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Generalized Jordan (σ, τ) - Higher Homomorphisms of a Γ-Ring M into a Γ Ring M'

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Abstract

Let M and M' be two prime Γ-ring and σ^n, τ^n be two higher homomorphism of a Γ-ring M, for all n ∈ N in the present paper we show that under certain conditions of M, every generalized Jordan (σ, τ) - higher homomorphism of a Γ-Ring M into a prime Γ -Ring M 'is either generalized (σ, τ) - higher homomorphism or (σ, τ) – higher anti- homomorphism.

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1- Introduction

Let M and Γ be two additive abelian groups, suppose that there is a mapping from M×Γ×M ⟶ M (the image of (a,α,b) being denoted by $a\alpha b$, a, b ∈ M and α ∈ Γ). Satisfying for all a, b, c ∈ M and α, β ∈ Γ:

(i) $(a + b)\alpha c = a\alpha c + b\alpha c$
$a(\alpha + \beta)c = a\alpha c + a\beta c$
$a\alpha(b + c) = a\alpha b + a\alpha c$

(ii) $(a\alpha b)\beta c = a\alpha(b\beta c)$

Then M is called a Γ-ring. This definition is due to Barnes [1], [8].

A Γ-ring M is called a prime if $a\Gamma M\Gamma b = (0)$ implies $a = 0$ or $b = 0$, where $a, b \in$ M, this definition is due to [5].

A Γ-ring M is called semiprime if $a\Gamma M\Gamma a = (0)$ implies $a = 0$, such that $a \in$ M, this definition is due to [7].

Let M be a 2-torsion free semiprime Γ-ring and suppose that $a, b \in$ M if $a\Gamma m\Gamma b + b\Gamma m\Gamma a = 0$ for all m $\in$ M, then $a\Gamma m\Gamma b = b\Gamma m\Gamma a = 0$ this definition is due to [11].

Let M be Γ-ring then M is called 2-torsion free if $2a = 0$ implies $a = 0$, for every $a \in$ M, this definition is due to [6].

Let σ^i, τ^i be two higher homomorphism of a Γ-ring M then σ^i, τ^i are called commutative if $\sigma^i\, \tau^i = \tau^i\, \sigma^i$, for all $i \in N$.

Let M be a Γ-ring and d: M $\longrightarrow$ M be an additive map (that is $d(a + b) = d(a) + d(b)$), then d is called a derivation on M if :

$d(a\alpha b) = d(a)\alpha b + a\alpha d(b)$, for all $a, b \in$ M and $\alpha \in \Gamma$.
d is called a Jordan derivation on Γ-ring if $d(a\alpha a) = d(a)\alpha a + a\alpha d(a)$, for all $a \in$ M and $\alpha \in \Gamma$, [4], [9].

Let M be a Γ-ring and f: M $\longrightarrow$ M be an additive map (that is $f(a + b) = f(a) + f(b)$), Then f is called a generalized derivation if there exists a derivation d: M $\longrightarrow$ M such that
$f(a\alpha b) = f(a)\alpha b + a\alpha d(b)$, for all $a, b \in$ M and $\alpha \in \Gamma$.
f is called a Jordan generalized derivation if there exists a Jordan derivation d: M $\longrightarrow$ M such that
$f(a\alpha a) = f(a)\alpha a + a\alpha\, d(a)$, for all $a \in$ M and $\alpha \in \Gamma$, [2], [3] .

Let θ be an additive mapping of Γ-ring M into a Γ-ring M', θ is called a homomorphism if for all $a, b \in$ M and $\alpha \in \Gamma$
$\theta(a\alpha b) = \theta(a)\, \alpha\theta(b)$, [1], [10].

Let θ be an additive mapping of Γ-ring M into a Γ-ring M'. θ is called a Jordan homomorphism if for all $a, b \in$ M and $\alpha \in \Gamma$
$\theta(a\alpha b + b\alpha a) = \theta(a)\alpha\theta(b) + \theta(b)\alpha\, \theta(a)$, [10] .

Let F be an additive mapping of a Γ-ring M into a Γ-ring M', F is called a generalized homomorphism if there exists a homomorphism θ from a Γ-ring M into a Γ'-ring M', such that $F(a\alpha b) = F(a)\alpha\theta(b)$, for all $a, b \in$ M and $\alpha \in \Gamma$, where θ is called a relating homomorphism.

F is called a generalized Jordan homomorphism if there exists a Jordan homomorphism θ from a Γ-ring M into a Γ-ring M', such that
F($a\alpha b$ + $b\alpha a$) = F(a)αθ(b) + F(b)α θ(a), for all a, b ∈ M and α ∈ Γ, where θ is called a relating Jordan homomorphism, [10] .

