International Mathematical Forum, Vol. 9, 2014, no. 1, 19 - 36
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/imf.2014.311225

Geo-informatics of Multi-scale Solutions in Navier-Stokes Equations for Deformed Porous Space

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Abstract

Within the framework of block self-organizing of geological bodies with use of deformation theory the mathematical solution of a problem for effective final speed is proposed. The analytical and numerical integrated solutions of Navier-Stokes equation for deformable porous space were obtained. The decisions of multi-scaled regional problems «on a flow basis» were also presented: from lithology of rock space - to a well and from a well - to petro-physics. The evolutionary transformation of the linear solution of the equation on mass conservation up to the energetically stable non-linear solution of the equation on preserving the number of movements is also offered.

Keywords: reservoir simulation, palaeo-genesis deformations, smart field development, segmental; well, complex-organized reservoir

1. Introduction

Now the oil companies more and more face the problems with new well production rate reduction. In order to maintain and increase the levels of oil production it is necessary to locate the wells in challenging geological conditions (compacted low-permeable rock; reservoirs with high clay contents; decompressed unconsolidated zones). All these result in high uncertainty of calculations, both for initial production rates and for long-term well operation forecast.

Thus the new technologies applied for the reservoirs, the development of means, technologies of control and production management as well as modeling the process of hard-to-recover field development had not found the adequate presentation in mass-scaled decisions inside the oil companies. It is related with the fact that very often there are no specialized physical/mathematical algorithms and techniques to be used for challenging cases of oil-saturated reservoir structures that finally allow improving the geophysical control and monitoring over the processes of their development.

The scientific support for the given problem is directly connected with the solution of important theoretical and applied tasks [1,2], such as:

- Final transfer of influence with the due account of rock petro-physical parameters: oil-saturation, compressibility, porosity, etc. New fundamental decisions on productive wells and base production from oil-saturated and condensate reservoirs.
- Contact between fluid flow and porous media in space being one of basic problems of mathematical theory to make mass-scaled averaging of volumetric mass transfer and wave transfer of movement amount;
- Interpretation of well test data with the use of conceptually new information on reservoir structure (dynamic skin factor, anisotropy, lamination, updating of knowledge on top and bottom location, etc.).

The result of these works was the development of new knowledge on challenging geological media, on system organization [12] of multi-scale phase movements, development of concepts on auto wave phenomena, principles synergy and self-organizing of geospheres of the Earth.

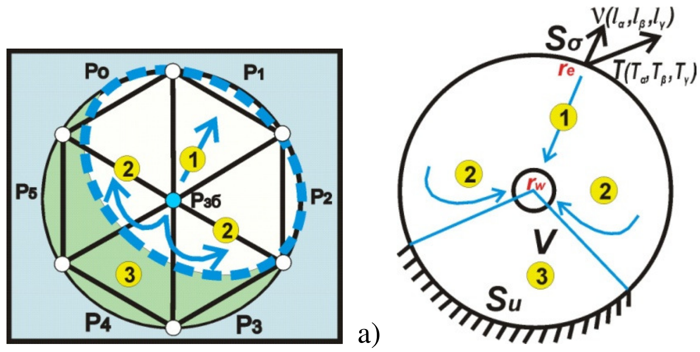
2. Theoretical Substantiation of a Problem

The complex phase trajectories fractured reservoir development process by and the respective formation pressure of consolidated matrix seismic emissions cannot be described by the parabolic equations of geological modeling and reservoir simulation. The geophysical fields of genesis and phase borders of inflow to wells are characterized by the following common abnormal features [2]:

- by complicated organization of space in macro- and micro-world with the absence of rigid structural control;
- by fluctuations of inflow profiles from zero up to hundreds of m3/day, that do not agree with low meanings of porosity and permeability;
- through restrictions of pools by faults, replacements, wave alluviums, consolidation, and for wells - by segments with no inflow from low-permeable, stagnant, capillary jammed zones;

- by tangential pressures, non-equilibrium temperature, viscosity, convective-diffusional shear deformations, non-equilibrium lithology, erosion.

During the injection of water that have formation temperature and hot water [14-18] at asynchronous multi-scale borders with various time delays there observed the power losses: water breakthroughs, slipping into high-permeable segments, self-organizing of man-made channels, suppression of filtration at the expense of energy cutting-off water-oil potential and deterioration in RPP (relative phase permeability), Fig.1(a). They arrange the model of tangential stresses of "glass – trap" type with areas of zero velocities and low-permeable jammed areas.



a) b)

Fig.1. (a) water-flooding element, (b) conformity conditions for deformational displacement: 1) channel-type, 2) porous-type, 3) diffusion-type.

