Traditional Mathematics in the Aomori Prefecture

Noriaki Nagase

Department of Mathematical Sciences
Faculty of Science and Technology
Hirosaki 036-8561, Japan

Hiroshi Nakazato

Department of Mathematical Sciences
Faculty of Science and Technology
Hirosaki University, Hirosaki 036-8561, Japan

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Abstract

In this note some geometrical problems in the traditional mathematics in Aomori, Japan are introduced. A new result inspired by the classical problems is also presented.

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1. Mathematical study in Japan before the Meiji restoration

The foundation of modern mathematical study in Japan was laid by Dairoku Kikuchi (1855-1917). He graduated from the university of Cambridge and introduced modern mathematics to Japan. European- American style arithmetics or mathematics became a formal subject of schools in Japan. The
period 1603-1868 was the golden age of Japanese traditional mathematics. Mikami’s [9] described the contributions of representative mathematicians T. Seki (1642?-1708) and K. Takebe (1664-1739) in the golden age. Their mathematical works were performed in the central city of Japan, Edo (Tokyo). Fukagawa and Pedoe [3] studied many mathematical tablets on which geometrical problems in the age were exhibited (cf. [4, 10]). An area problem on a mathematical tablet dedicated to Atsuta Shrine in Nagoya was solved by using an infinite series in 1844 (cf. [7]). A modern method to solve this problem depends on a hyperelliptic integral. You may recognize the top level of the traditional mathematics in Japan by these problems. In this note, we introduce some mathematical works performed in the Aomori prefecture during 1603-1892.

2. A temple geometry in the Aomori prefecture

The Aomori prefecture lies at the north end of Honshu (the biggest island of Japan). In 1939, Yoshichiro Haga found some mathematical documents at Hachinohe city in the Aomori prefecture. The documents were written by students of a mathematical school at Hachinohe city in nearly 1750. The contents of the documents were studied by Y. Haga and H. Kuwabara [8]. In 1949, Hirosaki University was founded. Professor Haga studied the history of science at this university. The authors of this note are studying mathematics at Hirosaki University. The aim of this paper is the introduction of the mathematical works in the Aomori prefecture during the Edo period 1603-1868 and the early times of the Imperial Japan, about 1870-1890.

In [8] Kuwabara studied the documents written by students of the Hachinohe mathematical school founded by Shinpou-Egen (1657-1753). Shinpou was a Buddhist priest and studied mathematics in Edo. He found regular dodecahedron and icosahedron by himself. He studied properties of these polyhedrons.

In 1979 Kuwabara dedicated a mathematical tablet to a temple in Hachinohe celebrating Shinpou’s works. In the tablet Kuwabara asked the following question and provided the answer. Suppose that $S$ is a sphere $x^2 + y^2 + z^2 = 6^2$. We consider a projection

$$
\Pi : (x, y, z) \mapsto \left( \frac{6x}{\sqrt{x^2 + y^2 + z^2}}, \frac{6y}{\sqrt{x^2 + y^2 + z^2}}, \frac{6z}{\sqrt{x^2 + y^2 + z^2}} \right)
$$

of $\mathbb{R}^3 \setminus \{(0, 0, 0)\}$ onto the sphere $S$. Suppose that a regular dodecahedron $D$ and a regular icosahedron $Ico$ are inscribed to the sphere. Find the length of $\Pi(E_D)$ and the length of $\Pi(E_{Ico})$ where $E_D$ is an edge of $D$ and $E_{Ico}$ is an edge
of Ico. Kuwabara gave the answer

$$\Pi(E_D) = 6\arccos\left(\frac{\sqrt{5}}{3}\right) \sim 4.37837,$$

$$\Pi(E_{Ico}) = 6\arccos\left(\frac{1}{\sqrt{5}}\right) \sim 6.64289.$$ 

Probably Kuwabara may have known that the original description by Shinpou’s student contained an error. Shinpou’s student Mitsusada Okumura asked the following question and gave his wrong answer in 1748.

Suppose that $D$ is a regular dodecahedron with an edge $E_D$ of length 35. Find the diameter $2r$ of the sphere circumscribed to $D$. We know that

$$x = 2r = \frac{\sqrt{3} + \sqrt{15}}{2} \times 35 \sim 98.0880977.$$ 

This value satisfies the quartic equation

$$x^4 - 11025x^2 + 13505625 = 0, \quad (2.1)$$

where $11025 = 9 \times 35^2$, $13505625 = 9 \times 35^4$. Okumura wrongly gave the value $96.34673443$ (cf. [8], pages 13, 84-85). He described the quartic equation