Let F = (f_i $)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ-ring M' and there exists a higher homomorphism θ = $(\phi_i)_{i\in N}$ from a Γ-ring M into a Γ-ring M' then f is said to be a generalized higher homomorphism (resp. generalized Jordan higher homomorphism) on a Γ-ring M into a Γ -ring M' if for all n∈N, we have

$$f_n(a\alpha b) = \sum_{i=1}^{n} f_i(a)\alpha\phi_i(b),$$

(respectively $f_n(a\alpha b + b\alpha a) = \sum_{i=1}^{n} f_i(a)\alpha\phi_i(b) + \sum_{i=1}^{n} f_i(b)\alpha\phi_i(a)$) , for all a, b ∈ M and α ∈ Γ, [10] .

Now, the main purpose of this paper is that every generalized Jordan (σ, τ) - higher homomorphism of a Γ-ring M into a prime Γ-ring M' is either generalized (σ, τ) - higher homomorphism or (σ, τ) - anti -higher homomorphism and every generalized Jordan (σ, τ)-higher homomorphism from a Γ-ring M into a 2-torsion free Γ-ring M' is a generalized Jordan triple (σ, τ) - higher homomorphism.

2- Generalized Jordan (σ, τ) - Higher Homomorphisms of a Γ-Rings

Definition(2.1):
Let F = (f_i $)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ -ring M' and σ,τ be two homomorphism of a Γ-ring M, F is called a generalized (σ, τ) - higher homomorphism if there exists a (σ, τ) - higher homomorphism θ = $(\phi_i)_{i\in N}$ from a Γ-ring M into a Γ-ring M', such that for all a, b ∈ M, α ∈ Γ and for every n ∈ N, we have:

$$f_n(a\alpha b) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))$$

Where θ is called the relating (σ, τ) - higher homomorphism.

Definition (2.2):
Let F = (f_i $)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ - ring M' and σ,τ be two homomorphism of a Γ-ring M, F is called a Jordan generalized (σ, τ) - higher homomorphism if there exists a (σ, τ) - higher homomorphism θ =

$(\phi_i)_{i\in N}$ from a Γ-ring M into a Γ - ring M', such that for all *a, b* ∈ M, α ∈ Γ and for every n ∈ N, we have:

$$f_n(a\alpha b+b\alpha a)=\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))+\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\tau^i(a))$$

Where θ is called the relating Jordan (σ, τ) - higher homomorphism.

Definition (2.3):
Let F = (f_i $)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ - ring M' and σ,τ be two homomorphism of a Γ-ring M, F is called a generalized Jordan triple (σ, τ) - higher homomorphism if there exists a Jordan triple (σ, τ) - higher homomorphism θ = $(\phi_i)_{i\in N}$ from a Γ-ring M into a Γ-ring M', such that for all *a, b* ∈ M, α, β ∈ Γ and for every n ∈ N:

$$f_n(a\alpha b\beta a)=\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))$$

Where θ is called the relating Jordan triple (σ, τ) - higher homomorphism.

Definition (2.4):
Let F = (f_i $)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ - ring M' and σ,τ be two homomorphism of a Γ-ring M, F is called a generalized (σ, τ) – higher anti- homomorphism if there exists a (σ, τ) - higher anti- homomorphism θ = $(\phi_i)_{i\in N}$ from a Γ-ring M into a Γ - ring M', such that for all *a, b* ∈ M, α ∈ Γ and for every n ∈ N we have:

$$f_n(a\alpha b)=\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\tau^i(a))$$

Where θ is called the relating (σ, τ) – higher anti - homomorphism.

Now, we present below an example of generalized higher (σ, τ) - homomorphism and it is clearly is a generalized Jordan (σ, τ) - higher homomorphism.

Example (2.5):
Let S_1, S_2 be two rings and *f* = (f_i $)_{i\in N}$ be a generalized higher (σ, τ) - homomorphism of a ring S_1 into a ring S_2 then there exist a relating higher (σ, τ) - homomorphism θ = $(\theta_i)_{i\in N}$ from a ring S_1 into S_2. Let M = $M_{1\times 2}(S_1)$, M' = $M_{1\times 2}(S_2)$ and $\Gamma=\left\{\begin{pmatrix} m \\ 0 \end{pmatrix} : m\in Z\right\}$ then M and M' be tow Γ-rings. F = $(F_i)_{i\in N}$ be a family of mappings from a Γ-ring M into Γ-ring M' defined by: $F_n((a\ \ b)) = (f_n(a)\ \ f_n(b))$ then there exists a relating higher (σ, τ) - homomorphism ϕ = $(\phi_i)_{i\in N}$ from a Γ-ring M into Γ-ring M', such that $\phi_n((a\ \ b)) = (\theta_n(a)\ \ \theta_n(b))$.