The method to calculate zonal non-uniform well inflow has been offered by M. Muskat [11]. The solutions is considered in generalized form as:

$$p = c_o\, ln\, r + \sum r^{\alpha}\,(a_{\alpha} sin\, \alpha\theta + b_{\alpha} cos\, \alpha\theta) + \sum r^{\alpha}\,(c_{\alpha} sin\, \alpha\theta + d_{\alpha} cos\, \alpha\theta).$$

The rate in view of static equilibrium conditions as per Masket is $Q=2\pi k(p_e/s-p_w)/\mu\ ln(r_e/r_w)$, where k is reservoir permeability, s – angle, p_e, p_w. r_e, r_w - pressures and radii accordingly at the well contour, μ - dynamic viscosity. However, this decision is not used in the analytical analysis of reservoir development, nor in geo-monitoring, nor in numerical modeling of porous media. That, actually, complicates interpretation of well geophysical studies, including seismic ones and deforms volumetric 3D multiphase structure of fluid movement and on self-organizing of geological structures in thermal, electro-magnetic and gravitational fields during the arrangement of deposits and geo-hydro-dynamical processes of modern geology, exploration and field development [18].

First of all let's enumerate the hydro-dynamic problems and geological risks in resolving the equations of thermal conductivity type, used in commercial 3D simulators that are eliminated by generalized solutions of Navier-Stokes equations and viscous-elastic deformation of porous space.

- The uncertainty with shift migration and ambiguity with top and bottom sides, intra-well space during various cycles of oil and gas field development, seismic study, search and exploration.

- The final velocity of effect transfer, the value of drainage and reservoir oil productivity; dependence on porosity, permeability and reservoir heterogeneity.
- The integrity and combination of hydro-dynamic solutions with geological and geo-physical information, growth in geo-information of numerical models.

The equations of equilibrium connecting stresses and volumetric forces F_α, Fig.1, (a,b), are as: $L_\alpha(\sigma)+F_\alpha=0$, где $L_\alpha(\sigma)=\partial\sigma_\alpha/\partial\alpha+\partial\tau_{\alpha\beta}/\partial\beta+\partial\tau_{\alpha\gamma}/\partial\gamma$, ($\alpha$=1-3). The normal and shear deformations ε_{ij} are connected with movements ξ_i through geometrical ratios:

$$\varepsilon_\alpha=\partial\xi_\alpha/\partial\alpha,\ \varepsilon_{\alpha\beta}=\partial\xi_\alpha/\partial\beta+\partial\xi_\beta/\partial\alpha,\ (1,2,3). \quad (1)$$

Excluding from (1) the movements, it is possible to get the equations of conformity for the deformations. The solution of a system with equations on the theory of elasticity should satisfy the boundary conditions given at the surface of the body (Fig.1,b). The geometrical boundary conditions determining the character of fixing the body are imposed directly upon the movements. The static boundary conditions determining the character of body stresses through superficial forces, are presented as follows:

$$t_\alpha=T_\alpha \quad (1\text{-}3). \quad (2)$$

where T_α - projection of superficial load at axis 1-3; $t_\alpha=\sigma_\alpha l_\alpha+\tau_{\alpha\beta}l_\beta+\tau_{\alpha\gamma}l_\gamma$, where σ_α, $\tau_{\alpha\beta}$, $\tau_{\alpha\gamma}$ – stresses at the boundary surface; l_α - directing cosines of standard v to this surface. From all possible options the true condition of a body having a place with given stresses and conditions of fixing may be selected with a help of power criterion of equilibrium. First of all, let's give definition of such possible condition.

Let's move u_α so that they are continuous functions of coordinates and correspond to conditions of fixing the examined body. It is obvious, that there is a set of moving systems that make a set of kinematically possible conditions of the body. For each of these conditions the geometrical ratios allow to find deformations and the physical ratios - to define the stresses. Generally these stresses will not satisfy the equations of equilibrium and static boundary conditions (2). The one of these possible moving systems at which the equations (1) and condition (2) are satisfied, is true one.

3. Polarization of Porous Space Microstructure, Symmetry of Viscous Drain and Elastic Deformation

3.1. Increase in Geo-information Knowledge on Geological Modeling and Reservoir Simulation of 3D Filtration

The availability of porous media radically changes thermodynamics of hydrocarbon mixes [4]. The existence of superficial layers having internal structure is proven experimentally and theoretically.

A.N. Dmitrievskiy has generated the concept «of vortex geo-dynamics» of the

Earth and block self-organizing of hydrocarbons (HC) [1]. The system multi-scaled approach in increasing geo-information knowledge during search, exploration and development of oil/gas fields, study of Palaeo- and Neo-tectonic evolution of lithosphere, effect from man-made processes upon biosphere and antroposphere is further development of V.I. Vernadskiy ideas.

In spherical shells of the Earth there occur the wave processes reflecting energy carry-on and transformation. These are electromagnetic, gravitational and thermal fields, elastic seismic waves, influence of phase transitions and more. Between the shells there are reflecting borders that create energy barriers between them. During the arrangement of elements of auto-wave system the cells capable to keep their own energy are formed. Concentrating in these cells the energy in a resonant mode interacts with adjacent cells and in case with “energy over-emission” increases its amplitude.