$$(3x^2 - 4 \cdot 35^2)^2 = 4x^2(x^2 + 4 \cdot 35^2)$$

or equivalently,

$$x^4 - 9800x^2 + 4802000 = 0, \quad (2.2)$$

to get the value, where $9800 = 8 \times 35^2$, $4802000 = 16 \times 5^3 \times 7^4$. But the authors can not decode Okumura’s process. His value 96.34673443 is a correct numerical solution of the equation (2.2). In [6] the second author of this note and T. Kimura analyzed Kepler’s models of planetary orbits. Kepler’s ideas were deeply related with the 5 regular polyhedrons. In [5] Kepler mentioned that the length of a regular dodecahedron inscribed to a sphere with radius 1000 is 714. He would know the fact

$$\sqrt{\frac{2}{3}}(3 - \sqrt{5}) \sim \frac{714}{1000}$$

by a commentary on Euclid’s “Elements of Geometry” published in 1566. In 1830, a Japanese mathematician Sadakazu Ikeda computed the volumes of the regular dodecahedron and icosahedron with an edge of length $a$ as

$$\frac{5(3 + \sqrt{5})a^3}{12}, \quad \frac{(15 + 7\sqrt{5})a^3}{4},$$

respectively([4]). You can meet some beautiful works in Japanese temple geometry in [10]. The authors guess that the tablet of Kuwabara indirectly
suggests an incomplete character of the works of Shinpou’s students at least in the solid geometry. It remains some possibility that the decoding of Okumura’s equation correct his errors.

After the Meiji restoration, some people in the Aomori prefecture studied mathematics in the traditional style. You can find a mathematical tablet in the Aomori prefectural Folk museum (1973-). In 1892, a tablet was dedicated by Yuisuke Kamiyama to a local temple in Hachinohe celebrating his grand father’s mathematical works. His grand father was a student of Shinpou’s school. Kamiyama asked 3 questions and gave the answers. The questions are rather elementary. The contents of the tablet were analyzed by H. Fukagawa at a lecture for the staffs of the folk museum. Kamiyama treated a typical plane geometry problem in his second question.

We shall consider a semi-circle \{ (b \cos \theta, b \sin \theta) : 0 \leq \theta \leq \pi \} in the \( x - y \) plane. Let \( A = (-b, 0), B = (b, 0), O = (0, 0) \). We consider a point \( P = (b \cos \phi, b \sin \phi) \) for some \( 0 < \phi < \pi \). Denote by \( a \) the length of the chord \( AP \). Denote by \( r \) the radius of the circle inscribed to the triangle \( OBP \). Denote by \( R \) the radius of the circle which is inscribed to the semi-circle and is tangent to \( OB \) and \( OP \). Kamiyama asked you to express \( r \) by using \( a, b \) and \( R \). Kamiyama gave the right answer \( r = (R/2) \times (a/b) \).

In the first question, Kamiyama treated algebraic equations. Suppose that \( R \) is a rectangular solid with height \( z \), width \( x \) and depth \( y \). We assume that the volume \( V \) of \( R \) is 140 and \( x + y = 9, \ z = y + 2 \). You shall answer the respective values of \( x, y, z \). We get a cubic equation

\[ x^3 - 20x^2 + 99x - 140 = (x - 4)(x^2 - 16x + 35) = 0 \]
in $x$. Since $x > 0, y > 0, z > 0$. We have two solutions $x = 4, y = 5, z = 7$ and $x = 8 - \sqrt{29}, y = 1 + \sqrt{29}, z = 3 + \sqrt{29}$. Essentially Kamiyama gave the right answer. But he gave a wrong evaluation $\sqrt{29} \approx 5.36655$. We can find $\sqrt{29} \approx 5.385164807$. So we find a minor error in his numerical computation.

In the third question, Kamiyama treated the maximum of a function. In this problem $\theta$ varies on the interval $0 < \theta \leq \pi/4$. Let $a > 0$ be a positive constant and $b = \cot \theta$. Let $B = (a + b, a), C = (a + b, 0), R = (a + b + p, 0), P = (a + b + p, a + p)$ where $p > 0$ and $a + p = (a + b + p) \tan \theta$. Let $y = BP = BR$. Kamiyama asks you to express $y$ as the function of $\theta$. He also asks the maximum of $y$ under the condition $0 < \theta \leq \pi/4$. We can give the answer $y = \sqrt{4 \tan^2 \theta + 1} a$ and the maximum $y = \sqrt{5}a$.

