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Complementary Nil Domination in Intuitionistic Fuzzy Graph

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Abstract

In this paper, We define complementary nil dominating set and its numbers of Intuitionistic Fuzzy Graph. The bound on this number are obtained for some standard Intuitionistic fuzzy graphs are derived. Some theorems related to the above concepts are studied.

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1. Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Research on the theory of intuitionistic fuzzy sets (IFSs) has been witnessing an exponential growth in Mathematics and its applications. R. Parvathy and M.G.Karunambigai's paper [7] introduced the concept of IFG and analyzed its components. Nagoor Gani, A and Sajitha Begum, S [5] defined degree, Order and Size in intuitionistic fuzzy graphs and extend the properties. The concept of Domination in fuzzy graphs is introduced by A. Somasundaram and S. Somasundaram [8] in the year 1998. Parvathi and Thamizhendhi[6] introduced the concepts of domination number in Intuitionistic fuzzy graphs. Study on domination concepts in Intuitionistic fuzzy graphs are more convenient than fuzzy graphs, which is useful in the traffic density and telecommunication systems. Complementary nil domination set in the crisp graph introduced and analyzed by Tamizh Chelvam et. al., [6] in 2009.

In this paper, We define complementary nil dominating set and its numbers of Intuitionistic Fuzzy Graphs. The bound on this number are obtained for some standard Intuitionistic fuzzy graphs. Some theorems related to the above concepts are studied and this concept is useful in networking analysis.

2. Preliminaries

Definition 2.1: A fuzzy graph $G = (\sigma,\mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where for all $u, v \in V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2: An Intuitionistic fuzzy graph is of the form G = (V, E) where

(i) $V=\{v1,v2,\ldots,vn\}$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $vi \in V$, respectively, and
$0 \leq \mu_1 (vi) + \gamma_1 (vi) \leq 1$ for every $vi \in V$, $(i = 1,2, \ldots n)$,

(ii) $E \subseteq V \times V$ where $\mu_2: VxV \to [0,1]$ and $\gamma_2: VxV \to [0,1]$ are such that
$\mu_2 (vi , vj) \leq \min [\mu_1(vi), \mu_1(vj)]$ and $\gamma_2 (vi , vj) \leq \max [\gamma_1(vi), \gamma_1(vj)]$
and $0 \leq \mu_2 (vi, vj) + \gamma_2 (vi, vj) \leq 1$ for every $(vi ,vj) \in E$, $(i, j = 1,2, \ldots n)$

Definition 2.3 An IFG H = < V', E' > is said to be an Intuitionistic fuzzy subgraph (IFSG) of the IFG, G = < V, E > if $V' \subseteq V$ and $E' \subseteq E$. In other words, if $\mu_{1i}' \leq \mu_{1i}$; $\gamma_{1i}' \geq \gamma_{1i}$ and

$\mu_{2ij}' \leq \mu_{2ij}$; $\gamma_{2ij}' \geq \gamma_{2ij}$ for every i, j = 1,2………n.

Definition 2.4: Let G = (V,E) be a IFG. Then the cardinality of G is defined as

$$|G| = \left|\sum_{vi\in V}\frac{1+\mu 1\ (vi)-\gamma 1\ (vi)}{2} + \sum_{vi,vj\in E}\frac{1+\mu 2\ (vi,vj)-\gamma 2(vi,vj)}{2}\right|$$

Definition 2.5: The vertex cardinality of IFG G is defined by $|V| = \left|\sum_{vi\in V}\frac{1+\mu 1\ (vi)-\gamma 1\ (vi)}{2}\right| = p$ and

The edge cardinality of IFG G is defined by $|E| = \left|\sum_{vi,vj\in E}\frac{1+\mu 2\ (vi,vj)-\gamma 2(vi,vj)}{2}\right| = q$.

The vertex cardinality of IFG is called the order of G and denoted by O(G). The cardinality of G is called the size of G, denoted by S(G).

Definition 2.6: An edge e = (x, y) of an IFG G = (V, E) is called an effective edge if $\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\gamma_2(x, y) = \gamma 1(x) \vee \gamma 1(y)$.

Definition 2.7: An Intuitionistic fuzzy graph is complete if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{2i}, \gamma_{2j})$ for all (vi , vj) $\in$ V.