The change in the level of dynamic heterogeneity of the media results in spasmodic qualitative changes with zonal heterogeneity expressing in processes that change their macroscopic structure. The massive transitions in heterogeneous media are based upon the combination of the mathematical theory of averaging for “operators” with high-oscillating properties, asymptomatic methods and theory of filtration [2,3,7].

The planetary stresses of lithosphere form dynamic structures of stretching and compression as well as diagonal structures of shears. Thus they form the pools with various structural confinements[1].

First, structures connected to erosive surfaces of the foundation and at their step-like immersing.

Second, with lateral distribution of stresses at stretching, there appear finer structures for which the influence of sedimentological factor (con-sedimentalogical processes) is characteristic.

Third, the stress of compression is distinctly seen arranging neotectonical structures.

The stress/deformed stratus of geo-media shows [13], that in inclined saturated porous space there is a 3D-scaled deformation of well inflow structure (Fig. 2). Along the main axis with normal loading the tangent stress lines can exceed normal one in several folds, forming abnormally high porous pressure, cavernous channels, super-reservoirs connected with decompressed rock and deformation of relief. Macroscopic channels (1), generically connected with the main hydro-dynamical channel have boundary (with reservoir) (3) shell of micro-level scale, momentary layer of shear stresses and deformations (2). True filtration velocity is a dynamic porous structure (decompressed, consolidated and stagnant diffusive zones is associated with effective permeability k_α (1,2,3) with various scales of anisotropism.

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[1] An L.Y. Paleochannel sands as conduits for hydrocarbon leakage across faults: an example from the Wilmington oil field. California. AAPG Bulletin. Amer.Assoc.of Petroleum Geologists. 2009.

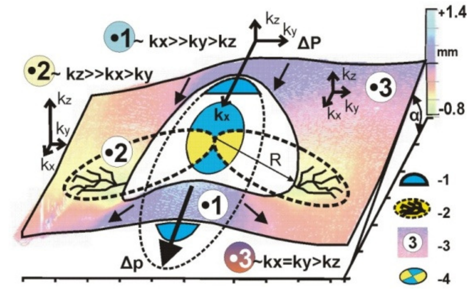

Fig 2. Relief of deformations of decompresses (1), consolidated (2) and stagnant (3) zones of loading porous structure permeability *k* (1,2,3)

Momentary phenomena of a micro-level, generation of elastic waves by a viscous well drain and borders of geo-media named as acoustic emission, caused by a sudden reorganization of structure in natural rock being at a state of stress-deformed status, are found out at the stages of search, exploration, micro-seismic jobs, hydro-fracturing and neo-tectonic of the reservoirs under development. Beside the occurrence of dissipative harbingers of geo-acoustic emission (sharp attenuation of longitudinal velocity) at a distance of about hundred meters away from a source deformation perturbance (wells) the signals have expressed distinct anisotropy. In [27] while resolving the problem of R.D.Mindlina (1936) it was shown that with object development stimulation there is always a seismic problem to evaluate superficial vertical fluctuations resulted with reservoir expansion caused by injection.

In case with zonal heterogeneity there are the areas with zero filtration velocity in capillary-jammed units (not as per Darcy), with no-through pressurized filtration. For arranging the displacement, i.e. drain from similar sealed cells ("glass – trap type") the limiting energy moment "of twisting" is necessary.

The same shear-type momentum filtration in porous space is described by system of Navier-Stokes equations consideration of mean macroscopic velocity *U* and impulse components *u*, v*, w*. Macroscopic velocity and pressure are defined as averaging by Steklov the appropriate fields continued by zero on the firm body and connected deformation in porous space. The inequality of these equations to Darcy system is also illustrated [8-10].

3.2. Energy Interface of Viscous Filtration and Pore Deformation

Darcy law is the first member of asymptotic decomposition of Navier-Stokes stationary equations. For rotating porous member located on a smooth rigid surface with radial blow-in (to a well) the solution of these Navier-Stocks equations was obtained ([8], 1977) under condition when $U(\eta)=\eta$. On a pliable surface with due account for cross displacement of a non-extendible membrane solutions was obtained by G.F. Trifonov (1978). For sheared layer of seismic emission in porous media the solution will be defined by the mixed boundary conditions at $\eta=0$, taking into account both cross and longitudinal displacements of deformations [19-24]

$$u=\partial\xi_1/\partial t,\ \mathrm{v}=\partial\xi_2/\partial t-\xi_2\partial U/\partial\eta,\ -p+\partial u/\partial\eta=\sigma_{11}/\rho u_*^2,\ \partial u/\partial\eta+ik\mathrm{v}=\sigma_{12}/\rho u_*^2 \qquad (3)$$

