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Results on Discrete Dynamical System

"The Map $f_{\mu b}(x) = \mu x(x - b)$ is Topologically Conjugate to the Logistic Map $f(x) = \mu x(1 - x)$"

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Abstract

This research includes four objects:

1- First of all prove that ((every eventually fixed point belong to a basin of attraction but the converse is not true)).

2- Defininig the map $f_{\mu b}(x) = \mu x(x - b)$ for the first time where $\boldsymbol{x} \in \mathbb{R}$ and prove it is topologically conjugate to the logistic map $f(x) = \mu x(1 - x)$ for $\boldsymbol{x} \in [0,1]$.

3- Show that the map $f_{\mu b}(x)$ is topologically conjugate to the map $f_{\mu}(x) = (2 - \mu)x(1 - x)$ for $\boldsymbol{x} \in [0,1]$.

4- Finally present the bifurcation diagram of the map $f_{\mu b}(x) = \mu x(x - b)$ with a referring to the effect of the parameter $\boldsymbol{b}$ with different values and fixing $\boldsymbol{x}$ and this result explained graphically.

Keywords: logistic map, eventually fixed point, topologically conjugate, bifurcation diagram

Introduction

Discrete dynamical systems refer to the study of dynamical systems where the space **X**, is typically compact continuous and the time is either $\mathbf{Z}$ or $\mathbf{N}$.
The word discrete in the phrase discrete dynamical system emphasizes the fact the time is not represented by a continuous like $\mathbb{R}$ or $\mathbb{R}^+$ as it happens in the case differential equations.
The low of evolution of a point x in the space **X** is given by $f(x)$, a self – map on **X** which is either homeomorphism or just continuous. A point x is a fixed point if $f(x) = x$, know if $f^{\ell}(x) = x$ i.e if $f^{\ell}(x)$ is a fixed point for some $\ell \geq 1$ (x it self need not to be fixed) .
Logistic map is defined by $f(x) = \mu x(1-x)$, where μ is the parameter for decades, several iterated functions have been extensively studied, and rich contents have been explored. Logistic map is one of the well-known maps and has become a standard for studding iterations , this map contains all the interesting subjects in nonlinear dynamics, in general the values of x and μ of logistic map restricted in the ranges , $0 \leq x \leq 1$, $0 \leq \mu \leq 4$ so that each x in the interval [0,1] is mapped on the same interval [0,1], its known that there is a stable fixed point $x_1 = 0$ in the range $0 \leq \mu \leq 1$, and another stable fixed point $x_2 = 1 - \frac{1}{\mu}$ in the range $1 \leq \mu \leq 3$.
In the research we discussed at the beginning of the search as a first step the truth of ((Every eventually fixed point belong to a basin of attraction but the inverse is not true)) in the context of this offering, we give two examples to demonstrate that the reverse is not true
and we proceed to the second step introduce the maps $f_{\mu b}(x) = \mu x(x-b)$ and $f_{\mu}(x) = (2-\mu)x(1-x)$ the first one $f_{\mu b}(x)$ defined in this research , while the second $f_{\mu}(x)$ had been defined before in [2] and both of them driven from the logistic map with a notion that b, μ are parameters controlling the dynamics the maps containing them and defined for $0 \leq b \leq 1$, This step included a study for some properties of the two examples above of providers such as ((***fixed points***)), ((***dynamics of fixed points***)) , ((***eventually fixed points***)) , and ((***basin of attraction***)).
Here the third step here comes the role of the third step, which came as the title of our research because I considered the focus of work which is that ***((*** $f_{\mu b}(x)$ ***and the logistic map are topologically conjugate))***, in [2] it have been proved for $f_{\mu}(x)$ i.e ***((*** $f_{\mu}(x)$ ***and the logistic map are topologically conjugate))***, this means they are on the one hand and the application for Logistics on the other hand are identical in terms of the dynamic behavior and as a result $f_{\mu b}(x)$ and $f_{\mu}(x)$ are topologically conjugate.
Finally we have the bifurcation diagram for $f_{\mu b}(x)$ and noticed the impact of parameter b on it (bifurcation diagram), this result explained graphically.

Definition 1:
A function whose domain (input) space and range (output) space are the same will be called a map , let x be a point and let f be a map , the orbit of x under f is the set of points

{ x , $f(x)$, $f^2(x)$,…} the starting point x for the orbit is called the initial value of the orbit. A point p is a fixed point of the map f if $f(p) = p$, [3] .