3. A modern question inspired by a classical model

J. Kepler considered that the ratio $r = r_J/r_S$ for the radius $r_J$ of Jupiter and the radius $r_S$ of the orbit of Saturn was expressed as $r_J/r_C$ by the radius $r_C$ of the circumscribed sphere of a cube and the radius of the inscribed sphere of the cube, that is, $r_J/r_C = 1/\sqrt{3}$. He supposed that the number of planets, Mercury, Venus, Earth, Mars, Jupiter, Saturn were related with the 5 regular polyhedrons. The authors wonder if mysterious characters of regular polyhedrons might have attracted Shinpou’s attention. Firstly Kepler considered a plane model. He considered a family of regular polygons inscribed to a circle. The envelop of the family of polygons is also a circle. This system make us call Poncelet’s closure theorem. Recently the second author of this note has found a new Poncelet curve (cf. [1], [2]).

For a fixed real number $-1 < a < 1, a \neq 0$. We consider an upper triangular $4 \times 4$ contraction $A = \begin{pmatrix} a_{ij} \end{pmatrix}_{i,j=1}^4$ with $a_{11} = a_{22} = a_{33} = a_{44} = a, a_{12} = a_{23} = a_{34} = 1 - a^2, a_{13} = a_{24} = -a(1 - a^2), a_{14} = a^2(1 - a^2)$. We also consider its 1-parameter family of $5 \times 5$ unitary dilations

$$U(\lambda) = \begin{pmatrix} a & 1 - a^2 & -a(1 - a^2) & a^2(1 - a^2) & -a^3\sqrt{1 - a^2} \\ 0 & a & 1 - a^2 & -a(1 - a^2) & a^2\sqrt{1 - a^2} \\ 0 & 0 & a & 1 - a^2 & -a\sqrt{1 - a^2} \\ 0 & 0 & 0 & a & \sqrt{1 - a^2} \\ \lambda & -a\sqrt{1 - a^2} & a^2\sqrt{1 - a^2} & -a^3\sqrt{1 - a^2} & a^4\lambda \end{pmatrix},$$

where $\lambda$ is an arbitrary complex number with $|\lambda| = 1$. We consider the numerical range $W(U(\lambda))$ of the $5 \times 5$ unitary matrices $U(\lambda)$ defined by

$$W(U(\lambda)) = \{ (U(\lambda)\xi, \xi) : \xi \in \mathbb{C}^5, \xi^*\xi = 1 \}.$$
Then the range \( W(U(\lambda)) \) is a closed convex set with vertices \( P_j(\lambda) = \exp(i\theta_j(\lambda)) \) for which

\[
\theta_1(\lambda) < \theta_2(\lambda) < \theta_3(\lambda) < \theta_4(\lambda) < \theta_5(\lambda) < \theta_1(\lambda) + 2\pi.
\]

We consider the intersection of the two lines \( P_j(\lambda)P_{j+1}(\lambda) \) and \( P_{j+2}(\lambda)P_{j+3}(\lambda) \) \((j = 1, 2, 3, 4, 5)\) where we use the convention: \( P_6(\lambda) = P_1(\lambda), \ P_7(\lambda) = P_2(\lambda), \ P_8(\lambda) = P_3(\lambda). \) The 5 intersections form a closed curve \( C \) when \( \lambda \) varies on the unit circle. This curve and the boundary of the numerical range \( W(A) = \{ \langle A\xi, \xi \rangle : \xi \in \mathbb{C}^4, \xi^*\xi = 1 \} \) form a new Poncelet pair. We ask you to compute the equation of the curve \( C \) in the case \( a = 1/8. \)

Here we shall provide the defining polynomial \( P(x, y) \), that is, \( C : P(x, y) = 0. \) It is given by

\[
P(x, y) = 21094820843683844 - 423644857803170368x + 2640506734000769544x^2 \\
+ 50356085343118368x^3 - 64876120392417999735x^4 \\
+ 258258179960432469216x^5 - 363223672051352395899x^6 \\
+ 162124200974583523104x^7 - 7048456812691041663x^8 \\
- 6882326983741014016x^9 + 864128247221190656x^{10} \\
+ 1089038644950992904y^2 - 7434900295179164640xy^2 \\
- 48012321941915346414x^2y^2 + 45261485290239945440x^3y^2 \\
- 899263548072368467569x^4y^2 + 434893671148137362784x^5y^2 \\
+ 13008700313694561616x^6y^2 - 31860118931808190464x^7y^2 \\
+ 3669334183605436416x^8y^2 + 9467970944029971849y^4 \\
+ 19715377559825122528xy^4 - 730284320288627440497x^2y^4 \\
+ 386978062128520869216x^3y^4 + 7381372965116797748x^4y^4 \\
- 53783817715895000800x^5y^4 + 5811710189925040128x^6y^4 \\
- 19370018404168748411y^6 + 114150467437774530336xy^6 \\
+ 80721462889035021060x^2y^6 - 39529450237826433024x^3y^6 \\
+ 4072125976937496576x^4y^6 + 26961546809522146689y^8 \\
- 10723327850413817856xy^8 + 1065620623885074432x^2y^8.
\]
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