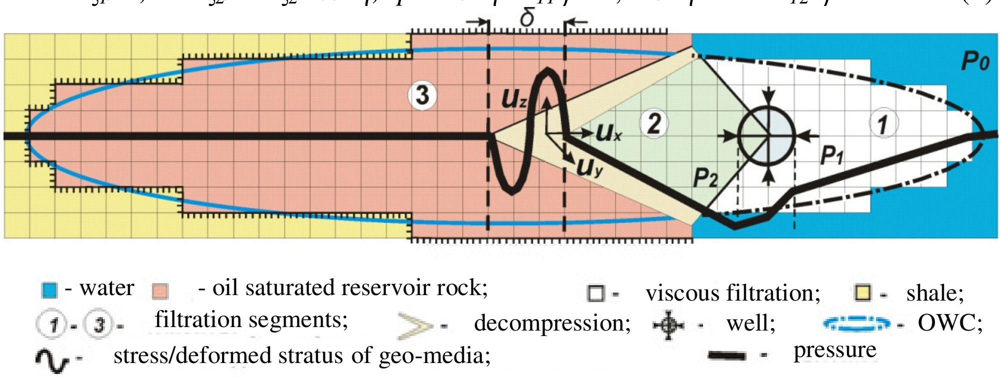

Fig.3. Interface of viscous drain (1) and deformations of equilibrium (2,3)

The task to determine the kinematic and dynamic characteristics of multi-layered viscoelastic sheared layer subject to normal pulse-type loads of pressure is arranged as follows [6, 20-23], Fig. 3.

The system of equations related to the movement of viscous-elastic geo-media and generalized Hook's low are resolved as

$$\sigma^q_{ij,j}=\rho^q\partial^2\xi_i/\partial t^2;\ \sigma^q_{ij}=\mu^q(\xi_{i,j}+\xi_{j,i})+\lambda^q\delta_{ij}\xi_{i,i}, \qquad (4)$$

where $\sigma^q_{ii}/3=p$ – effective stress (pressure), ξ_i - shifting; λ^q, μ^q, ρ^q – generalized Lame parameters, density, $q=1\text{-}N$, N – number of layers, $i,j=1\text{-}3$.

Geo-media is simulated by the generalized model of viscous elasticity jointly with relaxation time spectrum. The generalized model of viscous elasticity is defined by the dynamic shear modulus

$$\mu^q(\omega)=\mu^q_o+\Sigma^n_{j=1}\mu_j(w\tau_j)^2/(1+(w\tau_j)^2)-i\Sigma^n_{j=1}\mu_j w\tau_j/(1+(w\tau_j)^2),$$

obtained at the basis of relaxation function, presented by a sum of exponent $\mu(t)=\mu_o+\Sigma^n_{j=1}\mu_j e^{-t/\tau j}$ at harmonic law on loadings. For viscous-elastic media the dynamic volumetric module is equal to static one, that is why

$$\lambda^q(\omega)=\lambda^q_o-2/3\Sigma^n_{j=1}\mu_j(\omega\tau_j)^2/(1+(\omega\tau_j)^2)+i2/3\Sigma^n_{j=1}\mu_j\omega\tau_j/(1+(\omega\tau_j)^2).$$

At the boundaries of the intermediate layers

$$\sigma_{ij}^{q-1}|_{x2=hq}=\sigma_{ij}^q|_{x2=hq};\ \xi_{ij}^{q-1}|_{x2=hq}=\xi_{ij}^q|_{x2=hq}.$$

External surface $\xi_i|_{x2=0}=0$ - motionless or free $\sigma_{ij}|_{x2=0}=0$. (5)

The movement of viscous incompressible liquid is described by the Navier-Stokes equations

$$\partial v_i/\partial t+v_j v_{i,j}-\langle v_j v_{i,j}\rangle+v_j U_{i,j}+U_j v_{i,j}=-1/\rho_f\,\partial p/\partial x_i+\nu\nabla^2 v_i \qquad (6)$$

and indissolubility: $\mathrm{v}_{i,i}=0;\ \nu U'=u_*^2+\langle\mathrm{v}_1\mathrm{v}_2\rangle$, where v_i, U_i - pulse and mean value of velocity, $\langle\mathrm{v}_j\mathrm{v}_{i,j}\rangle=(\mathrm{v}_{i,j}\mathrm{v}_j^*+\mathrm{v}_{i,j}^*\mathrm{v}_j)/4$, $*$- complex interface, ν - kinematic viscosity, ρ_f – fluid density. While neglecting the quadratic terms of velocity pulsations, the equations of movement for viscous incompressible fluid on a deformable surface supposes to get the solution as waves:

$$\mathrm{v}_i=\mathrm{v}_i(x)\exp i(k_j x_j-wt),\quad p=p(x)\exp i(k_j x_j-wt). \qquad (7)$$