Example

Let f:[0,1]⟶[0,1], define the logistic map $f(x) = \mu x\,(1- x)$ for μ= 2.8 the orbit of x = 0.01 under f is { 0.01, 0.0277, 0.07547, 0.0166695, …} this set is the orbit of 0.01 under f and 0.01 is the initial value of the orbit , and f has two fixed point 0 and o.6428571.

Definition 2

Suppose x_0 is a fixed point of f , then $x0$ is an attracting fixed point if | $f'(x_0)$ | < 1, the point $x0$ is repelling fixed point if | $f'(x_0)$ | > 1, finally if | $f'(x_0)$ |= 1, then the the fixed point $x0$ is called neutral or indifferent .[4]

Definition 3

Let x be in the domain of f, then x is an eventually fixed point of f if there is a positive integer n such that $f^n(x)$ is a fixed point of f .

Theorem (Attracting Fixed Point):

Suppose x_0 is an attracting fixed point for f , then there is an interval I contains x_0 in its interior and in which the following condition is satisfied if $x \in I$, then $f^n(x) \in I$ for all n and more over, $f^n(x) \longrightarrow x_0$ as $n \longrightarrow \infty$. prove see [5] **.**

Definition 4

The set of initial conditions whose orbits converge to the attracting fixed point is called the basin of attraction [3].

From the definitions and previous theorem we get the following result:

Proposition 1

Every eventually fixed point belong to a basin of attraction but the converse is not true.

proof: let p be a fixed point for a map f and I = { x : $f^k(x) \to p$ } as $k \to \infty$ from def. 4 the basin of attraction of p is I such that p is an attracting fixed point for f , if $x\,0$ is an eventually fixed point of f such that $f^n(x_0) = p$ then from def.3 $x_0 \in I$.

Now suppose A= { x_0 : $f^n(x_0) = p$ } and $x\,0$ is eventually fixed points , we observe that A is the set of all eventually fixed point and A$\subset I$.

In order to explain that the invers is not true we introduce the following two basic examples:

First Example *(the family* $f_{\mu b}(x)$ *)*:

The map $f_{\mu b}(x)$ is a new family of parametric functions driven from the well-known logistic map $f(x) = \mu x(1-x)$ where $f_{\mu b}(x)$: ℝ⟶ ℝ is one-one, onto map with $\mu \in [0,4]$ and $b \in [0,1]$ according this new family $f_{\mu b}(x)$ the following analysis have been done .

Fixed points: $f_{\mu b}(x)$ have two fixed points by setting $f_{\mu b}(x) = \boldsymbol{x}$ and they are $x_1 = 0$, $x_2 = b+(1/\mu)$.

Dynamics of fixed points x_1 and x_2 of $f_{\mu b}$:

From def.2, $f'_{\mu b}(x) = 2\mu x - \mu b$ thus, we get $f'_{\mu b}(x_1) = f'_{\mu b}(0) = -\mu b$ and $f'_{\mu b}(x_2) = f'_{\mu b}(b + 1/\mu)$ = $\mu b+2$ this happens next :

a- In relation to the first fixed point $x1 = 0$, $f'_{\mu b}(x_1) = f'_{\mu b}(0) = -\mu b$ and we know the $\mu \in [0,4]$ and $b \in [0,1]$ we see that:
  - **i-** for $|-\mu b|<1$, ***x1*** is attracting fixed point .
  - **ii-** for $|-\mu b|=1$, ***x1*** is neutral fixed point .
  - **iii-** for $|-\mu b|>1$, ***x1*** is repelling fixed point .

b- for the fixed point $x2 = b + 1/\mu$, we find that $f'_{\mu b}(x_2) = \mu b + 2$ as in (a-) for $\mu \in [0,4]$ and $b \in [0,1]$ we get :
  - **i-** for $|\mu b+2| < 1$, x_2 is attracting fixed point .
  - **ii-** for $|\mu b+2| = 1$, x_2 is neutral fixed point .
  - **iii-** for $|\mu b+2| > 1$, x_2 is repelling fixed point .