Let σ_1^n, τ_1^n be two homomorphism of a Γ-ring M, such that $\sigma_1^n, \tau_1^n : M \longrightarrow M \ni$ $\sigma_1^n((a \quad b)) = (\sigma^n(a) \quad \sigma^n(b)), \tau_1^n((a \quad b)) = (\tau^n(a) \quad \tau^n(b))$ then F_n is a generalized higher (σ, τ) - homomorphism.

Lemma (2.6):
Let F = (f_i)$_{i\in N}$ be a generalized Jordan (σ, τ) - higher homomorphism of a Γ-ring M into a Γ-ring M', then for all *a*, *b*, *c* ∈ M, α, β ∈ Γ and for every n ∈ N
If $\sigma^{i^2} = \sigma^i, \tau^{i^2} = \tau^i, \sigma^i\tau^i = \sigma^i\tau^{n-i}$ and σ^i, τ^i are commutative

(i)
$$f_n(a\alpha b\beta a + a\beta b\alpha a) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a)) + \sum_{i=1}^{n} f_i(\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a))$$

(ii)
$$f_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c)) + \sum_{i=1}^{n} f_i(\sigma^i(c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))$$

If M' is a 2-torsion free commutative Γ-ring

(iii)
$$f_n(a\alpha b\alpha c) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(c))$$

(iv)
$$f_n(a\alpha b\alpha c + c\alpha b\alpha a) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(c)) + \sum_{i=1}^{n} f_i(\sigma^i(c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a))$$

Proof:
(i) Replacing *a*β*b* + *b*β*a* for *b* in the definition (2.2), we get:

$$f_n(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(a\beta b + b\beta a)) + \sum_{i=1}^{n} f_i(\sigma^i(a\beta b + b\beta a))\alpha\phi_i(\tau^i(a))$$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(a)\beta\tau^i(b) + \tau^i(b)\beta\tau^i(a)) + \sum_{i=1}^{n} f_i(\sigma^i(a)\beta\sigma^i(b) + \sigma^i(b)\beta\sigma^i(a))\alpha\phi_i(\tau^i(a))$$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\left(\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a)\beta\phi_j(\tau^{j^2}(b)) + \sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b)\beta\phi_j(\tau^{j^2}(a))\right) +$$
$$\sum_{i=1}^{n}\left(\sum_{j=1}^{i} f_j(\sigma^{j^2}(a)\beta\phi_j(\tau^j\sigma^j(b)) + \sum_{j=1}^{i} f_j(\sigma^{j^2}(b)\beta\phi_j(\tau^j\sigma^j(a)\right)\alpha\phi_i(\tau^i(a))$$

Since $\sigma^{i^2} = \sigma^i$, $\tau^{i^2} = \tau^i$, $\sigma^i\tau^i = \sigma^i\tau^{n-i}$ and σ^i, τ^i are commutative

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(a))\beta\phi_i(\tau^i(b)) + \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a)) +$$
$$\sum_{i=1}^{n} f_i(\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a)) + \sum_{i=1}^{n} f_i(\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(a))\alpha\phi_i(\tau^i(a))$$

…(1)

On the other hand:

$$f_n(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a) = f_n(a\alpha a\beta b + a\alpha b\beta a + a\beta b\alpha a + b\beta a\alpha a)$$
$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(a))\beta\phi_i(\tau^i(b)) + \sum_{i=1}^{n} f_i(\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(a))\alpha\phi_i(\tau^i(a)) +$$
$$f_n(a\alpha b\beta a + a\beta b\alpha a)$$

...(2)

Comparing (1) and (2), we get:

$$f_n(a\alpha b\beta a + a\beta b\alpha a) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a)) +$$
$$\sum_{i=1}^{n} f_i(\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a))$$

(ii) Replace $a + c$ for a in the definition (2.3), we get:

$$f_n((a+c)\alpha b\beta(a+c)) = \sum_{i=1}^{n} f_i(\sigma^i(a+c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a+c))$$
$$= \sum_{i=1}^{n} f_i(\sigma^i(a) + \sigma^i(c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a) + \tau^i(c))$$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a)\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a)) + \sum_{i=1}^{n} f_i(\sigma^i(a)\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c))$$
$$= \sum_{i=1}^{n} f_i(\sigma^i(c)\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a)) + \sum_{i=1}^{n} f_i(\sigma^i(c)\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c))$$

…(1)

On the other hand:

$$f_n((a+c)\alpha b\,\beta(a+c)) = f_n(a\alpha b\beta a + a\alpha b\beta c + c\alpha b\beta a + c\,\alpha b\beta a)$$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\,\phi_i(\tau^i(a)) + \sum_{i=1}^{n} f_i(\sigma^i(c))\alpha\,\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c)) +$$
$$f_n(a\alpha b\beta c + c\alpha b\beta a)$$

…(2)

Comparing (1) and (2), we get:

$$f_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\,\phi_i(\tau^i(c)) +$$
$$\sum_{i=1}^{n} f_i(\sigma^i(c))\alpha\,\phi_i(\sigma^i\tau^{n-i}(b))\beta\,\phi_i(\tau^i(a))$$