Definition 2.8: An Intuitionistic fuzzy graph G is said to be strong IFG if $\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\gamma_2(x, y) = \gamma 1(x) \vee \gamma 1(y)$ for all (vi , vj) $\in$ E. That is every edge is effective edge.

Definition 2.9 : The complement of an IFG G = < V, E > is denoted by $\overline{G} = (\overline{V}, \overline{E})$ and is defined as i) $\bar{\mu}_1(v) = \mu_1(v)$ and $\bar{\gamma}_1(v) = \gamma_1(v)$

ii) $\bar{\mu}_2(u,v) = \mu_1(u) \wedge \mu_1(v) - \mu_2(u,v)$ and $\bar{\gamma}_2(u,v) = \gamma_1(u) \vee \gamma_1(v) - \gamma_2(u,v)$ for u,v in V

Definition 2.10: Let G = (V,E) be an IFG. The neighbourhood of any vertex v is defined as

$N(v) = (N_\mu(v), N_\gamma(v))$, Where $N_\mu(v) = \{w \in V ;\ \mu_2(v,w) = \mu_1(v) \wedge \mu_1(w)\}$ and $N_\gamma(v) = \{w \in V ;\ \gamma_2(v,w) = \gamma_1(v) \vee \gamma_1(w)\}$. $N[v] = N(v) \cup \{v\}$ is called the closed neighbourhood of v.

Definition 2.11: The neighbourhood degree of a vertex is defined as $d_N(v) = (d_{N\mu}(v), d_{N\gamma}(v))$ where $d_{N\mu}(v) = \sum_{w\in N(v)} \mu_1(w)$ and $d_{N\gamma}(v) = \sum_{w\in N(v)} \gamma_1(w)$.

The minimum neighbourhood degree is defined as $\delta_N(G) = (\delta_{N\mu}(v), \delta_{N\gamma}(v))$, where $\delta_{N\mu}(v) = \wedge \{ d_{N\mu}(v): v\in V\}$ and $\delta_{N\gamma}(v) = \wedge \{ d_{N\gamma}(v): v\in V\}$.

Definition 2.12: The effective degree of a vertex v in a IFG. G = (V, E) is defined to be sum of the effective edges incident at v, and denoted by $d_E(v)$. The minimum effective degree of G is $\delta_E(G) = \wedge \{d_E(v)/v \in V\}$

Definition 2.13: Let G= (V, E) be an IFG. Let u,v ∈ V, we say that u dominated v in G if there exist a strong arc between them. A subset D ⊆V is said to be dominating set in G if for every $v \in V-D$, there exist u in D such that u dominated v. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by γ. The maximum scalar cardinality of a minimal domination set is called upper domination number and is denoted by the symbol Γ .

Definition 2.14: An independent set of an Intuitionistic fuzzy graph G = (V, E) is a subset S of V such that no two vertices of S are adjacent in G.

3. Complementary nil domination set in IFG

Definition 3.1: Let G = (V, E) be an IFG. A set $S \subset V$ is said to be a complementary nil domination set (or simply cnd-set) of an IFG G, if S is a dominating set and its complement V-S is not a dominating set. The minimum scalar cardinality over all cnd-set is called a complimentary nil domination number and is denoted by the symbol γ_{cnd}, the corresponding minimum cnd –set is denote by γ_{cnd}-set.

Example 3.2: Let G = (V,E) be IFG be defined as follows

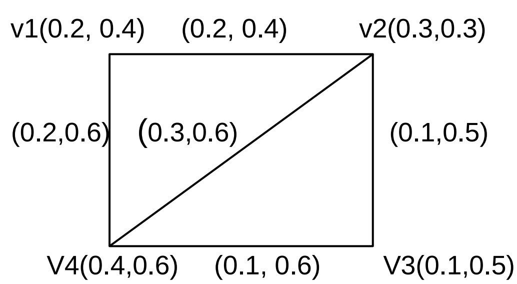

Fig- 1: Intuitionistic fuzzy graph

Here S1 = { v1, v2, v4} and S2 = { v2, v3, v4} are minimal cnd-sets. Here S2 is minimum cardinality and S1 is maximum cardinality. Hence, $\gamma_{cnd}(G) = 1.2$ and $\Gamma_{cnd}(G) = 1.3$

Definition 3.3: Let $S \subseteq V$ in the connected IFG G = (V, E). A vertex $u \in S$ is said to be an enclave of S if $\mu_2(u,v) < \min[\mu_1(u), \mu_1(v)]$ and $\gamma_2(u, v) < \max[\gamma_1(u), \gamma_1(v)]$ for all $v \in V-S$. That is $N[u] \subseteq S$.

