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An Efficient Technique for Solving Gas Dynamics Equation Using the Natural Decomposition Method

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Abstract

In this paper, we introduce a new technique called the Natural Decomposition Method (NDM) for solving homogeneous and inhomogeneous gas dynamic equations. The new method is an elegant combination of the Natural transform Method (NTM) and Adomian Decomposition Method (ADM). The proposed method is applied directly without using linearization, transformation, discretization or taking some restrictive assumptions. Using the new method, we successfully obtain exact solution of two illustrative examples, and the results are compared with the results of the existing methods. This shows the reliability, accuracy, and efficiency of the new method.

Mathematics Subject Classification: 35Q51, 74G15

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1 Introduction

Gas dynamic equations are the mathematical expressions of conservation laws which exist in physical and applied science practices such as conservation of momentum, conservation of mass, conservation of energy and so on. The standard gas dynamic equation in one spacial dimension is given by:

$$v_t(x,t) - v_{xx}(x,t) + \frac{1}{2}F_1(v(x,t)) + F_2(v(x,t)) = g(x,t), \quad (1.1)$$

subject to the initial condition

$$v(x,0) = f(x), \quad (1.2)$$

where $g(x,t)$ is the source term, and $F_1(v(x,t))$ and $F_2(v(x,t))$ are representing the nonlinear terms, $v_x^2(x,t)$ and $v^2(x,t)$ of the unknown function $v(x,t)$. Many numerical techniques has been used to solve gas dynamic equations such as Decomposition Method (DM) [1], Reduce Differential Transform Method [2], Reconstruction of variational Iteration Method (RVIM) [3], Fourier Transform Adomian Decomposition Method (FTADM) [4], Homotopy Perturbation Method (HPM) [5], Variational Iterative Method (VIM) [6], El–Zaki Transform Homotopy Perturbation Method (ETHPM) [7], and so on.
In this paper, we introduce a new technique called the Natural Decomposition Method (NDM) for solving homogeneous and inhomogeneous gas dynamic equations. The new technique is a combination of the Natural transform Method (NTM) and Adomian Decomposition Method (ADM). It does not require any unnecessary linearization, discretization, transformation or taking some restrictive assumptions and it avoids round off errors. The new technique lead to exact or approximate solution in form of a rapidly convergence series with elegant computational terms. Exact solution of two illustrative examples are successfully fund using the new technique. Hence, the Natural Decomposition Method is a powerful mathematical technique for solving gas dynamic equations and can easily be applied to solve many nonlinear partial differential equations.

The remaining part of this paper is organized as follows: In Section 2, we begin with the Mathematical Preliminaries. In section 3, we present the Analysis of the Natural Decomposition Method. In Section 4, we present some applications of the (NDM) to show its simplicity, effectiveness and accuracy. Finally in Section 5 conclusion and section 6, we gave the references of this paper.

2 Mathematical Preliminaries

In this section, we present some definitions and properties of the Natural transform.

Definition: The Natural transform of the function $f(t) \in A$, for $t \in (0, \infty)$ is defined by [14, 15]:

$$\mathbb{N}^+ [f(t)] = V(s,u) = \int_0^\infty e^{-st} f(ut)\, dt; \quad s,\, u \in (0, \infty)\,, \tag{2.1}$$

where $\mathbb{N}^+ [f(t)]$ is the Natural transformation of the time function $f(t)$, $A = \left\{ f(t) : \exists\, M,\, \tau_1,\, \tau_2 > 0,\ |f(t)| < M e^{\frac{|t|}{\tau_j}},\ where\ t \in (-1)^j \times [0, \infty) \right\}$, where j is a non-negative integer, and the variables s and u are the Natural transform variables.

Basic theorem of the Natural transform method are given below. See[15, 16, 17]:

Theorem 1 *If $V(s,u)$ is the Natural transform and $F(s)$ is the Laplace transform of the function $f(t) \in A$, then $\mathbb{N}^+ [f(t)] = V(s,u) = \frac{1}{u} \int_0^\infty e^{-\frac{st}{u}} f(t)\, dt = \frac{1}{u} F\left(\frac{s}{u}\right)$.*

Theorem 2 *If $V(s,u)$ is the Natural transform and $G(u)$ is the Sumudu transform of the function $f(t) \in A$, then $\mathbb{N}^+ [f(t)] = V(s,u) = \frac{1}{s} \int_0^\infty e^{-t} f\left(\frac{ut}{s}\right)\, dt = \frac{1}{s} G\left(\frac{u}{s}\right)$.*