(iii) Replace α for β in (ii), we get:

$$f_n(a\alpha b\,\alpha c + c\,\alpha b\,\alpha a) = f_n(a\alpha b\,\alpha c + a\alpha b\,\alpha c) = 2f_n(a\alpha b\,\alpha c)$$
$$= 2\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\,\phi_i(\sigma^i\tau^{n-i}(b))\alpha\,\phi_i(\tau^i(c))$$

Since M' is a 2-torsion free Γ-ring

$$f_n(a\alpha b\alpha c) = f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\,\phi_i(\tau^i(c))$$

(iv) Replace α for β in (ii), we get:

$$f_n(a\alpha b\alpha c + c\alpha b\alpha a) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\,\phi_i(\tau^i(c)) +$$
$$\sum_{i=1}^{n} f_i(\sigma^i(c))\alpha\,\phi_i(\sigma^i\tau^{n-i}(b))\alpha\,\phi_i(\tau^i(a))$$

Definition (2.7):

Let F = (f_i)$_{i\in N}$ be a generalized Jordan triple (σ, τ) - higher homomorphism from a Γ-ring M into a Γ-ring M', then for all a, b ∈ M, α ∈ Γ and n ∈ N, we define $\delta_n(a, b)_\alpha$: M×Γ×M ⟶ M' by:

$$\delta_n(a,b)_\alpha = f_n(a\alpha b) - \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))$$

Lemma (2.8):

Let F = (f_i)$_{i\in N}$ be a generalized Jordan (σ, τ) - higher homomorphism from a Γ-ring M into a Γ-ring M', then for all a, b, c ∈ M, α, β ∈ Γ and n ∈ N:

(i) $\delta_n(a,b)_\alpha = -\delta_n(b,a)_\alpha$

(ii) $\delta_n(a + b,c)_\alpha = \delta_n(a,c)_\alpha + \delta_n(b,c)_\alpha$

(iii) $\delta_n(a,b + c)_\alpha = \delta_n(a,b)_\alpha + \delta_n(a,c)_\alpha$

(iv) $\delta_n(a,b)_{\alpha+\beta} = \delta_n(a,b)_\alpha + \delta_n(a,b)_\beta$

Proof:

(i) $$f_n(a\alpha b+b\alpha a)=\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))+\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\tau^i(a))$$

$$f_n(a\alpha b)-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))=-f_n(b\alpha a)-\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\tau^i(a))$$

$\delta_n(a,b)_\alpha = -\,\delta_n(b,a)_\alpha$

(ii) $$\delta_n(a+b,c)_\alpha = f_n((a+b)\alpha c)-\sum_{i=1}^{n} f_i(\sigma^i(a+b))\alpha\phi_i(\tau^i(c))$$

$$=f_n(a\alpha c+b\alpha c)-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(c))-\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\tau^i(c))$$

$$=f_n(a\alpha c)-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(c))+f_n(b\alpha c)-\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\tau^i(c))$$

$= \delta_n(a,c)_\alpha + \delta_n(b,c)_\alpha$

(iii) $$\delta_n(a,b+c)_\alpha = f_n(a\alpha(b+c))-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b+c))$$

$$=f_n(a\alpha b+a\alpha c))-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(c))$$

$$=f_n(a\alpha b)-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))+f_n(a\alpha c))-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(c))$$

$= \delta_n(a,b)_\alpha + \delta_n(a,c)_\alpha$

(iv) $$\delta_n(a,b)_{\alpha+\beta} = f_n(a(\alpha+\beta)b)-\sum_{i=1}^{n} f_i(\sigma^i(a))(\alpha+\beta)\phi_i(\tau^i(b))$$

$$=f_n(a\alpha b)-\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))+f_n(a\beta b)-\sum_{i=1}^{n} f_i(\sigma^i(a))\beta\phi_i(\tau^i(b))$$

$= \delta_n(a,b)_\alpha + \delta_n(a,b)_\beta$

Remark (2.9):
Note that $F = (f_i)_{i\in N}$ is a generalized (σ, τ) - higher homomorphism from a Γ-ring M into a Γ-ring M' if and only if $\delta_n(a,b)_\alpha = 0$ for all $a, b \in M$, $\alpha \in \Gamma$ and $n \in N$.