In the above Fig-1, v1 is enclave of the set S1 and v3 is enclave of the set S2.

Theorem 3.4: A dominating set S is a cnd-set if and only if it contains at least one enclave.

Proof: Let S be a cnd-set of a IFG G = (V, E). The V-S is not a dominating set which implies that there exist a vertex u∈S such that μ_2 (u,v) < min [μ_1(u), μ_1(v)] and γ_2 (u, v) < max [γ_1(u), γ_1(v)] for all v∈V-S. Therefore u is an enclave of S.
Hence S contains at least one enclave.

Conversely, Suppose the dominating set S contains enclaves. Without loss of generality let us take u be the enclave of S. That is μ_2 (u,v) < min [μ_1(u), μ_1(v)] and γ_2 (u, v) < max [γ_1(u), γ_1(v)] for all v∈V-S. Hence V-S is not a dominating set.
Hence dominating set S is cnd-set.

Remark 3.5:

For any IFG G = (V,E)

1. Every super set of a cnd-set is also a cnd-set.
2. Complement of a cnd-set is not a cnd-set.
3. Complement of a domination set is not a cnd-set.
4. A cnd-set is not independent set.

Theorem 3.6: In any Intuituionistic fuzzy graph G = (V, E) , every complementary nil dominating set of G intersects with every dominating set of G.

Proof: Let S be γ_{cnd}-set and D be a γ-set of G = (V, E).
Suppose S ∩ D = ∅, then D ⊆ V-S and V-S contains a dominating set D.
Therefore V-S, a super set of D, is a dominating set.
Which is contradiction to our assumption.
Hence S ∩ D ≠ ∅.

Theorem 3.7: If S is a cnd-set of an IFG G = (V,E), then there is a vertex u∈ S such that S – {u} is a dominating set.

Proof: Let S be a cnd-set. By theorem 3.4, Every cnd-set contains at least one enclave of S.
Let u∈ S be an enclave of S. Then μ_2 (u,v) < min [μ_1(u), μ_1(v)] and γ_2 (u, v) < max [γ_1(u), γ_1(v)] for all v∈V-S.
Since G is connected IFG, there exist at least a vertex w ∈ S such that μ_2 (u,v) = min [μ_1(u), μ_1(v)] and γ_2 (u, v) = max [γ_1(u), γ_1(v)].
Hence S – {u} is a dominating set.

Theorem 3.8: A cnd-set in an IFG G = (V, E) is not a singleton.

Proof: Let S be a cnd-set. By theorem 3.4, Every cnd-set contains at least one enclave of S.
Let u∈ S be an enclave of S. Then μ_2 (u,v) < min [μ_1(u), μ_1(v)] and γ_2 (u, v) < max [γ_1(u), γ_1(v)] for all v∈V-S.
Suppose S contains only one vertex u, then it must be isolated in G.

This is contradiction to connectedness.
Hence cnd-set contains more than one vertex.

Theorem 3.9: Let G = (V, E) be an IFG and S be a γ_{cnd}-set of G. If u and v are two enclaves in S, then (i) $N[u] \cap N[v] \neq \emptyset$ and
(ii) u and v are adjacent. That is, $\mu_2 (u,v) = \min [\mu_1(u), \mu_1(v)]$ and $\gamma_2 (u, v) = \max [\gamma_1(u), \gamma_1(v)]$.

Proof: Let S be a minimum cnd-set of Intuitionistic fuzzy graph G = (V,E).
Let u and v are two enclaves of S.
Suppose $N[u] \cap N[v] = \emptyset$, then u is an enclave of S – N(v) which implies that V- (S – N(v)) is not a dominating set.
Therefore S- N(v) is a cnd-set of G and $|S – N(v)| < |S| = \gamma_{cnd}(G)$.
Which is contradiction to the minimality of S.
Then $N[u] \cap N[v] \neq \emptyset$.