Theorem 3 *If $\mathbb{N}^+ [f(t)] = V(s,u)$, then $\mathbb{N}^+ [f(at)] = \frac{1}{a} V(s,u)$.*

Theorem 4 *If $\mathbb{N}^+ [f(t)] = V(s,u)$, then $\mathbb{N}^+ [f'(t)] = \frac{s}{u} V(s,u) - \frac{f(0)}{u}$.*

Theorem 5 *If $\mathbb{N}^+ [f(t)] = V(s,u)$, then $\mathbb{N}^+ [f''(t)] = \frac{s^2}{u^2} V(s,u) - \frac{s}{u^2} f(0) - \frac{f'(0)}{u}$.*

Remark The Natural transform is a linear operator. That is, if α and β are non–zero constants, then
$\mathbb{N}^+ [\alpha\, f(t) \pm \beta\, g(t)] = \alpha \mathbb{N}^+ [f(t)] \pm \beta \mathbb{N}^+ [g(t)] = \alpha F^+(s,u) \pm \beta G^+(s,u)$.
Moreover, $F^+(s,u)$ and $G^+(s,u)$ are the Natural transforms of $f(t)$ and $g(t)$, respectively.

Table 1. List of some special Natural transforms of some functions. See[15]

Functional Form	Natural Transform Form
1	$\frac{1}{s}$
t	$\frac{u}{s^2}$
e^{at}	$\frac{1}{s-au}$
$\frac{t^{n-1}}{(n-1)!}, n = 1, 2, ...$	$\frac{u^{n-1}}{s^n}$
$\sin(t)$	$\frac{u}{s^2+u^2}$

3 Analysis of the Method

In this section, we illustrate the basic idea of the Natural Decomposition Method (NDM) for general nonlinear gas dynamic equation of the form (1.1)–(1.2):

$$v_t(x,t) - v_{xx}(x,t) + \frac{1}{2}F_1(v(x,t)) + F_2(v(x,t)) = g(x,t), \tag{3.1}$$

subject to the initial condition

$$v(x,0) = f(x), \tag{3.2}$$

where $g(x,t)$ is the source term, and $F_1(v(x,t))$ and $F_2(v(x,t))$ represent the nonlinear terms $v_x^2(x,t)$ and $v^2(x,t)$ of the unknown function $v(x,t)$.
Applying the Natural transform on both sides of Eq. (3.1), we obtain:

$$\frac{s}{u}V(x,s,u) - \frac{1}{u}v(x,0) - \mathbb{N}^+\left[\ v(x,t)\right] + \mathbb{N}^+\left[\ \frac{1}{2}F_1(v(x,t)) + F_2(v(x,t))\right] = \mathbb{N}\left[g(x,t)\right]. \tag{3.3}$$

Substituting the given initial condition of Eq. (3.2) into Eq. (3.3), we obtain:

$$V(x,s,u) = \frac{1}{s}f(x) + \frac{u}{s}\mathbb{N}^+\left[g(x,t)\right] + \frac{u}{s}\mathbb{N}^+\left[\ v(x,t) - \frac{1}{2}F_1(v(x,t)) - F_2(v(x,t))\right]. \tag{3.4}$$

Taking the inverse Natural transform of Eq. (3.4), we obtain

$$v(x,t) = G(x,t) + \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^+\left[\ v(x,t) - \frac{1}{2}F_1(v(x,t)) - F_2(v(x,t))\right]\right], \tag{3.5}$$

where $G(x,t)$ is a term arising from the source terms.

Now we assume an infinite series solutions for the unknown functions $v(x,t)$ of the form:

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t). \tag{3.6}$$

The nonlinear terms $F_1(v(x,t))$ and $F_2(v(x,t))$ can easily be represented by decomposition series of the form:

$$F_1(v(x,t)) = \sum_{n=0}^{\infty} A_n(x,t), \tag{3.7}$$

and

$$F_2(v(x,t)) = \sum_{n=0}^{\infty} B_n(x,t), \tag{3.8}$$

where A_n and B_n are Adomian polynomials which represent the nonlinear terms $v_x^2(x,t)$ and $v^2(x,t)$, and can easily be computed using the following formulas:

$$A_n = \frac{1}{n!}\frac{d^n}{dx^n}\left[F_1\left(\sum_{i=0}^{n} \lambda^i v_i(x,t)\right)\right]_{\lambda=0}, \tag{3.9}$$

and

$$B_n = \frac{1}{n!}\frac{d^n}{dx^n}\left[F_2\left(\sum_{i=0}^{n} \lambda^i v_i(x,t)\right)\right]_{\lambda=0}, \tag{3.10}$$

where $n = 0, 1, 2, \cdots$
Below are some few components of A_n and B_n.