Basin of attraction of $f_{\mu b}(x)$:

There is a quicker way to find out the basin of attraction of the attracting fixed point using algebra to compare $|f_{\mu b}(x) - x_1|$ to $|x - x_1|$ or $|f_{\mu b}(x) - x_2|$ to $|x - x_2|$, if the former is smaller than the latter, it mean the orbit is getting closer to x_1 or x_2 and this happens only when $x1$ or $x2$ an attracting fixed point.
It is the application of the rule above we get the attraction basin for point x_1, which is the set $\{x : |x-b| < 1/\mu\}$. As for the point x_2 attractions Basin is the set $\{x : -(1/\mu)+b < x < (1/\mu) + b\}$.

Eventually fixed point of $f_{\mu b}(x)$:

a- To evaluate the eventually fixed point of $x_1 = 0$ we have to solve the equation $f^n_{\mu b}(x) = f_{\mu b}(f_{\mu b}(...f_{\mu b}(x)))$ = $x_1 = 0$, n – times.
  * the eventually fixed points of $f_{\mu b}(x) = x_1$ are $p_0 = 0$ and $p_1 = b$, This procedure is called ***first cycle*** of $f_{\mu b}(x)$ for the fixed point 0 .
  * the eventually fixed points of $f^2_{\mu b}(x) = f_{\mu b}(f_{\mu b}(x))$ = $x_1 = 0$ are $p_0 = 0$, $p_1 = b$, $p_2 = \dfrac{\mu b + \sqrt{\mu^2 b^2 + 4\mu b}}{2\mu}$ and $p_3 = \dfrac{\mu b - \sqrt{\mu^2 b^2 + 4\mu b}}{2\mu}$ This procedure is called ***second cycle*** of $f_{\mu b}(x)$ for the fixed point 0.

* the eventually fixed points of $f^3_{\mu b}(x)$ =0 are $p_0 = 0$, $p_1 = b$, $p_2 = \frac{\mu b + \sqrt{\mu^2 b^2 + 4\mu b}}{2\mu}$, $p_3 = \frac{\mu b - \sqrt{\mu^2 b^2 + 4\mu b}}{2\mu}$, $p_4 = \frac{\mu b + \sqrt{\mu^2 b^2 + 8\mu b}}{2\mu}$ and $p_5 = \frac{\mu b - \sqrt{\mu^2 b^2 + 8\mu b}}{2\mu}$ This procedure is called ***third cycle*** of $f_{\mu b}(x)$ for the fixed point 0.

* Thus, we continue the eventually fixed point of $f^4_{\mu b}(x) = 0$ are $p_0 = 0$, $p_1 = b$, $p_2 = \frac{\mu b + \sqrt{\mu^2 b^2 + 4\mu b}}{2\mu}$, $p_3 = \frac{\mu b - \sqrt{\mu^2 b^2 + 4\mu b}}{2\mu}$, $p_4 = \frac{\mu b + \sqrt{\mu^2 b^2 + 8\mu b}}{2\mu}$, $p_5 = \frac{\mu b - \sqrt{\mu^2 b^2 + 8\mu b}}{2\mu}$, $p_6 = \frac{\mu b + \sqrt{\mu^2 b^2 + 12\mu b}}{2\mu}$ and $p_7 = \frac{\mu b - \sqrt{\mu^2 b^2 + 12\mu b}}{2\mu}$ the ***fourth cycle*** for the fixed point 0 .

Thus, we continue to any finite number of iterations ***n*** of the function $f_{\mu b}(x)$ for the so-called eventually fixed points for the fixed point ***x1 .***

b- As we did in the preceding paragraph (a) we will the same thing for the fixed point ***x₂ = b +(1/μ)*** we will make $f_{\mu b}(x)$ = ***b +(1/μ)*** which we will get two eventually fixed points $f^n(-0.1) \neq 0$ this procedure is called the ***first cycle*** of $f_{\mu b}(x)$ for the fixed point ***b+(1/μ)***, but for $f^2_{\mu b}(x) = f_{\mu b}(f_{\mu b}(x))$ = ***b +(1/μ)*** we know that we will get four fixed points by solving the equation :

$\mu^3 x^4 - \mu^2 b x^3 - x^2(\mu^3 b + \mu^2 b) + x(\mu^2 b^2 + \mu b^2) = b + (1/\mu)$ which is more complicated than we could solve it using regular methods and it could be solved numerically by replacing certain values for μ and b Committed to their intervals which belong to .