Substituting (6) in system of equations (5) and neglecting quadratic terms of velocity pulsations we receive the system of ordinary differential equations for

Fourier factors v_i and p:

$$u''\text{-}k^2u=G_1(\eta); \quad (8)$$

$$\mathrm{v}''\text{-}k^2\mathrm{v}=G_2(\eta); \quad (9)$$

$$\omega''\text{-}k^2\omega=G_3(\eta); \quad (10)$$

$$ik_y u+ik_z\omega + \mathrm{v}' = 0, \quad U^{\mathrm{I}} = 1 + \langle u\mathrm{v}\rangle. \quad (11)$$

where $G_1(\eta)=ik_y(U\text{-}c)u+\mathrm{v}U'+ik_y p$; $G_2(\eta)=ik_y(U\text{-}c)\mathrm{v}+p'$; $G_3(\eta)= ik_y(U\text{-}c)\omega+ik_z p$. Here the variable values are dimensionless with minimal speed of capillary impregnation u_* and length $l^*=\nu/u_*$.

Differentiating the equation (9) by η and by combining it with equations (8,10), previously multiplied accordingly by ik_y and ik_z, we shall get the equation for pressure

$$p''+k^2p=G_4(\eta), \text{ where } G_4(\eta)=-ik_y U'\mathrm{v}. \quad (12)$$

For the solution of equation (8) we shall take advantage of "variations of constant" method. The solution is searched as $u=A(\eta)\varphi_1\text{-}B(\eta)\varphi_2$, where A, B are defined from conditions $A'\varphi_1\text{-} B'\varphi_2=0$; $A'\varphi'_1\text{-} B'\varphi'_2= G_1(\eta)$. This means, if $\varphi_1(\eta)=e^{-k\eta}$, $\varphi_2(\eta)=e^{k\eta}$ – is a fundamental system of solutions of appropriate homogeneous differential equations, then $u=\varphi_2\int\frac{\varphi_1 G_1}{W}dt - \varphi_1\int\frac{\varphi_2 G_1}{W}dt +c_1\varphi_1 +\bar{c}_1\varphi_2$, where $W(t)= \varphi_1\varphi_2' - \varphi_1'\varphi_2$, there is a solution for the equation (8). In our case $\varphi_1\varphi_2' - \varphi_1'\varphi_2 = k$. Accepting $\bar{c}_1=0$, i.e. considering only the decreasing η wave, the general solution of the equation (8) will be written down as

$$u= \frac{1}{k}\int G_1(t)\, sh\, k(\eta-t)\, dt +c_1e^{-k\eta}. \quad (13)$$

Similarly we shall write down the solutions of the equations (9,10) and pressure

$$\mathrm{v}(\eta)= \frac{1}{k}\int G_2(t)\, sh\, k(\eta-t)\, dt +c_2e^{-k\eta}, \quad (14)$$

$$w(\eta) = \frac{1}{k}\int G_3(t)\, sh\, k(\eta-t)\, dt +c_3e^{-k\eta}, \quad (15)$$

$$p(\eta)= \frac{1}{ik}\int G_4(t)\, sh\, ik(\eta-t)\, dt +c_4e^{-ik\eta}. \quad (16)$$

Let's introduce the function of viscosity $\theta(\eta)$:

$$\theta(\eta) = v'' - k^2v, \text{тогда } P(\eta) = 1/k^2\{\theta' - i\alpha[(U-c)v' - U'v]\}. \quad (17)$$

Substituting (17) in the equation of cross pulsation (9) we get

$$\theta'' - [k^2 + i\alpha(U-c)]\theta + i\alpha vU'' = 0. \quad (18)$$

If to consider a layer of constant stress and to take into account the fact that in viscous sheared layer $U(\eta)=\eta$, we shall get the system of linear differential equations (8-12), the solutions of which can be written down in quadratures:

$$\theta'' - \left[k^2 + ik_y(\eta-c)\right]\theta = 0, \quad (18)$$

$$\mathrm{v}'' - k^2\mathrm{v} = \theta, \quad (19)$$

$$w'' - \left[k^2 + ik_y(\eta-c)\right]w = ik_z p, \quad (20)$$

$$p = 1/k^2\{\theta' + \left[ik_y(\eta-c)\,\mathrm{v}' - \mathrm{v}\right]\}. \quad (21)$$

Let's write down the solution of equation (18) as decreasing function of Airy:

$$\theta(\eta) = C_1 Ai\big(k_v(\eta)\big), \text{где } k_v(\eta) = (ik)^{-2/3}\left[k^2 + ik_y(\eta-c)\right], \quad (22)$$

which with large values of η has asymptotic presentation as

$$\theta(\eta) \sim C_1 \text{?xp}[-1/3\sqrt{2}\alpha(1+i)(\eta-C)^{3/2}]. \quad (23)$$