$$\begin{aligned}
A_0 &= F_1(v_0) \\
&= v_{0x}^2, \\
A_1 &= v_{1x}F_1'(v_0) \\
&= 2v_{0x}v_{1x}, \\
A_2 &= v_{2x}F_1'(v_0) + \frac{1}{2!}v_{1x}^2 F_1''(v_0) \\
&= 2v_{0x}v_{2x} + v_{1x}^2, \\
&\vdots
\end{aligned}$$

$$\begin{aligned} B_0 &= F_2(v_0) \\ &= v_0^2, \\ B_1 &= v_1 F_2'(v_0) \\ &= 2v_0 v_1, \\ B_2 &= v_2 F_2'(v_0) + \frac{1}{2!} v_1^2 F_2''(v_0) \\ &= 2v_0 v_2 + v_1^2, \\ &\vdots \end{aligned}$$

and so on.
By substituting Eq. (3.6) and Eq. (3.7) into Eq. (3.5), we obtain:

$$\sum_{n=0}^{\infty} v_n(x,t) = G(x,t) + \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[\sum_{n=0}^{\infty} v_n(x,t) - \frac{1}{2}\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n\right]\right] \quad (3.11)$$

Then by comparing both sides of Eq. (3.9) above, we can easily generate the recursive relation as follows:

$$\begin{aligned} v_0(x,t) &= G(x,t), \qquad (3.12) \\ v_1(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[v_0(x,t) - \frac{1}{2}A_0 - B_0\right]\right], \\ v_2(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[v_1(x,t) - \frac{1}{2}A_1 - B_1\right]\right], \\ v_3(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[v_2(x,t) - \frac{1}{2}A_2 - B_2\right]\right], \end{aligned}$$

and so on.
Thus, the general recursive relation is given by:

$$v_{n+1}(x,t) = \mathbb{N}^{-1}\left[\frac{u}{s}\mathbb{N}^{+}\left[v(x,t) - \frac{1}{2}A_n - B_n\right]\right], \ n \geq 0. \quad (3.13)$$

Hence, the exact or approximate solutions of the unknown function $v(x,t)$ is given by:

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t).$$

4 Applications

In this section, we illustrate the applicability of the Natural Decomposition Method to homogeneous and inhomogeneous gas dynamics equations.

Example 4.1 *Consider the homogeneous gas dynamic equation of the form [1-9]:*

$$v_t(x,t) + \frac{1}{2}(v^2(x,t))_x = v_{xx}(x,t) - v^2(x,t), \tag{4.1}$$

subject to the initial condition:

$$v(x,0) = e^{-x}. \tag{4.2}$$

Applying the Natural transform on both sides of Eq. (4.1), we obtain:

$$\frac{s}{u}V(x,s,u) - \frac{1}{u}v(x,0) + \frac{1}{2}\mathbb{N}^+\left[(v^2(x,t))_x\right] = \mathbb{N}^+\left[v(x,t) - v^2(x,t)\right]. \tag{4.3}$$

Substituting the given initial condition of Eq. (4.2) into Eq. (4.3), we obtain:

$$V(x,s,u) = \frac{1}{s}e^{-x} + \frac{u}{s}\left[\mathbb{N}^+\left[v(x,t) - \frac{1}{2}(v^2(x,t))_x - v^2(x,t)\right]\right]. \tag{4.4}$$

Then by taking the inverse Natural transform of Eq. (4.4), we have

$$v(x,t) = e^{-x} + \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[v(x,t) - \frac{1}{2}(v^2(x,t))_x - v^2(x,t)\right]\right]\right]. \tag{4.5}$$

We now assume an infinite series solution of the unknown function $v(x,t)$ of the form:

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t). \tag{4.6}$$

Then by using Eq. (4.6), we can re-write Eq. (4.5) in the form:

$$\sum_{n=0}^{\infty} v_n(x,t) = e^{-x} + \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[\sum_{n=0}^{\infty} v_n(x,t) - \frac{1}{2}\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n\right]\right]\right], \tag{4.7}$$

where A_n and B_n are the Adomian polynomial which represent the nonlinear terms $(v^2)_x(x,t)$ and $v^2(x,t)$ respectively.
By comparing both sides of Eq. (4.7), we can easily generate the recursive relation as follows:

$$\begin{aligned}
v_0(x,t) &= e^{-x}, \\
v_1(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[v_0 - \frac{1}{2}A_0 - B_0\right]\right]\right] \\
v_2(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[v_1 - \frac{1}{2}A_1 - B_1\right]\right]\right] \\
v_3(x,t) &= \mathbb{N}^{-1}\left[\frac{u^2}{s^2}\left[\mathbb{N}^+\left[v_2(x,t) - \frac{1}{2}A_2 - B_2\right]\right]\right], \\
&\vdots
\end{aligned}$$

Therefore, the general recursive relation of the unknown function $v(x,t)$ is given by:

$$v_{n+1}(x,t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_n(x,t) - \frac{1}{2}A_n - B_n\right]\right]\right], \quad n \geq 0. \tag{4.8}$$

Using the general recursive relation of Eq. (4.8), we can easily compute the remaining components of the unknown function $v(x,t)$ as follows:

$$\begin{aligned}
v_1(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_0(x,t) - \frac{1}{2}A_0 - B_0\right]\right]\right] \\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_0(x,t) - \frac{1}{2}(v^2)_{0x} - v_0^2\right]\right]\right] \\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[e^{-x}\right]\right]\right] \\
&= e^{-x}\mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[1\right]\right]\right] \\
&= e^{-x}\mathbb{N}^{-1}\left[\frac{u}{s^2}\right] \\
&= \frac{t}{1!}e^{-x},
\end{aligned}$$

$$\begin{aligned}
v_2(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_1(x,t) - \frac{1}{2}A_1 - B_1\right]\right]\right] \\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_1(x,t) - (v_0v_1)_x - 2v_0v_1\right]\right]\right] \\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[te^{-x}\right]\right]\right] \\
&= e^{-x}\mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[t\right]\right]\right] \\
&= e^{-x}\mathbb{N}^{-1}\left[\frac{u^2}{s^3}\right] \\
&= \frac{t^2}{2!}e^{-x},
\end{aligned}$$

$$\begin{aligned}
v_3(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_2(x,t)-\frac{1}{2}A_2-B_2\right]\right]\right]\\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_2(x,t)-(v_0v_2)_x-\frac{1}{2}v_{1x}-2v_0v_2-v_1^2\right]\right]\right]\\
&= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[\frac{t^2}{2!}e^{-x}\right]\right]\right]\\
&= \frac{1}{2!}e^{-x}\mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[t^2\right]\right]\right]\\
&= \frac{1}{2!}e^{-x}\mathbb{N}^{-1}\left[\frac{2u^3}{s^4}\right]\\
&= \frac{t^3}{3!}e^{-x},\\
&\vdots
\end{aligned}$$

and so on.
Hence, the approximate series solution of the unknown function $v(x,t)$ is given by:

$$\begin{aligned}
v(x,t) &= \sum_{n=0}^{\infty} v_n(x,t)\\
&= v_0(x,t)+v_1(x,t)+v_2(x,t)+v_3(x,t)+\cdots\\
&= e^{-x}+\frac{t}{1!}e^{-x}+\frac{t^2}{2!}e^{-x}+\frac{t^3}{3!}e^{-x}+\cdots\\
&= e^{t-x}.
\end{aligned}$$

Therefore, the exact solution of the gas dynamic equation (4.1)–(4.2) is given by:

$$v(x,t) = e^{t-x}.$$

The exact solution is in closed agreement with the result obtained by (DM) [2], (RDTM) [2], (RVIM) [3], (FTADM) [4], (HPM) [5], and (VIM) [6]

Example 4.2 *Consider the inhomogeneous gas dynamics equation of the form [3, 7]:*

$$v_t(x,t)+\frac{1}{2}(v^2(x,t))_x = v_{xx}(x,t)-v^2(x,t)-e^{t-x}, \tag{4.9}$$

subject to the initial condition:

$$v(x,0) = 1-e^{-x}. \tag{4.10}$$

Applying the Natural transform on both sides of Eq. (4.9), we obtain:

$$\frac{s}{u}V(x,s,u) - \frac{1}{u}v(x,0) + \frac{1}{2}\mathbb{N}^+\left[(v^2(x,t))_x\right] = \frac{e^{-x}}{s-u} + \mathbb{N}^+\left[v(x,t) - v^2(x,t)\right]. \tag{4.11}$$

