Complementary Nil Domination in

Intuitionistic Fuzzy Graph

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Abstract

In this paper, We define complementary nil dominating set and its numbers of Intuitionistic Fuzzy Graph. The bound on this number are obtained for some standard Intuitionistic fuzzy graphs are derived. Some theorems related to the above concepts are studied.

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1. Introduction


In this paper, We define complementary nil dominating set and its numbers of Intuitionistic Fuzzy Graphs. The bound on this number are obtained for some standard Intuitionistic fuzzy graphs. Some theorems related to the above concepts are studied and this concept is useful in networking analysis.

2. Preliminaries

**Definition 2.1:** A fuzzy graph \( G = (\sigma, \mu) \) is a pair of functions \( \sigma : V \rightarrow [0,1] \) and \( \mu : V \times V \rightarrow [0,1] \), where for all \( u, v \in V \), we have \( \mu(u,v) \leq \sigma(u) \wedge \sigma(v) \).

**Definition 2.2:** An Intuitionistic fuzzy graph is of the form \( G = (V, E) \) where

(i) \( V=\{v_1,v_2,\ldots,v_n\} \) such that \( \mu_1 : V \rightarrow [0,1] \) and \( \gamma_1 : V \rightarrow [0,1] \) denote the degree of membership and non-membership of the element \( v_i \in V \), respectively, and \( 0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1 \) for every \( v_i \in V \), \( (i = 1,2, \ldots, n) \),

(ii) \( E \subseteq V \times V \) where \( \mu_2 : V \times V \rightarrow [0,1] \) and \( \gamma_2 : V \times V \rightarrow [0,1] \) are such that

\[
\mu_2(v_i, v_j) \leq \min \{\mu_1(v_i), \mu_1(v_j)\} \quad \text{and} \quad \gamma_2(v_i, v_j) \leq \max \{\gamma_1(v_i), \gamma_1(v_j)\}
\]

and \( 0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1 \) for every \( (v_i, v_j) \in E \), \( (i, j = 1,2, \ldots, n) \)

**Definition 2.3** An IFG \( H = < V', E' > \) is said to be an Intuitionistic fuzzy subgraph (IFSG) of the IFG, \( G = < V, E > \) if \( V' \subseteq V \) and \( E' \subseteq E \). In other words, if \( \mu_{1i}' \leq \mu_{1i} ; \gamma_{1i}' \geq \gamma_{1i} \) and
Definition 2.4: Let $G = (V, E)$ be a IFG. Then the cardinality of $G$ is defined as

$$|G| = \left| \sum_{v \in V} \frac{1+\mu_1(v) - \gamma_1(v)}{2} + \sum_{v \in V, \forall \in E} \frac{1+\mu_2(\forall v \in E) - \gamma_2(v \in E)}{2} \right|$$

Definition 2.5: The vertex cardinality of IFG $G$ is defined by $|V| = \left| \sum_{v \in V} \frac{1+\mu_1(v) - \gamma_1(v)}{2} \right| = p$ and

The edge cardinality of IFG $G$ is defined by $|E| = \left| \sum_{v \in V, \forall \in E} \frac{1+\mu_2(\forall v \in E) - \gamma_2(v \in E)}{2} \right| = q$

The vertex cardinality of IFG is called the order of $G$ and denoted by $O(G)$. The cardinality of $G$ is called the size of $G$, denoted by $S(G)$.

Definition 2.6: An edge $e = (x, y)$ of an IFG $G = (V, E)$ is called an effective edge if $\mu_2(x, y) = \mu_1(x) \land \mu_1(y)$ and $\gamma_2(x, y) = \gamma_1(x) \lor \gamma_1(y)$.

Definition 2.7: An Intuitionistic fuzzy graph is complete if $\mu_2(v) = \mu_1(v) \land \mu_1(v)$ for all $(v_i, v_j) \in E$. That is every edge is effective edge.