The solution for component velocities v and w we shall write down as quadratures:

$$v(\eta) = \frac{1}{k}\int \theta(t)\, sh\, k(\eta - t)dt + C_2 e^{-k\eta}, \tag{24}$$

$$w(\eta) = C_3 Ai\big(k_v(\eta)\big) + i\beta \int_0^\eta P(t)G(\eta,t)\, dt, \tag{25}$$

$$u(\eta) = [iv' - \beta w]/\alpha, \tag{26}$$

where $G(\eta,t) = \pi\big[Bi\big(k_v(\eta)\big)Ai\big(k_v(t)\big) - Ai\big(k_v(\eta)\big)Bi\big(k_v(t)\big)\big]$.

For Airy function of the second sort $Bi\big(k_v(\eta)\big)$ the varying branch is chosen.

The conditions on the surface of deformable layer (3) is defined to within the accuracy of up to normalized multiplier C_1, determined by the intensity of pulsing pressure at the external boundary of a layer: $p=p_\delta$ at $\eta=\delta$.

$$\frac{\sigma_{12}}{\rho u_*^2} - i[k_y + k(1 - Ai'\big(k_v(0)\big)](\frac{\partial \xi_2}{\partial t} - \xi_2 U'(0)) + Ai'\big(k_v(0)\big)k_y \frac{\partial \xi_1}{\partial t} = \frac{iC_1}{k_y},$$

$$\frac{\sigma_{11}}{\rho u_*^2} + (\frac{ik_y(1-ck)}{k^2} + k)[\frac{\partial \xi_2}{\partial t} - \xi_2 U'_\eta(0)] = -\frac{Ai'(k_v(0))}{k^2}C_1 \tag{27}$$

If to consider the rigid border of reservoir replacement then the condition of adhesion is represented as $C_2=C_3=0$, the profiles of pulsing velocities are presented as:

$$\mathrm{v}(\eta) = C_1/k \int_0^\eta Ai\big(k_v(t)\big) sh\, k(\eta - t)\, dt, \tag{28}$$

$$w(\eta) = C_1 \int_0^\eta G(\eta,t)p(t)\, dt, \tag{29}$$

$$u(\eta) = C_1/k_y\{i\int_0^\eta Ai\big(k_v(t)\big) ch\, k(\eta - t)\, dt - k_z \int_0^\eta G(\eta,t)p(t)dt\} \tag{30}$$

wh?r? $p(t) = 1/k^2\{Ai'\big(k_v(t)\big) - ik_y(t-c)\int_0^t Ai\big(k_v(\xi)\big) ch\, k(t-\xi)\, d\xi -$

$$-1/k\int_0^t Ai\big(k_v(\xi)\big) sh\, k(t-\xi)\, d\xi\}.$$

Following the conditions of interface with asymptotic dependence $U/u_*=Q/(2\pi RH\phi u_*)=1/æ\ ln\ \eta+C_æ$, where m - porosity, H – reservoir thickness and smooth interface with diffusion area in some point R_o, we find the unknown parameters *æ* and $C_æ$ of the mean viscous inflow to a wells. The parameters of movement in a buffer zone are found using the formulas $æ=\phi u_*/R_o \cdot U^I(R_o)$; $C_æ=U(R_o)/u_* - ln\ R_o/æ$.

Basing upon the analysis of the designed stress values that are responsible for stability of mean velocity it is possible to make the following conclusion: at any phase velocity of the stress they have a steady type till $\eta\sim35$. This means that point of interface with asymptotic continuation of profiles is in the area of $\eta\sim35$.

The solutions for $U(\eta)$ are used to get the first evolutionary approximation in numerical process to account for the profiles of mean velocity at various geological borders of the facies with different filtration conditions: from free elastic permeable like membrane up to capillary jamming at stagnant zones.

3.3. Mean Well Inflow Profile Characteristics in Deformed Porous Space

The superficial waves bring in such perturbance in near-surface layer of mean mass transfer, that the dynamic values in them cannot be submitted in a simple superposition for mean and fluctuating with frequency waves of fields, i.e. as interference picture of wave interaction.

The nonlinear and viscous effects in hydro-dynamic waves are illustrated as a property to raise directed mean energy transfer, pulse of heat and mass. This is the feature of the physical media by itself, where the reaction cannot be instantaneous. Therefore, there is an integrated reaction (adaptation), at which pulse and energy are dispersed in final volume, i.e. there is their transfer into large spatial-time scales of movements that is display of inertial media property and impossibility to distribute the perturbance with infinite velocity.

The decisions for pulsing pressure fields and velocity allow us to define the mean pulse flow for the specified period of time in a vertical direction caused by correlations between fluctuating field of velocity and pressure, i.e. finally, by waves.