The various results support our goal which we rich that: $|f^n_{\mu b}(x) - x_1| < |f^{n-1}_{\mu b}(x) - x_1|$ *and* $|f^n_{\mu b}(x) - x_2| < |f^{n-1}_{\mu b}(x) - x_2|$ then ***each eventually fixed point belong to the basin of attraction of an attracting fixed point***, this means the basin of attraction fo the two points ***0*** , ***b +(1/μ)*** Is all the initial points that belong to the interval containing they.

Application Example $f_{\mu b}(x)$:

Let ***μ = 2*** and ***b = 0.1*** then $f_{\mu b}(x) =$ ***2x(x- 0.1)*** have two fixed points $x_1 = 0, x_2 = 0.6$, x_1 is attracting fixed point and x_2 is repelling, the basin of attraction for x_1 is I = (-0.4 ,0.6) but x_2 has no basin of attraction because it is repelling.

$f^2_{\mu b}(x) = f_{\mu b}(f_{\mu b}(x))$ have four eventually fixed points $p_0 = 0,\ p_1 = 0.1,$ $p_2 = 0.27912879, p_3 = -0.1791288$ and to chick this result we will do the following simple iteration:

$f(0) = 0, f^2(0) = 0, ...$ the fixed point it self is eventually fixed point.

$f(0.1) = 0, f(0) = f^2(0.1) = 0, f(0) = 0, ...$ (two cycle)

$f(0.27912879)=0.1, f(0.1)=0, f(0)=0,...., f^3(0.27912879)=0$ (three cycle)
$f(-0.1791288)=0.1, f(0.1)=0, f(0)=0,..... f^3(-0.1791288)=0$ (three cycle)
If we select randomly a point belong to the basin of attraction I= (-0.4,0.6) such as x=-0.1 we get $f(-0.1)=0.04$,
$f^2(-0.1)=f(0.04)=0.0048$,
$f^3(-0.1)=f(0.0048)=-0.00091392$
$f^4(-0.1)=f(-0.00091392)=0.0001844544995$
$f^5(-0.1)=f(0.0001844544995)=-0.000036822853$
i.e $f^n(-0.1)\to 0$ but $f^n(-0.1)\neq 0$.
That is: ***((the convers is not true))*** in (proposition1).

Second Example (*the family* $f_\mu(x)=(2-\mu)x(1-x)$*) :*

This example borrowed from [2] and we studied its dynamics.
The family of function $f_\mu(x)=(2-\mu)x(1-x)$, 0≤x≤1 and 0≤μ≤4 where $f_\mu(x)$ **:**[0,1]⟶[0,1] , let us do the following analyses.

Fixed points : $f_\mu(x)$ has two fixed points $x_1=0$ and $x_2=\dfrac{1+\mu}{2-\mu}$.

Dynamics of fixed points of $f_\mu(x)$ **:**

From the definition of attracting and repelling fixed point $f'_\mu(x)=(2-\mu)-2(2-\mu)x$

a - for the fixed point x=0 , $f'_\mu(0)=2-\mu$ we get the following region :

i- if $1<\mu<3$, x_1 is an attracting fixed point .
ii- if $\mu<1$ or $\mu>3$, x_1 is repelling fixed point.
iii- if μ= 1 or μ= 3 , x_1 is neutral fixed point .

b - for the second fixed point $x_2=\dfrac{1+\mu}{2-\mu}$, $f'_\mu(\dfrac{1+\mu}{2-\mu})=5+2\mu$ we found out that:

i- if -3 < μ < -2 , x_2 is is an attracting fixed point .
ii- if μ < -3 , μ < -2 , x_2 is repelling fixed point .
iii- if μ = -3 or μ= -2 , x_2 is neutral fixed point .

Basin attraction of $f_\mu(x)$:

As we explain in example the ***family*** $f_{\mu b}(x)$ in order to calculate the basin of attraction of $f_\mu(x)$ we use the same way in $f_{\mu b}(x)$ we get:

* For $x_1 = 0$ the basin of attraction is $[0, \frac{3-\mu}{2-\mu}] \cup [\frac{1-\mu}{2-\mu}, 1]$.