Lemma (2.10):
Let $F = (f_i)_{i\in N}$ be a generalized Jordan (σ, τ) - higher homomorphism of a 2-torsion free Γ-ring M into a Γ-ring M', such that $\sigma^{n^2}=\sigma^n$, $\tau^n\sigma^n=\sigma^n$, $\sigma^i\tau^{n-i}=\tau^i\sigma^i$ and σ^i, τ^i for all $i \in N$ are commutative, then for all $a, b, m \in M$, $\alpha, \beta \in \Gamma$ and $n \in N$

(i) $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$
$\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$

(ii) $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(b),\tau^n(a))_\alpha +$
$\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(a),\tau^n(b))_\alpha = 0$

(iii) $\delta_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(b),\tau^n(a))_\beta +$
$\delta_n(\sigma^n(b),\sigma^n(a))_\beta\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(a),\tau^n(b))_\beta = 0$

Proof:

(i) We prove by using the induction, we can assume that:
$\delta_s(\sigma^s(a),\sigma^s(b))_\alpha\beta\phi_s(\sigma^s(m))\beta G_s(\tau^s(b),\tau^s(a))_\alpha +$
$\delta_s(\sigma^s(b),\sigma^s(a))_\alpha\beta\phi_s(\sigma^s(m))\beta G_s(\tau^s(a),\tau^s(b))_\alpha = 0$ for all a, b, m$\in$ M, and s, n $\in$ N, s < n.

Let w = $a\alpha b\beta m\beta b\alpha a + b\alpha a\beta m\beta a\alpha b$, since F is a generalized Jordan (σ, τ) - higher homomorphism
$f_n(w) = f_n(a\alpha(b\beta m\beta b)\alpha a + b\alpha(a\beta m\beta a)\alpha b)$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b\beta m\beta b))\alpha\phi_i(\tau^i(a)) +$$
$$\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\sigma^i\tau^{n-i}(a\beta m\beta a))\alpha\phi_i(\tau^i(b))$$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\left(\sum_{j=1}^{i}\phi_j(\sigma^j(\sigma^j\tau^{n-j}(b)))\beta\phi_j(\sigma^j\tau^{n-j}(\sigma^j\tau^{n-j}(m)))\beta\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\right)\alpha\phi_i(\tau^i(a)) +$$
$$\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\left(\sum_{j=1}^{i}\phi_j(\sigma^j(\sigma^j\tau^{n-j}(a)))\beta\phi_j(\sigma^j\tau^{n-j}(\sigma^j\tau^{n-j}(m)))\beta\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\right)\alpha\phi_i(\tau^i(b))$$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(b)))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a)) +$$
$$\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(a)))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(\sigma^i\tau^{n-i}(a))\alpha\phi_i(\tau^i(b))$$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(b)))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\alpha\phi_j(\tau^j(a)) +$$
$$\sum_{i=1}^{n} f_i(\sigma^i(b))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(a)))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\alpha\phi_j(\tau^j(b))$$

$$=f_n(\sigma^n(a))\alpha\phi_n(\sigma^n(\sigma^n(b)))\beta\phi_n(\sigma^n(\sigma^n(m)))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\alpha\phi_j(\tau^j(a))+$$

$$\sum_{i=1}^{n-1}f_i(\sigma^i(a))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\alpha\phi_j(\tau^j(a))+$$

$$f_n(\sigma^n(b))\alpha\phi_n(\sigma^n(\sigma^n(a)))\beta\phi_n(\sigma^n(\sigma^n(m)))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\alpha\phi_j(\tau^j(b))+$$

$$\sum_{i=1}^{n-1}f_i(\sigma^i(b))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(a))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\alpha\phi_j(\tau^j(b)) \quad \ldots(1)$$

On the other hand:

$f_n(w) = f_n((a\alpha b)\beta m\beta(b\alpha a) + (b\alpha a)\beta m\beta(a\alpha b))$

$$=\sum_{i=1}^{n}f_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(b\alpha a))+$$

$$\sum_{i=1}^{n}f_i(\sigma^i(b\alpha a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))$$

$$=\sum_{i=1}^{n}f_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^j\tau^{n-j}(m))\beta\left(\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a))\alpha\phi_j(\tau^{j^2}(b))+\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b))\alpha\phi_j(\tau^{j^2}(a))-\right.$$

$$\left.\phi_i(\tau^i(a\alpha b))\right)+\sum_{i=1}^{n}\left(\sum_{j=1}^{i}f_j(\sigma^{j^2}(a))\alpha\phi_j(\tau^j\sigma^j(b))+\sum_{j=1}^{i}f_j(\sigma^{j^2}(b))\alpha\phi_j(\tau^j\sigma^j(a))-\right.$$

$$\left.f_i(\sigma^i(a\alpha b))\right)\beta\phi_i((\sigma^j\tau^{n-j}(m))\beta\phi_i(\tau^i(a\alpha b))$$

$$=\sum_{i=1}^{n}f_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^j\tau^{n-j}(m))\beta\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a))\alpha\phi_j(\tau^{j^2}(b))+\sum_{i=1}^{n}f_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^j\tau^{n-j}(m))\beta\cdot$$

$$\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b))\alpha\phi_j(\tau^{j^2}(a))-\sum_{i=1}^{n}f_i(\sigma^i(a\alpha b)\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))+$$