Definition 2.9: The complement of an IFG $G = (V, E)$ is denoted by $\overline{G} = (\overline{V}, \overline{E})$ and is defined as

i) $\overline{\mu}_1(v) = \mu_1(v)$ and $\overline{\gamma}_1(v) = \gamma_1(v)$

ii) $\overline{\mu}_2(u, v) = \mu_1(u) \land \mu_1(v) - \mu_2(u, v)$ and $\overline{\gamma}_2(u, v) = \gamma_1(u) \lor \gamma_1(v) - \gamma_2(u, v)$ for $u,v \in V$

Definition 2.10: Let $G = (V, E)$ be an IFG. The neighbourhood of any vertex $v$ is defined as

$N(v) = (N_\mu(v), N_\gamma(v))$, Where $N_\mu(v) = \{w \in V ; \mu_2(v, w) = \mu_1(v) \land \mu_1(w)\}$ and $N_\gamma(v) = \{w \in V ; \gamma_2(v, w) = \gamma_1(v) \lor \gamma_1(w)\}$. $N[v] = N(v) \cup \{v\}$ is called the closed neighbourhood of $v$.

Definition 2.11: The k-neighbourhood degree of a vertex is defined as $d_N(v) = (d_{N_\mu}(v), d_{N_\gamma}(v))$ where $d_{N_\mu}(v) = \sum_{w \in \overline{E}(v)} \mu_1(w)$ and $d_{N_\gamma}(v) = \sum_{w \in \overline{E}(v)} \gamma_1(w)$.

The minimum neighbourhood degree is defined as $\delta_N(G) = (\delta_{N_\mu}(v), \delta_{N_\gamma}(v))$, where $\delta_{N_\mu}(v) = \Lambda \{ d_{N_\mu}(v) ; v \in V \}$ and $\delta_{N_\gamma}(v) = \Lambda \{ d_{N_\gamma}(v) ; v \in V \}$.

Definition 2.12: The effective degree of a vertex $v$ in a IFG $G = (V, E)$ is defined to be sum of the effective edges incident at $v$, and denoted by $d_{E}(v)$. The minimum effective degree of $G$ is $\delta_E(G) = \Lambda \{ d_{E}(v) / v \in V \}$

Definition 2.13: Let $G = (V, E)$ be an IFG. Let $u,v \in V$, we say that $u$ dominated $v$ in $G$ if there exist a strong arc between them. A subset $D \subseteq V$ is said to be dominating...
set in $G$ if for every $v \in V-D$, there exist $u$ in $D$ such that $u$ dominated $v$. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by $\gamma$. The maximum scalar cardinality of a minimal domination set is called upper domination number and is denoted by the symbol $\Gamma$.

**Definition 2.14:** An independent set of an Intuitionistic fuzzy graph $G = (V, E)$ is a subset $S$ of $V$ such that no two vertices of $S$ are adjacent in $G$.

### 3. Complementary nil domination set in IFG

**Definition 3.1:** Let $G = (V, E)$ be an IFG. A set $S \subseteq V$ is said to be a complementary nil domination set (or simply cnd-set) of an IFG $G$, if $S$ is a dominating set and its complement $V-S$ is not a dominating set. The minimum scalar cardinality over all cnd-set is called a complimentary nil domination number and is denoted by the symbol $\gamma_{cnd}$, the corresponding minimum cnd -set is denote by $\gamma_{cnd}$-set.

**Example 3.2:** Let $G = (V,E)$ be IFG be defined as follows

Here $S_1 = \{v_1, v_2, v_4\}$ and $S_2 = \{v_2, v_3, v_4\}$ are minimal cnd-sets. Here $S_2$ is minimum cardinality and $S_1$ is maximum cardinality. Hence, $\gamma_{cnd}(G) = 1.2$ and $\Gamma_{cnd}(G) = 1.3$

**Definition 3.3:** Let $S \subseteq V$ in the connected IFG $G = (V, E)$. A vertex $u \in S$ is said to be an enclave of $S$ if $\mu_2(u,v) < \min[\mu_1(u), \mu_1(v)]$ and $\gamma_2(u, v) < \max[\gamma_1(u), \gamma_1(v)]$ for all $v \in V-S$. That is $N[u] \subseteq S$.

In the above Fig-1, $v_1$ is enclave of the set $S_1$ and $v_3$ is enclave of the set $S_2$.

