}]^A [\Omega(f_1, \dots, \widehat{f_i}, \dots, f_{p+1})] \\ &\quad + \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} \Omega(\{f_i, f_j\}, f_1, \dots, \widehat{f_i}, \dots, \widehat{f_j}, \dots, f_{p+1}) \end{aligned}$$

where $\widehat{f_i}$ means that the term f_i is omitted.

When $\eta \in \Lambda_{Pois}^p(M)$, then

$$\eta^A : C^\infty(M) \times \dots \times C^\infty(M) \longrightarrow C^\infty(M^A, A), (f_1, \dots, f_p) \longmapsto [\eta(f_1, \dots, f_p)]^A$$

is skew-symmetric multilinear forms of degree p from $C^\infty(M)$ into $C^\infty(M^A, A)$ i.e

$$\eta^A \in \Lambda_{Pois}^p(M^A, \sim).$$

Thus

Proposition 5 *For any $\eta \in \Lambda_{Pois}^p(M)$, we have $\tilde{d}\eta^A = (d\eta)^A$.*

Proof. For any $f_1, \dots, f_{p+1} \in C^\infty(M)$, we have

$$\begin{aligned}
 (\widetilde{d}\eta^A)(f_1, \dots, f_{p+1}) &= \sum_{i=1}^{p+1} (-1)^i [\widetilde{ad}(f_i)]^A \left(\eta^A(f_1, \dots, \widehat{f_i}, \dots, f_{p+1}) \right) \\
 &\quad + \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} \eta^A \left(\{f_i, f_j\}, f_1, \dots, \widehat{f_i}, \dots, \widehat{f_j}, \dots, f_{p+1} \right) \\
 &= \sum_{i=1}^{p+1} (-1)^i [\widetilde{ad}(f_i)]^A \left(\eta(f_1, \dots, \widehat{f_i}, \dots, f_{p+1}) \right)^A \\
 &\quad + \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} [\eta(\{f_i, f_j\}, f_1, \dots, \widehat{f_i}, \dots, \widehat{f_j}, \dots, f_{p+1})]^A \\
 &= \sum_{i=1}^{p+1} (-1)^i \{f_i, \eta(f_1, \dots, \widehat{f_i}, \dots, f_{p+1})\}^A \\
 &\quad + \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} [\eta(\{f_i, f_j\}, f_1, \dots, \widehat{f_i}, \dots, \widehat{f_j}, \dots, f_{p+1})]^A \\
 &= [(d\eta)(f_1, f_2, \dots, f_{p+1})]^A.
 \end{aligned}$$

That ends the proof. ■

Corollary 6 *The 1-form η^A is $\widetilde{d}$ -closed i.e ($\widetilde{d}\eta^A = 0$), if and only if $d\eta = 0$. In particular when η is a derivation of Poisson algebra $C^\infty(M)$.*

Proof. Indeed, for $p = 1$, we have

$$\begin{aligned}
 (\widetilde{d}\eta^A)(f, g) &= [\widetilde{ad}(f)]^A [\eta^A(g)] - [\widetilde{ad}(g)]^A [\eta^A(f)] - \eta^A(\{f, g\}) \\
 &= \{f, \eta(g)\}^A - \{g, \eta(f)\}^A - [\eta(\{f, g\})]^A \\
 &= (\{f, \eta(g)\} - \{g, \eta(f)\} - \eta(\{f, g\}))^A \\
 &= [d\eta(f, g)]^A.
 \end{aligned}$$

for any $f, g \in C^\infty(M)$.

Thus $\widetilde{d}\eta^A = 0$ if and only if $d\eta = 0$.

When η is a derivation of Poisson algebra $C^\infty(M)$, we have $f, g \in C^\infty(M)$,

$$\begin{aligned}
 \eta(\{f, g\}) &= \{\eta(f), g\} + \{f, \eta(g)\} \\
 &= \{f, \eta(g)\} - \{g, \eta(f)\}
 \end{aligned}$$

i.e

$$\begin{aligned}
 (\widetilde{d}\eta^A)(f, g) &= [d\eta(f, g)]^A \\
 &= 0.
 \end{aligned}$$

That ends the proof. ■

Proposition 7 *If η and η' both are cohomologous d -closed p -forms then η^A and η'^A both are cohomologous $\tilde{d}$ -closed p -forms.*

Proof. For any $f_1, \dots, f_p \in C^\infty(M)$ we have

$$\begin{aligned} [\eta^A - \eta'^A](f_1, \dots, f_p) &= \eta^A(f_1, \dots, f_p) - \eta'^A(f_1, \dots, f_p) \\ &= [\eta(f_1, \dots, f_p)]^A - [\eta'(f_1, \dots, f_p)]^A \\ &= [\eta(f_1, \dots, f_p) - \eta'(f_1, \dots, f_p)]^A \\ &= [(\eta - \eta')(f_1, \dots, f_p)]^A. \end{aligned}$$

If there exists $\nu \in \Lambda_{Pois}^{p-1}(M)$ such that

$$\eta - \eta' = d\nu$$

then

$$\begin{aligned} [\eta^A - \eta'^A](f_1, \dots, f_p) &= [(\eta - \eta')(f_1, \dots, f_p)]^A \\ &= [d\nu(f_1, \dots, f_p)]^A \\ &= \tilde{d}\nu^A(f_1, \dots, f_p). \end{aligned}$$

i.e

$$\eta^A - \eta'^A = \tilde{d}\nu^A.$$

■

The cohomology class of the d -closed p -form η induces the cohomology class of the $\tilde{d}$ -closed p -form η^A .