$$\sum_{i=1}^{n}f_i(\sigma^{i^2}(a))\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))+\sum_{i=1}^{n}f_i(\sigma^{i^2}(b))\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\cdot$$

$$\phi_i(\tau^i(a\alpha b))-\sum_{i=1}^{n}f_i(\sigma^i(a\alpha b)\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))$$

$$= -\sum_{i=1}^{n} f_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^j\tau^{n-j}(m))\beta(\phi_i(\tau^i(a\alpha b)) - \sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a))\alpha\phi_j(\tau^{j^2}(b)) -$$

$$\sum_{i=1}^{n} f_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta(\phi_i(\tau^i(a\alpha b)) - \sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b))\alpha\phi_j(\tau^{j^2}(a)) +$$

$$\sum_{i=1}^{n} f_i(\sigma^{i^2}(a))\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b)) +$$

$$\sum_{i=1}^{n} f_i(\sigma^{i^2}(b))\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))$$

$$= -\sum_{i=1}^{n} f_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a), \tau^i(b))_\alpha -$$

$$\sum_{i=1}^{n} f_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b), \tau^i(a))_\alpha +$$

$$\sum_{i=1}^{n} f_i(\sigma^{i^2}(a))\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b)) +$$

$$\sum_{i=1}^{n} f_i(\sigma^{i^2}(b))\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))$$

$$= -f_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a), \tau^n(b))_\alpha -$$

$$\sum_{i=1}^{n-1} f_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a), \tau^i(b))_\alpha -$$

$$f_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b), \tau^n(a))_\alpha -$$

$$\sum_{i=1}^{n-1} f_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b), \tau^i(a))_\alpha +$$

$$f_n(\sigma^{n^2}(a))\phi_n(\tau^n\sigma^n(b))\beta\phi_n((\sigma^n(m))\beta\phi_n(\tau^n(a\alpha b)) +$$

$$\sum_{i=1}^{n-1} f_i(\sigma^{i^2}(a)\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b)) +$$

$$f_n(\sigma^{n^2}(b))\alpha\phi_n(\tau^n\sigma^n(a))\beta\phi_n((\sigma^n(m))\beta\phi_n(\tau^n(a\alpha b)) +$$

$$\sum_{i=1}^{n-1} f_i(\sigma^{i^2}(b)\alpha\beta\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b)) \qquad \ldots(2)$$

Compare (1), (2) and since $\sigma^{n^2} = \sigma^n$, $\tau^n\sigma^n = \sigma^n$, $\sigma^i\tau^{n-i} = \tau^i\sigma^i$ and σ^i, τ^i are commutative

$$
\begin{aligned}
0 = &-f_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha - \\
&f_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha + \\
&f_n(\sigma^n(a))\alpha\phi_n(\sigma^n(b))\beta\phi_n((\sigma^n(m))\beta(\phi_n(\tau^n(a\alpha b)) - \\
&\sum_{i=1}^{n}\phi_i(\tau^i(\sigma^i\tau^{n-i}(a)))\alpha\phi_i(\tau^i(b)) + f_n(b)\alpha\phi_n(\sigma^n(a))\beta(\phi_n((\sigma^n(m))\beta(\phi_n(\tau^n(a\alpha b)) - \\
&\sum_{i=1}^{n}\phi_i(\tau^i(\sigma^i\tau^{n-i}(a)))\alpha\phi_i(\tau^i(b)) - \sum_{i=1}^{n-1} f_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha - \\
&\sum_{i=1}^{n-1} f_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha + \\
&\sum_{i=1}^{n-1} f_i(\sigma^i(a)\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta(\phi_i(\tau^i(a\alpha b)) - \\
&\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\alpha\phi_j(\tau^i(a))) + \sum_{i=1}^{n-1} f_i(\sigma^i(b)\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta(\phi_i(\tau^i(a\alpha b)) - \\
&\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\alpha\phi_j(\tau^i(b)))
\end{aligned}
$$

$$
\begin{aligned}
= &-f_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha - \\
&f_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha + \\
&f_n(\sigma^n(a))\alpha\phi_n(\sigma^n(b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha + \\
&f_n(\sigma^n(b))\alpha\phi_n(\sigma^n(a))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha - \\
&\sum_{i=1}^{n-1} f_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha - \\
&\sum_{i=1}^{n-1} f_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha + \\
&\sum_{i=1}^{n-1} f_i(\sigma^i(a)\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha + \\
&\sum_{i=1}^{n-1} f_i(\sigma^i(b)\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha
\end{aligned}
$$

$$
\begin{aligned}
= &-\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha - \\
&\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha - \\
&\sum_{i=1}^{n-1}\delta_i(\sigma^i(b),\sigma^i(a))_\alpha\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha - \\
&\sum_{i=1}^{n-1}\delta_i(\sigma^i(a),\sigma^i(b))_\alpha\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha
\end{aligned}
$$

$$= -(\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha +$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha -$$
$$\sum_{i=1}^{n-1} f_i(\sigma^i(b),\sigma^i(a))_\alpha\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha +$$
$$\sum_{i=1}^{n-1} f_i(\sigma^i(a),\sigma^i(b))_\alpha\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha)$$

By our hypothesis, we have:
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$$

(ii) Replace β by α in (i), we get (ii).