Suppose $\mu_2 (u,v) < \min [\mu_1(u), \mu_1(v)]$ and $\gamma_2 (u, v) < \max [\gamma_1(u), \gamma_1(v)]$, that is u and v are non-adjacent. Then $u \notin N(v)$ and u is an enclave of S-{v}
ie) V- (S- {v}) is not a dominating set.
Hence S-{v} is a cnd-set, which is contradiction to minimality of S.
Hence u and v are adjacent.

Theorem 3.10: For any Intutitionistic fuzzy graph G= (V, E),

$$\Gamma(G) + \gamma_{cnd}(G) \leq O(G) + \text{Max}\{ |v_i| \} \text{ for every } v_i \in V$$

Proof:
If G is a Intuitionistic fuzzy graph then cnd-set ⊆V and super set of γ-set.
i.e) $\gamma_{cnd}(G) \leq O(G)$ and $\Gamma(G) = \text{Max}\{\text{minimal dominating set}\}$
i.e) $\Gamma(G) \leq \text{Max}\{| v_i| \}$
Therefore, we have $\Gamma(G) + \gamma_{cnd}(G) \leq O(G) + \text{Max}\{ | v_i| \}$

Example 3.11:
Let G= (V, E) be IFG with V={ a, b, c, d, e, f} and defined by $(\mu_1(a), \gamma_1(a)) = (0.2,0.4)$, $(\mu_1(b), \gamma_1(b)) = (0.1,0.5)$, $(\mu_1(c), \gamma_1(c)) = (0.4,0.5)$,$(\mu_1(d), \gamma_1(d)) = (0.3,0.6)$, $(\mu_1(e), \gamma_1(e)) = (0.4,0.6)$, $(\mu_1(f), \gamma_1(f)) = (0.3,0.5)$, and
$(\mu_2(a, b), \gamma_2(a, b))= (0.1,0.5)$, $(\mu_2(a, d), \gamma_2 (a, d))= (0.2,0.6)$, $(\mu_2(a, f), \gamma_2 (a, f))= (0.1,0.2)$, $(\mu_2(b, e), \gamma_2 (b, e))= (0.1,0.6)$, $(\mu_2(c, d), \gamma_2 (c, d))= (0.3,0.6)$, $(\mu_2(c, f), \gamma_2 (c, f))= (0.3,0.5)$, $(\mu_2(f, e), \gamma_2 (f, e))= (0.3,0.6)$.

Here, S1 = { a, b c, d} and S2= { b, c, e, f} are minimal complementary nil dominating set. Also minimal γ-set = { b, c } and $\delta_N(G) = \{ 0.4, 0.9\}$

(i) |a| = 0.4, |b| =0.3 , |c| = 0.45 , |d| =0.35, |e| = 0.4 and |f| = 0.4. O(G) = p = 2.3

(ii) $\gamma_{cnd}(G) = 1.5$ and $\Gamma_{cnd}(G) = 1.55$

(iii) $\gamma(G) = \Gamma(G) = 0.75$

(iv) The vertices a and d are two enclaves with respect to S1. The vertices e and f are two enclaves with respect to S2.

(v) N[a] = {a, b, d} N[d] = { a, c, d} i.e) $N[a] \cap N[d] \neq \emptyset$. Also a and d are adjacent

(vi) The vertex $d \in S1$ and S1 – d is a dominating set.

(vii) Min{ | v_i|} = 0.3, Max{ | v_i|} = 0.45 for every $v_i \in V$. Also $\Gamma(G) + \gamma_{cnd}(G)$ = 0.75+ 1.5 =2.25 and O(G) + Max{ | v_i| } = 2.3+ 0.45 = 2.75.
i.e) $\Gamma(G) + \gamma_{cnd}(G) \leq O(G) + Max\{ | v_i| \}$

4. Conclusion

Here We defined complementary nil dominating set and its numbers of Intuitionistic Fuzzy Graphs. The bound on this number are obtained for some standard Intuitionistic fuzzy graphs. Some results related to the above concepts are studied.

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