**Theorem 3.4:** A dominating set $S$ is a cnd-set if and only if it contains at least one enclave.
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Proof: Let $S$ be a cnd-set of a IFG $G = (V, E)$. The $V-S$ is not a dominating set which implies that there exist a vertex $u \in S$ such that $\mu_2(u, v) < \min \left[ \mu_1(u), \mu_1(v) \right]$ and $\gamma_2(u, v) < \max \left[ \gamma_1(u), \gamma_1(v) \right]$ for all $v \in V-S$. Therefore $u$ is an enclave of $S$.

Hence $S$ contains at least one enclave.

Conversely, Suppose the dominating set $S$ contains enclaves. Without loss of generality let us take $u$ be the enclave of $S$. That is $\mu_2(u, v) < \min \left[ \mu_1(u), \mu_1(v) \right]$ and $\gamma_2(u, v) < \max \left[ \gamma_1(u), \gamma_1(v) \right]$ for all $v \in V-S$. Hence $V-S$ is not a dominating set.

Hence dominating set $S$ is cnd-set.

Remark 3.5:

1. For any IFG $G = (V, E)$
2. Every super set of a cnd-set is also a cnd-set.
3. Complement of a cnd-set is not a cnd-set.
4. Complement of a domination set is not a cnd-set.
5. A cnd-set is not independent set.

Theorem 3.6: In any Intuitionistic fuzzy graph $G = (V, E)$, every complementary nil dominating set of $G$ intersects with every dominating set of $G$.

Proof: Let $S$ be a $\gamma$-cnd-set and $D$ be a $\gamma$-set of $G = (V, E)$.

Suppose $S \cap D = \emptyset$, then $D \subseteq V-S$ and $V-S$ contains a dominating set $D$.

Therefore $V-S$, a super set of $D$, is a dominating set.

Which is contradiction to our assumption.

Hence $S \cap D \neq \emptyset$.

Theorem 3.7: If $S$ is a cnd-set of an IFG $G = (V, E)$, then there is a vertex $u \in S$ such that $S - \{u\}$ is a dominating set.

Proof: Let $S$ be a cnd-set. By theorem 3.4, Every cnd-set contains at least one enclave of $S$.

Let $u \in S$ be an enclave of $S$. Then $\mu_2(u, v) < \min \left[ \mu_1(u), \mu_1(v) \right]$ and $\gamma_2(u, v) < \max \left[ \gamma_1(u), \gamma_1(v) \right]$ for all $v \in V-S$.

Since $G$ is connected IFG, there exist at least a vertex $w \in S$ such that $\mu_2(u, v) = \min \left[ \mu_1(u), \mu_1(v) \right]$ and $\gamma_2(u, v) = \max \left[ \gamma_1(u), \gamma_1(v) \right]$.

Hence $S - \{u\}$ is a dominating set.

Theorem 3.8: A cnd-set in an IFG $G = (V, E)$ is not a singleton.

Proof: Let $S$ be a cnd-set. By theorem 3.4, Every cnd-set contains at least one enclave of $S$.

Let $u \in S$ be an enclave of $S$. Then $\mu_2(u, v) < \min \left[ \mu_1(u), \mu_1(v) \right]$ and $\gamma_2(u, v) < \max \left[ \gamma_1(u), \gamma_1(v) \right]$ for all $v \in V-S$.

Suppose $S$ contains only one vertex $u$, then it must be isolated in $G$. 
This is contradiction to connectedness.
Hence cnd-set contains more than one vertex.

**Theorem 3.9:** Let G = (V, E) be an IFG and S be a $\gamma_{cnd}$-set of G. If u and v are two enclaves in S, then
(i) $N[u] \cap N[v] \neq \emptyset$ and
(ii) u and v are adjacent. That is, $\mu_2 (u,v) = \min [\mu_1(u), \mu_1(v)]$ and $\gamma_2 (u,v) = \max [\gamma_1(u), \gamma_1(v)]$.

**Proof:** Let S be a minimum cnd-set of Intuitionistic fuzzy graph G = (V,E).
Let u and v are two enclaves of S.
Suppose $N[u] \cap N[v] = \emptyset$, then u is an enclave of S – N(v) which implies that $V - (S - N(v))$ is not a dominating set.
Therefore $S - N(v)$ is a cnd-set of G and $|S - N(v)| < |S| = \gamma_{cnd}(G)$.
Which is contradiction to the minimality of S.
Then $N[u] \cap N[v] \neq \emptyset$.