** For $x_2 = \frac{1+\mu}{2-\mu}$ the basin of attraction is $[0, \frac{\mu-1}{\mu-2}]$.

Eventually fixed points of $f_\mu(x)$:

In order to find the eventually fixed points for the first fixed point $x_1 = 0$ we have to solve the equation $(2-\mu)^3(x(1-x))\big(1-(2-\mu)x(1-x)\big)\Big(1-((2-\mu)^2 x(1-x))\Big)\big(1-(2-\mu)x(1-x)\big)$ for n = 1, 2, 3,…, k with k∈N, thus for $f_\mu(x)$ we have two eventually fixed points $q_0 = 0$ and $q_1 = 1$, but for the map $f_\mu^2(x) = x_1$,

i.e. $(2-\mu)^2(x(1-x))(1-(2-\mu)x(1-x)) = 0$, we get four fixed points: $q_0 = 0$, $q_1 = 1$, $q_2 = \frac{(2-\mu)+\sqrt{\mu^2-4}}{2(2-\mu)}$, $q_3 = \frac{(2-\mu)-\sqrt{\mu^2-4}}{2(2-\mu)}$, one of q_2, q_3 or both of them is eventually .imposing that $\mu^2 - 4 > 0$, and continuo for $f_\mu^3(x) = x_1$,

i.e. $(2-\mu)^3(x(1-x))\big(1-(2-\mu)x(1-x)\big)\Big(1-((2-\mu)^2 x(1-x))\Big)\big(1-(2-\mu)x(1-x)\big) = 0$

if we make $f_\mu^3(x) = 0$ will be resolved very complex traditional methods, but it can be solved numerically , The same approach will be followed with a fixed point $x_2 = \frac{1+\mu}{2-\mu}$.

We found that the eventually fixed points of $f_\mu(x) = x_2$ are exactly two , and they are $q_0 = \frac{2-\mu+\sqrt{(2-\mu)^2-4(1+\mu)}}{2(2-\mu)}$ and $q_1 = \frac{(2-\mu)-\sqrt{(2-\mu)^2-4(1+\mu)}}{2(2-\mu)}$ but for $f_\mu^2(x) = x_2$ and the highest iterations will be very complex solved in regular methods, while can be solved numerically by replacing certain values for μ depending on the previous conditions that determine the dynamic of the fixed points x_2.

Application Example $f_\mu(x)$:

Put μ = 2.8 ,the map $f_\mu(x) = (2-\mu)x(1-x)$ will $f_{2.8}(x) = -0.8x(1-x)$ and $f_{2.8}(x)$ will give us two fixed points $x_1 = 0$, $x_2 = -4.25$ because 1< μ < 3 we conclude that x_1 is an attracting fixed point with basin of attraction (-0.25,0)∪(1,2.25) and (0,2.75)∪(0.25,1). While x_1 has no basin of attraction because it is repelling fixed point when 1< μ < 3. Back to the point $x_1 = 0$, the eventually fixed points for $f_{2.8}(x)$ are $q_0 = 0$ and $q_1 = 1$ while for the map $f_{2.8}^2(x)$ in addition to q_0, q_1 we get $q_2 = -0.7247448731$ and $q_3 = 1.724744873$.

Know let us test q_2 iterations:
$f_{2.8}(q_2) = 0.159591793$
$f_{2.8}^2(q_2) = -0.107297802$
$f_{2.8}^3(x) = 0.095048496$
$f_{2.8}^4(x) \neq 0$
In the same way, we get:
$f_{2.8}(q_3) = 1.000000003$
$f_{2.8}^2(q_3) = 0.000000002$
$f_{2.8}^3(q_3) = -0.000000001$
$f_{2.8}^4(q_3) \approx 0$
We can summarize the above result and conclude that belong to the basin of attraction and its eventually fixed point but q_2 doesn't belong to the basin of attraction and its not eventually fixed point.

Now let us select any arbitrary point in the basin of attraction such as -0.2 and iterate it under
$f_{2.8}(-0.2) = 0.192$
$f_{2.8}^2(-0.2) = -0.1241088$
$f_{2.8}^3(-0.2) = 0.02482176$
$f_{2.8}^4(-0.2) = -0.019364512$
$f_{2.8}^5(-0.2) = 0.015791597$
.
.
.
And so on, we noticed that $f_{2.8}^n(-0.2) \to 0$ as $n \to \infty$ but ≠0, i.e. ***not any point in the basin of attraction of*** $f_\mu(x)$ ***is an eventually fixed point.***

Definition 5: The two maps ϕ:$\boldsymbol{A}$⟶$\boldsymbol{A}$ and ψ:$\boldsymbol{B}$⟶$\boldsymbol{B}$ are said to be topologically conjugates if thire is a homeomorphism map $\boldsymbol{h}$: $\boldsymbol{A}$⟶$\boldsymbol{B}$ such that $\boldsymbol{h}(\phi(\mathrm{x}))=\psi(\boldsymbol{h}(\mathrm{x}))$, we can also write $\boldsymbol{h}\mathrm{o}\phi=\psi\mathrm{o}\boldsymbol{h}$.[6]

The map $f_{\mu,b}(x)$ is topologically conjugate to the logistic map $f(x) = \mu x(1-x)$
Proof : we are going to prove that the map $f_{\mu,b}(x) = \mu x(x-b)$ is topologically conjugate to logistic map $f(x) = \mu x(1-x)$.