The waves change the stationary part of mean flow well profile. This is related with the existence of stationary pulse and energy transfer from an average field of velocities to superficial waves or from them back. The physics of this phenomenon is as follows: the emerging phase shear between wave components of velocity due to inertness of wave field results in occurrence Reynold's pressure (distinct from zero) which carry out negative in relation to the work with mean velocity fields causing a flow of energy coming from mean filtration velocity of well inflow to superficial waves.

Shear stresses coinciding in their direction with the pressure wave distribution velocity, i.e. if the torque coincides with a direction of filtration flow - increase the diffusional flow in a layer and the velocity of impregnation. Thus the oil flow from viscous-elastic porous media determined by $\langle \tau v \rangle$. is increased. So, the viscous-elastic layer at the displacement front becomes redistributed one.

$$\overline{uv} = Re\{\frac{1}{k^2}\iint G_1(t_1)\, sh\, k(\eta - t_1) G_2(t_2) sh\, k(\eta - t_2) dt_1 dt_{\,2} + C_1 C_2\, e^{-2k\eta} + \\ + \frac{C_2}{k}\int G_1(t_1)\, sh\, k(\eta - t_1)\, e^{-k\eta} dt_1 \, + \frac{C_1}{k}\int G_2(t_2)\, sh\, k(\eta - t_2)\, e^{-k\eta} dt_2\}$$

At the surface of viscous-elastic film layer of capillary jammed oil phase the correlation of pressure pulses with longitudinal speed $\overline{pv} = \left(\frac{\partial \xi_1}{\partial t} p^* + \frac{\partial \xi_1^*}{\partial t} p\right)/4$ defines the diffusion of pulsing energy at the border. On a rigid and ideally pliable surface the correlation of these sizes is equal to zero.

$$\overline{pv} = Re\{\frac{1}{ik^2}\iint G_2(t_2)\, sh\, k(\eta - t_2) G_4(t_4)\, sh\, ik(\eta - t_4) dt_2 dt_{\,4} + C_2 C_4\, e^{-2k\eta} + \\ + \frac{C_4}{k}\int G_2(t_2)\, sh\, k(\eta - t_2)\, e^{-ik\eta} dt_2 \, + \frac{C_2}{ik}\int G_4(t_4)\, sh\, ik(\eta - t_4)\, e^{-k\eta} dt_4\}$$

Factor of diffusion flow $Y_q=\overline{pv}/\Phi$, where

$$\Phi = \frac{w}{4}\int_0^h i\{[\lambda^*(w) - \lambda(w)]\varepsilon_{ii}\varepsilon_{ii}^* + [\mu^*(w) - \mu(w)]\varepsilon_{ij}\varepsilon_{ij}^*\}\,dx.$$

in case with no shear stress is counterbalanced by dissipation velocity Φ , i.e. $Y_q=1$ with all the frequencies.

Basing upon the solution of the task we have obtained the combined stress-deformation status of homogenous layer of seismic emission, dissipation velocity, potential and kinetic energy of elastic boundary layer with dissipative structure of the generalized model of viscous-elastic material depending on wave number, velocity of shear movement, thickness of layer and its mechanical parameters.

3.4. Natural Properties of Mixed Deformed Viscous-Elastic Layer

Equations (4), presented in a form of Helmholtz, in a cylindrical system of coordinates [20-23,26] has the type of Bessels' differential equation. Ther solution of the resulted system is written down in a form of longitudinal and transversal waves with complex wave number of $k=\alpha+i\beta$:

$\varphi(z_\varphi) = a_1 J_0(z_\varphi) + a_2 Y_0(z_\varphi)$; $\psi(z_\psi) = a_3 J_1(z_\psi) + a_4 Y_1(z_\psi)$; где $z_j=k_jR$, $j=\varphi,\psi$.

While satisfying the boundary conditionsm we get the transcendal system of equations of the 4th order. This gives the characteristic equation to find the nominal frequencies of sheared layer $det\{A\}=0$.

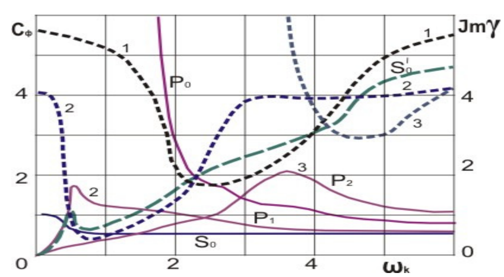

Fig. 4. Phase velocity and attenuation factor for bended and transversal waves of permeable viscous-elastic porous layer

Fig.4. Contains the values of dimensionless phase velocity $c_\Phi=c/c_o$ (continuous line), where $c_o=\sqrt{\mu(3\lambda+2\mu)/(\lambda+\mu)\rho_s}$ and attenuation factor δ (dotted line) for the free layer depending on frequency $w_\kappa=wh/c_o$. Practically from zero frequency there is a bended wave extending with velocity as defined using Young's modulus. With frequency growth the phase velocity aspires to the velocity of transverse wave. With natural frequencies the sharp drop in attenuation factors is observed.