Suppose $\mu_2 (u,v) < \min [\mu_1(u), \mu_1(v)]$ and $\gamma_2 (u,v) < \max [\gamma_1(u), \gamma_1(v)]$, that is u and v are non-adjacent. Then u $\notin N(v)$ and u is an enclave of S-{v}.

Therefore, we have $\Gamma(G) + \gamma_{cnd}(G) \leq O(G) + \max \{|v_i|\}$ for every $v_i \in V$.

**Theorem 3.10:** For any Intuitionistic fuzzy graph G= (V, E),

$\Gamma(G) + \gamma_{cnd}(G) \leq O(G) + \max \{|v_i|\}$ for every $v_i \in V$.

**Proof:**
If G is a Intuitionistic fuzzy graph then cnd-set $\subseteq V$ and super set of $\gamma$-set.

i.e) $\gamma_{cnd}(G) \leq O(G)$ and $\Gamma(G) = \max \{|v_i|\}$

i.e) $\Gamma(G) \leq \max \{|v_i|\}$

Therefore, we have $\Gamma(G) + \gamma_{cnd}(G) \leq O(G) + \max \{|v_i|\}$.

**Example 3.11:**
Let G= (V, E) be IFG with V={a, b, c, d, e, f} and defined by $(\mu_1(a), \gamma_1(a)) = (0.2, 0.4), (\mu_1(b), \gamma_1(b)) = (0.1, 0.5), (\mu_1(c), \gamma_1(c)) = (0.4, 0.5), (\mu_1(d), \gamma_1(d)) = (0.3, 0.6), (\mu_2(e), \gamma_2(e)) = (0.4, 0.6), (\mu_1(f), \gamma_1(f)) = (0.3, 0.5)$, and

$(\mu_2(a, b), \gamma_2(a, b)) = (0.1, 0.5), (\mu_2(a, d), \gamma_2(a, d)) = (0.2, 0.6), (\mu_2(a, f), \gamma_2(a, f)) = (0.1, 0.2), (\mu_2(b, e), \gamma_2(b, e)) = (0.1, 0.6), (\mu_2(c, d), \gamma_2(c, d)) = (0.3, 0.6), (\mu_2(c, f), \gamma_2(c, f)) = (0.3, 0.5), (\mu_2(f, e), \gamma_2(f, e)) = (0.3, 0.6)$.

Here, S1 = {a, b, c, d} and S2= {b, c, e, f} are minimal complementary nil dominating set. Also minimal $\gamma$-set = {b, c} and $\delta_N(G) = \{0.4, 0.9\}$

(i) $|a| = 0.4, |b| = 0.3, |c| = 0.45, |d| = 0.35, |e| = 0.4$ and $|f| = 0.4$. $O(G) = p = 2.3$
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(ii) $\gamma_{cnd}(G) = 1.5$ and $\Gamma_{cnd}(G) = 1.55$
(iii) $\gamma(G) = \Gamma(G) = 0.75$
(iv) The vertices a and d are two enclaves with respect to $S_1$. The vertices $e$ and $f$ are two enclaves with respect to $S_2$.
(v) $N[a] = \{a, b, d\}$ $N[d] = \{a, c, d\}$ i.e) $N[a]\cap N[d] \neq \emptyset$. Also a and d are adjacent
(vi) The vertex $d \in S_1$ and $S_1 - d$ is a dominating set.
(vii) $\min \{ |v_i| \} = 0.3, \max \{ |v_i| \} = 0.45$ for every $v_i \in V$. Also $\Gamma(G) + \gamma_{cnd}(G) = 0.75 + 1.5 = 2.25$ and $O(G) + \max \{ |v_i| \} = 2.3 + 0.45 = 2.75$. i.e) $\Gamma(G) + \gamma_{cnd}(G) \leq O(G) + \max \{ |v_i| \}$

4. Conclusion

Here we defined complementary nil dominating set and its numbers of Intuitionistic Fuzzy Graphs. The bound on this number are obtained for some standard Intuitionistic fuzzy graphs. Some results related to the above concepts are studied.

References


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