Thus we need to find an $\boldsymbol{h}$ relating the following two maps $f_{\mu,b}(x)$ and $f(x) = \mu x(1-x)$ such that $f_{\mu,b}(x) = \boldsymbol{h}\mathrm{ofo}\boldsymbol{h}^{-1}$ …(1)

We consider a linear conjugate, h(x) = px+q , where p and q are properly chosen with such an h(x) we get :

$f_{\mu,b}(x) = h \circ f \circ h^{-1} = A + Bx + Cx^2$, with $A = \frac{-\mu q^2}{p} + q - \mu q$, $B = \mu + \frac{2\mu q}{p}$, $C = \frac{-\mu}{q}$ we choose p = -1 and q = 1-1/μ thus $\boldsymbol{h}$(x) = -x + (1- 1/μ) .

The map $f_{\mu,b}(x) = \mu x(x-b)$ and $f_\mu(x) = (2-\mu)x(1-x)$ are topologically conjugate .

Proof: in the previous proof demonstrate that $f_{\mu,b}(x)$ and $f(x) = \mu x(1-x)$ are topologically conjugate, in[2] researchers proof that ∃ g is bijective map and continuous such that

$f_\mu(x) = gofog$ …(2)

and $g(x) = \frac{\mu}{2-\mu}x + \frac{1-\mu}{2-\mu}$

Thus from (1) we get $\boldsymbol{f} = \boldsymbol{h}^{-1}\, \mathrm{o} f_{\mu,b}(x)\, \mathrm{o}\boldsymbol{h}$ …(3)

and from (2) we get $\boldsymbol{f} = \boldsymbol{g}^{-1} \mathrm{o} f_\mu(x)\, \mathrm{o}\boldsymbol{g}$ …(4)

but ***(3) = (4)*** , then:

$\boldsymbol{h}^{-1} f_{\mu,b}(x)\, \mathrm{o}\boldsymbol{h} = \boldsymbol{g}^{-1} \mathrm{o} f_\mu(x)\, \mathrm{o}\boldsymbol{g}$ thus $f_{\mu,b}(x) = \boldsymbol{h}\mathrm{o}\boldsymbol{g}^{-1} \mathrm{o} f_\mu(x)\, \mathrm{o}\boldsymbol{g}\mathrm{o}\boldsymbol{h}^{-1}$ …(5)

that is ∃ bijctive and continuous map $\boldsymbol{g}\mathrm{o}\boldsymbol{h}^{-1}$ satisfy (5), this mean $f_{\mu,b}(x)$ is topologically conjugate to $f_\mu(x)$.

Bifurcation diagram of $f_{\mu,b}(x)$

Bifurcation diagrams provide a useful method to show how a system's behavior changes according to the value of a control parameter. A classic example is the logistic map, where the systems shows both periodic and chaotic behavior, and periodic orbits appear as a discrete set of points with flows, it is more difficult to present the bifurcations, due to the continuous nature of the flow. While a periodic orbit in a map is an oscillation between a discrete numbers of points, in a flow a variable will sweep out a continuous range of values, [7].

In this paper, we will introduce the bifurcation diagram graphically by using matlab for the t map $f_{\mu,b}(x)$ with different values for μ and b as follows:

(Note: μ=m)