Substituting the given initial condition of Eq. (4.10) into Eq. (4.11), we obtain:

$$V(x,s,u) = \frac{1-e^{-x}}{s} + \frac{ue^{-x}}{s(s-u)} + \frac{u}{s}\left[\mathbb{N}^+\left[v(x,t) - \frac{1}{2}(v^2(x,t))_x - v^2(x,t)\right]\right]. \tag{4.12}$$

Then by taking the inverse Natural transform of Eq. (4.12), we have

$$v(x,t) = 1 - e^{t-x} + \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[v(x,t) - \frac{1}{2}(v^2(x,t))_x - v^2(x,t)\right]\right]\right], \tag{4.13}$$

since, $\mathbb{N}^{-1}\left[\frac{ue^{-x}}{s(s-u)}\right] = e^{-x}\mathbb{N}^{-1}\left[\frac{u}{s(s-u)}\right] = e^{-x}(e^t - 1)$.
We now assume an infinite series solution of the unknown function $v(x,t)$ *of the form:*

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t). \tag{4.14}$$

Then by using Eq. (4.14), we can re-write Eq. (4.13) in the form:

$$\sum_{n=0}^{\infty} v_n(x,t) = 1 - e^{t-x} + \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[\sum_{n=0}^{\infty} v_n(x,t) - \frac{1}{2}\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n\right]\right]\right], \tag{4.15}$$

where A_n *and* B_n *are the Adomian polynomial which represent the nonlinear terms* $(v^2)_x(x,t)$ *and* $v^2(x,t)$ *respectively.*
By comparing both sides of Eq. (4.15), we can easily generate the recursive relation as follows:

$$\begin{aligned}
v_0(x,t) &= 1 - e^{t-x}, \\
v_1(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[v_0 - \frac{1}{2}A_0 - B_0\right]\right]\right] \\
v_2(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^+\left[v_1 - \frac{1}{2}A_1 - B_1\right]\right]\right] \\
v_3(x,t) &= \mathbb{N}^{-1}\left[\frac{u^2}{s^2}\left[\mathbb{N}^+\left[v_2(x,t) - \frac{1}{2}A_2 - B_2\right]\right]\right], \\
&\vdots
\end{aligned}$$

Thus, the general recursive relation of the unknown function $v(x,t)$ *is given by:*

$$v_{n+1}(x,t) = \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_n(x,t) - \frac{1}{2}A_n - B_n\right]\right]\right], \quad n \geq 0. \tag{4.16}$$

By using the general recursive relation of Eq. (4.16), we can easily compute the remaining components of the unknown function $v(x,t)$ *as follows:*

$$\begin{aligned} v_1(x,t) &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_0(x,t) - \frac{1}{2}A_0 - B_0\right]\right]\right] \\ &= \mathbb{N}^{-1}\left[\frac{u}{s}\left[\mathbb{N}^{+}\left[v_0(x,t) - \frac{1}{2}(v^2)_{0x} - v_0^2\right]\right]\right] \\ &= 0. \end{aligned}$$

Thus,

$$v_{n+1}(x,t) = 0, \quad n \geq 0.$$

Hence, the approximate series solution of the unknown function $v(x,t)$ *is given by:*

$$\begin{aligned} v(x,t) &= \sum_{n=0}^{\infty} v_n(x,t) \\ &= v_0(x,t) + v_1(x,t) + v_2(x,t) + v_3(x,t) + \cdots \\ &= 1 - e^{t-x} + 0 + 0 + 0 + \cdots \\ &= 1 - e^{t-x}. \end{aligned}$$

Therefore, the exact solution of the inhomogeneous gas dynamics equation (4.9)–(4.10) is given by:

$$v(x,t) = 1 - e^{t-x}.$$

The exact solution is in closed agreement with the result obtained by (RVIM) [3] and (ETHPM) [7]

5 Conclusion

In this paper, we introduce an efficient technique called the Natural Decomposition Method for solving gas dynamics equation without using any transformation, linearization, discritization or taking some restrictive assumptions.

The new technique provides an elegant series solution which converge very rapidly with reduced computational size, and avoids round off errors. The exact solution obtained by the new technique is in excellent agreement with the results of the existing methods. Thus, the proposed technique is a powerful, reliable and efficient mathematical tool for solving nonlinear partial differential equations.

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