Let $Z_{Pois}^p(M^A, \sim)$ be the set of $\tilde{d}$ -closed p -forms from $C^\infty(M)$ into $C^\infty(M^A, A)$ and $B_{Pois}^p(M^A, \sim)$ be the set of $\tilde{d}$ -exact p -forms from $C^\infty(M)$ into $C^\infty(M^A, A)$. We denote

$$H_{Pois}^p(M^A, \sim) = Z_{Pois}^p(M^A, \sim) / B_{Pois}^p(M^A, \sim).$$

For $p = 0$, $\Lambda_{Pois}^0(M^A, \sim) = C^\infty(M^A, A)$. It is obvious that $H^0(M^A, \sim)$ is the center of $C^\infty(M^A, A)$ i.e the set

$$\left\{ \phi \in (M^A, A); [\widetilde{ad(f)}]^A(\phi) = 0 \text{ for every } f \in C^\infty(M) \right\}.$$

For $p = 1$, we have

$$H_{Pois}^1(M^A, \sim) = 0.$$

4 $\widetilde{d}_A$ -Poisson cohomology

The map

$$C^\infty(M^A, A) \longrightarrow \text{Der}_A[C^\infty(M^A, A)], \varphi \longmapsto \widetilde{\tau}_\varphi,$$

is a representation of $C^\infty(M^A, A)$ into $C^\infty(M^A, A)$. We denote $\widetilde{d}_A$ the cohomology operator associated to this representation.

For any $p \in \mathbb{N}$, $\Lambda_{Pois}^p(M^A, \sim_A) = \mathcal{C}^p[C^\infty(M^A, A), C^\infty(M^A, A)]$ denotes the $C^\infty(M^A, A)$ -module of skew-symmetric multilinear forms of degree p on $C^\infty(M^A, A)$ into $C^\infty(M^A, A)$. We have

$$\Lambda_{Pois}^0(M^A, \sim_A) = C^\infty(M^A, A).$$

We denote

$$\Lambda_{Pois}(M^A, \sim_A) = \bigoplus_{p=0}^n \Lambda_{Pois}^p(M^A, \sim_A).$$

For $\Omega \in \Lambda_{Pois}^p(M^A, \sim_A)$ and $\varphi_1, \varphi_2, \dots, \varphi_{p+1} \in C^\infty(M^A, A)$, we have

$$\begin{aligned} \widetilde{d}_A \Omega(\varphi_1, \dots, \varphi_{p+1}) &= \sum_{i=1}^{p+1} (-1)^{i-1} \widetilde{\tau}_{\varphi_i} [\Omega(\varphi_1, \dots, \widehat{\varphi}_i, \dots, \varphi_{p+1})] \\ &+ \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} \Omega(\{\varphi_i, \varphi_j\}_A, \varphi_1, \dots, \widehat{\varphi}_i, \dots, \widehat{\varphi}_j, \dots, \varphi_{p+1}) \end{aligned}$$

i.e

$$\begin{aligned} \widetilde{d}_A \Omega(\varphi_1, \varphi_2, \dots, \varphi_{p+1}) &= \sum_{i=1}^{p+1} (-1)^{i-1} \{\varphi_i, \Omega(\varphi_1, \dots, \widehat{\varphi}_i, \dots, \varphi_{p+1})\}_A \\ &+ \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} \Omega(\{\varphi_i, \varphi_j\}_A, \varphi_1, \dots, \widehat{\varphi}_i, \dots, \widehat{\varphi}_j, \dots, \varphi_{p+1}). \end{aligned}$$

For $p = 1$, we have

$$\widetilde{d}_A \Omega(\varphi, \psi) = \{\Omega(\varphi), \psi\}_A + \{\varphi, \Omega(\psi)\}_A - \Omega(\{\varphi, \psi\}_A)$$

for any $\varphi, \psi \in C^\infty(M^A, A)$. Thus

Corollary 8 *The 1-form Ω is $\widetilde{d}_A$ -closed i.e $\widetilde{d}_A \Omega = 0$ if, and only if,*

$$\Omega(\{\varphi, \psi\}_A) = \{\Omega(\varphi), \psi\}_A + \{\varphi, \Omega(\psi)\}_A$$

i.e Ω is a derivation of the algebra $C^\infty(M^A, A)$.

Let $Z_{Pois}^p(M^A, \sim_A)$ be the set of $\widetilde{d}_A$ -closed p -forms from $C^\infty(M^A, A)$ into $C^\infty(M^A, A)$ and $B_{Pois}^p(M^A, \sim_A)$ be the set of $\widetilde{d}_A$ -exact p -forms from $C^\infty(M)$ into $C^\infty(M^A, A)$. We denote

$$H_{Pois}^p(M^A, \sim_A) = Z_{Pois}^p(M^A, \sim_A) / B_{Pois}^p(M^A, \sim_A).$$

For $p = 0$, $\Lambda_{Pois}^0(M^A, \sim_A) = C^\infty(M^A, A)$. It is obvious that $H^0(M^A, \sim_A)$ is the center of $C^\infty(M^A, A)$ i.e the set

$$\{\varphi \in C^\infty(M^A, A); \{\varphi, \phi\}_A = 0 \text{ for every } \phi \in C^\infty(M^A, A)\}.$$

For $p = 1$, we have

$$H_{Pois}^1(M^A, \sim_A) = 0.$$

Acknowledgements: The first author thanks Deutscher Akademischer Austauschdienst (DAAD) for their financial support.

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Received: August 15, 2013