(iii) Interchanging α and β in (i), we get (iii).

Lemma (2.11):
Let F = (f_i)$_{i\in N}$ be a generalized Jordan (σ, τ) - higher homomorphism from a Γ-ring M into a prime Γ-ring M', then for all *a*, *b*, m ∈ M, α, β∈Γ and n ∈ N

(i) $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha =$
$\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$

(ii) $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(b),\tau^n(a))_\alpha =$
$\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(a),\tau^n(b))_\alpha = 0$

(iii) $\delta_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(b),\tau^n(a))_\beta =$
$\delta_n(\sigma^n(b),\sigma^n(a))_\beta\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(a),\tau^n(b))_\beta = 0$

Proof:
(i) By lemma (2.10) (i), we have:
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$$
Since by lemma (let M be a 2-torsion free semiprime Γ-ring and suppose that *a*, *b* ∈ M if *a*ΓmΓ*b* + *b*ΓmΓ*a* = 0 for all m ∈ M, then *a*ΓmΓ*b* = *b*ΓmΓ*a* = 0), we get:
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha =$$
$$\delta_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$$

(ii) Replace α for β in (i), we obtain (ii).

(iii) Interchanging α and β in (i), we obtain (iii).

Theorem (2.12):
Let F = (f_i)$_{i\in N}$ be a generalized Jordan (σ, τ) - higher homomorphism from a Γ-ring M into a prime Γ-ring M', then for all *a, b, c, d*, m ∈ M, α, β∈Γ and n ∈ N

(i) $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(c))_\alpha=0$

(ii) $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha=0$

(iii) $\delta_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta=0$

Proof:

(i) Replacing $a+c$ for a in lemma (2.11) (i), we get:

$$\delta_n(\sigma^n(a+c),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a+c))_\alpha=0$$

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha+$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(c))_\alpha+$$
$$\delta_n(\sigma^n(c),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha+$$
$$\delta_n(\sigma^n(c),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(c))_\alpha=0$$

By lemma (2.11)(i), we get:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(c))_\alpha+$$
$$\delta_n(\sigma^n(c),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha=0$$

Therefore, we get:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(c))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(c))_\alpha=0$$

$$=-\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(c))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta$$
$$\delta_n(\sigma^n(c),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha=0$$

Since M' is a prime Γ-ring and therefore:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(c))_\alpha=0 \qquad \ldots(1)$$

Replacing $b+d$ for b in lemma (2.11) (i), we get:

$$\delta_n(\sigma^n(a),\sigma^n(b+d))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b+d),\tau^n(a))_\alpha=0$$

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha+$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(a))_\alpha+$$
$$\delta_n(\sigma^n(a),\sigma^n(d))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha+$$
$$\delta_n(\sigma^n(a),\sigma^n(d))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(a))_\alpha=0$$

By lemma (2.11) (i), we get:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(a))_\alpha+$$
$$\delta_n(\sigma^n(a),\sigma^n(d))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha=0$$

Therefore, we get:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(a))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(a))_\alpha=0$$

$$= -\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(a))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta$$
$$\delta_n(\sigma^n(a),\sigma^n(d))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha = 0$$

Since M' is a prime Γ-ring and therefore:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(a))_\alpha = 0 \qquad \ldots(2)$$

Now, $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b+d),\tau^n(a+c))_\alpha = 0$

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(b),\tau^n(c))_\alpha +$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(a))_\alpha +$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(c))_\alpha = 0$$

Since by lemma (2.11) and (1) and (2), we get:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(\mathrm{m}))\beta G_n(\tau^n(d),\tau^n(c))_\alpha = 0\cdot$$

(ii) Replace α for β in (1), we obtain (ii).

(iii) Replacing α + β for α in (ii), we get:

$$\delta_n(\sigma^n(a),\sigma^n(b))_{\alpha+\beta}\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_{\alpha+\beta} = 0$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha +$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta +$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha +$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0$$

By (i) and (ii), we get:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta +$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha = 0$$

Therefore, we have:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0$$

$$= -\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha$$
$$\delta_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0$$

Since M' is a prime Γ-ring, then:

$$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(\mathrm{m}))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0\cdot$$

3- The Main Results

Theorem(3.1):
Every generalized Jordan (σ, τ) - higher homomorphism from a Γ-ring M into a prime Γ-ring M' is either generalized (σ, τ) - higher homomorphism or (σ, τ) - higher anti- homomorphism.