Results

** In (fig.1) there is one bifurcation point when 3.7<μ<3.8 and x = 0.7, b = 0.8 .

*** In (fig.2) there are two bifurcation point when 3.9<μ<4 and x = 0.7, b = 0.87.

**** In (fig.3) there are uncountable number of bifurcaton point whene 3.88<μ<4 and x=0.7, b=0.92.

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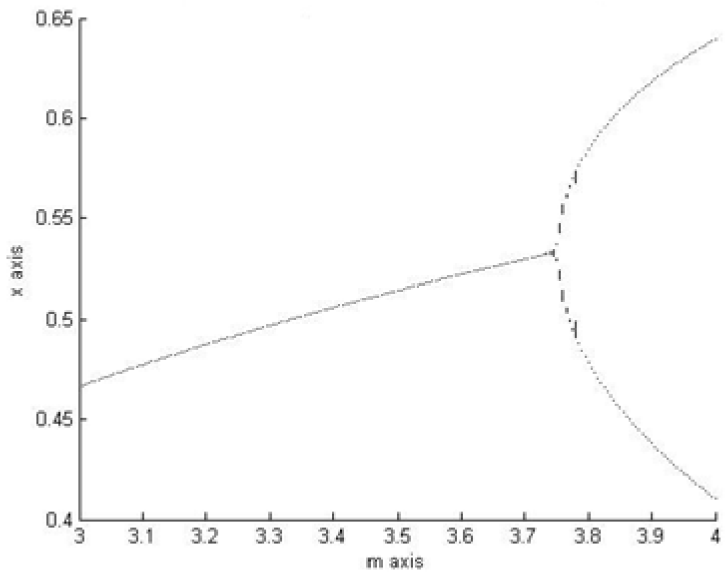

Fig. 1 the bifurcation diagram of $f_{\mu,b}(x)$ with b = 0.8 , x = 0.7 , μ = 3 to 4

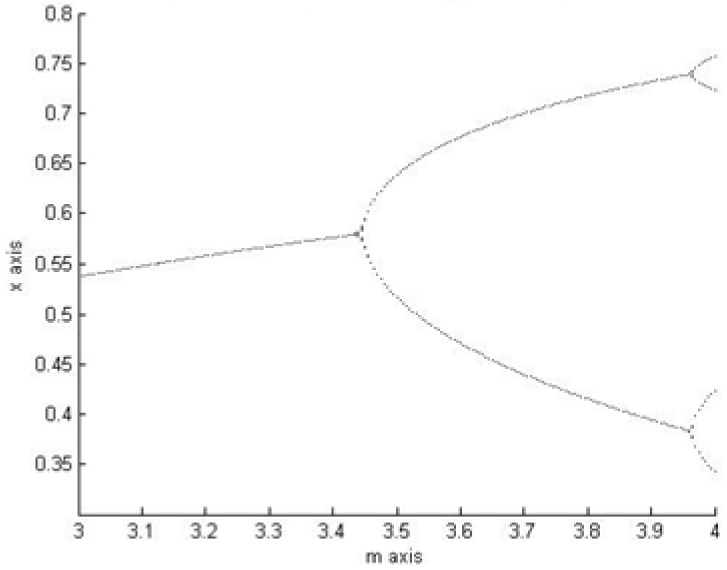

Fig.2 the bifurcation diagram of $f_{\mu,b}(x)$ with b = 0.87 ,x = 0.7 , μ = 3 to 4 .

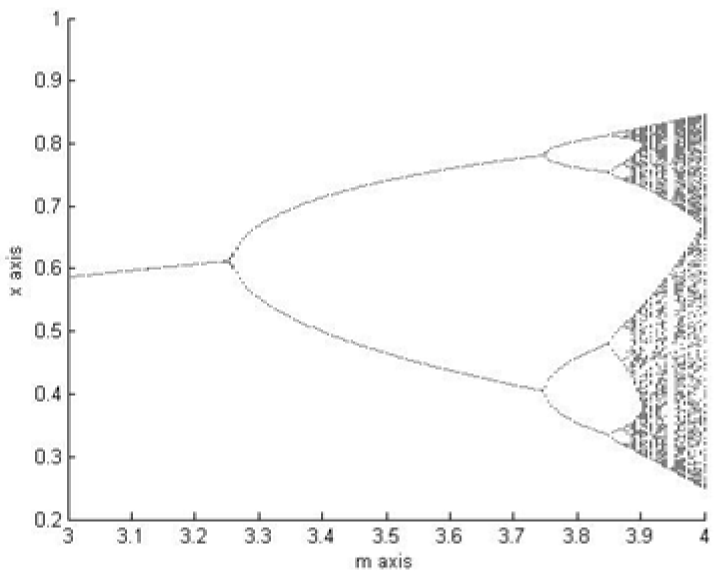

Fig.3 the bifurcation diagram of $f_{\mu,b}(x)$ with b = 0.92 , x = 0.7 , μ = 3 to 4

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