Proof:
Let F = (f_i)$_{i\in N}$ be a generalized Jordan (σ, τ) - higher homomorphism of a Γ-ring M into a prime Γ-ring M', then by theorem (2.12) (i);
$\delta_n(\sigma^n(a),\sigma^n(b))_\alpha \beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(c))_\alpha = 0$.
Since M' is a prime Γ-ring therefore either $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha = 0$ or $G_n(\tau^n(d),\tau^n(c))_\alpha = 0$, for all *a, b, c, d* ∈ M, α, β∈Γ and n ∈ N.
If $G_n(\tau^n(d),\tau^n(c))_\alpha \neq 0$ for all *c, d* ∈ M, α∈Γ and n ∈ N then $\delta_n(\sigma^n(a),\sigma^n(b))_\alpha = 0$.Hence, we get F is a generalized (σ, τ) - higher homomorphism.
But if $G_n(\tau^n(d),\tau^n(c))_\alpha = 0$ for all *c, d* ∈ M, α∈Γ and n ∈ N then we get F is a (σ, τ) - higher anti- homomorphism.

Proposition (3.2):
Let F = (f_i)$_{i\in N}$ be a generalized Jordan (σ, τ) - higher homomorphism from a Γ-ring M into 2-torsion free Γ-ring M', such that *a*α*b*β*a* = *a*β*b*α*a*, for all *a*, *b* ∈ M and α, β∈Γ, *a*'α*b*'β*a*' = *a*'β*b*'α*a*', for all *a*', *b*' ∈ M' and α, β ∈ Γ, $\sigma^{i^2} = \sigma^i$, $\tau^{i^2} = \tau^i$, $\sigma^i\tau^i = \sigma^i\tau^{n-i}$ and σ^i, τ^i are commutative for all i∈ N, then F is a generalized Jordan triple (σ, τ) - higher homomorphism.

Proof:
Replace *a*β*b* + *b*β*a* for *b* in the definition (2.2), we get:

$$f_n(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a)$$
$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(a\beta b + b\beta a)) + \sum_{i=1}^{n} f_i(\sigma^i(a\beta b + b\beta a))\alpha\phi_i(\tau^i(a))$$
$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\tau^i(a)\beta\tau^i(b) + \tau^i(b)\beta\tau^i(a)) + \sum_{i=1}^{n} f_i(\sigma^i(a)\beta\sigma^i(b) + \sigma^i(b)\beta\sigma^i(a))\alpha\phi_i(\tau^i(a))$$
$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\left(\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a)\beta\phi_j(\tau^{j^2}(b)) + \sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b)\beta\phi_j(\tau^{j^2}(a))\right) +$$
$$\sum_{i=1}^{n}\left(\sum_{j=1}^{i} f_j(\sigma^{j^2}(a)\beta\phi_j(\tau^j\sigma^j(b)) + \sum_{j=1}^{i} f_j(\sigma^{j^2}(b)\beta\phi_j(\tau^j\sigma^j(a))\right)\alpha\phi_i\tau^i(a))$$

Since $a'\alpha b'\beta a' = a'\beta b'\alpha a'$, for all a', $b' \in$ M' and α, $\beta \in \Gamma$, $\sigma^{i^2} = \sigma^i$, $\tau^{i^2} = \tau^i$, $\sigma^i\tau^i = \sigma^i\tau^{n-i}$ and σ^i, τ^i are commutative.

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(a)\beta\phi_i(\tau^i(b)) + 2\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b)\beta\phi_i(\tau^i(a)) + \sum_{i=1}^{n} f_i(\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(a)\alpha\phi_i(\tau^i(a) \quad \ldots(1)$$

On the other hand:

$$f_n(a\,\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a) = f_n(a\,\alpha a\beta b + a\,\alpha b\beta a + a\beta b\,\alpha a + b\beta a\alpha a)$$

Since $a\alpha b\beta a = a\beta b\alpha a$, for all a, $b \in$ M and α, $\beta \in \Gamma$

$$= \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(a)\beta\phi_i(\tau^i(b)) + \sum_{i=1}^{n} f_i(\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(a)\alpha\phi_i(\tau^i(a)) + 2f_n(a\alpha b\beta a) \quad \ldots(2)$$

Compare (1) and (2), we get:

$$2f_n(a\alpha b\beta a) = 2\sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b)\beta\phi_i(\tau^i(a))$$

Since M' is a 2-torsion free Γ-ring, we get:

$$f_n(a\alpha b\beta a) = \sum_{i=1}^{n} f_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b)\beta\phi_i(\tau^i(a)).$$

Hence, F is a generalized Jordan triple (σ,τ) - higher homomorphism.

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