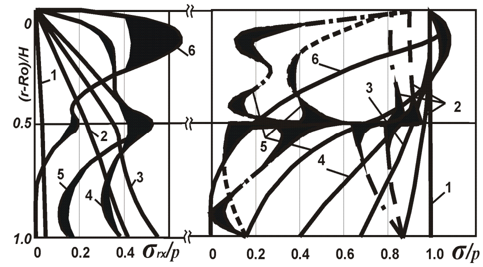

Fig. 5. Polarization of normal stress for viscous drain in porous geo-subsurface, self-organizing of channels of vertical shear deformations

In area of natural frequency ω_κ~1 the amplitude becomes several dozen times larger than thickness of a shift layer, forming transversal fluid inflows and attached lithology. Anisotropy of permeability becomes <1, increasing vertical displacement, water cross-flows, meniscus of inclined gas-oil and water-oil contacts, wave alluviums of carbon/micas clay particles at the borders of oil-containing rock.

The profiles of transversal and attenuated normal stresses (Fig. 5) in the fixed layer with parameters: $\mu_0^2/\mu_0^1=5$, $\rho^1=\rho^2$, $\tilde{\alpha}$ =0,04 (1); 0,5 (2); 1 (3); 2,5 (4); 5 (5); 10 (6), where $\tilde{\alpha}=kh$, show the structure of the layer and are used at inversion of 3D seismic. As is seen in Figure the greatest normal stress occurs at the surface, and inside the layer has the minimal value, i.e. the radial component of displacement naturally fades. For tangential stresses this dependence is of the other kind: the maximum values are at some depth of the cover or at the surface, i.e. there is the regeneration of a radial component of moving vector into a tangential one to turn fluid component round the axis, the normal stress is split and the velocity changes the phase.

3.5. Stability of Interaction of Mean Characteristics of Mass Transfer and Pulsing Deformations

To understand the thin equilibrium between a viscous layer and non-viscous internal area as well as to confirm a principle of maximal stability for mean characteristics of pulsing boundary layer, we should consider a task to simulate the pulsing current on a viscoelastic porous surface. Thus by an iterative way we take into account the non-linearity of a profile $U(\eta)$ at the basis of the decision in quadrature.

The system of non-linear differential equations (6) to simulate the pulsing of a boundary layer at filtration is resolved. The profile of mean velocity $U(\eta)$ is set on the basis of a principle of maximal stability for mean characteristics received after resolution of a system of linear differential equations (21-26), proceeding from approximation in viscous sub-layer $U(\eta)=\eta$.

The given system of non-linear differential equations is resolved numerically, by iterative approximation in final differences. To determine the function of viscosity $\theta(\eta)$ and pressure $p(\eta)$ while resolving the equations (8-12) the value of pulsing v(η) is taken from the previous iterative step.

Conditions of the interface which have been written down in the form that doesn't contain the unknown parameters, look like: $\eta\tau^{I} +\tau=1,\ \eta\tau^{II} +2\tau^{I}=0,\ \eta\tau^{III} +3\tau^{II}=0,\ \eta E^{II} +2E^{I}=0,$ where $\tau=-\rho_f$<uv>, $E=(u^2+v^2+w^2)/2$. First of them is a condition final normalizing. The requirement to have maximum curvature in a point of interface $|E^{II}(R)|$, which provides the maximal stability of the considered profile and is a final precondition in defining the pulsing amplitudes and unknown parameters k, C, R.

At each iterative stage the task is to pick up the unknown parameters of k and c for the pulsing movement, so that to make smooth interface in some point R with the profile of mean velocity down to U^{IV} and by mean pulsing energy - down to E^{II} with asymptotic dependences $U=1/æ\ ln\ \eta +C_æ$, $E=E_1(1+B/\eta)$.was performed.

The parameters of movement in a buffer zone are under established by the formulas: $æ=ф/R_o \cdot U^I(R)$; $C_æ=U(R)-lnR/æ$; $-B=R^2E^I(R)/(E(R)+RE^I(R))$; $E_1=RE(R)/(B+R)$.

Thus, the profile of mean velocity is determined in complete zone of viscoelastic properties' influence for the sheared layer of porous border. Distribution of pressure (by Reynolds) τ/ρ_f=-<uv> shows, that the non-linearity stabilizes the profile in relation to small perturbance both at a rigid and viscoelastic basis. The task then becomes correct.

The calculation has shown such an important feature as the leap by 180° for the relative shear between pulsing pressure and tangential component of a pulsing velocity upon the transition through a layer of concurrence, where the local vector of velocity is equal to phase velocity of waves, i.e. a point of phase leap $U(\eta_K)=c$, that is determined by the initial profile $U(\eta)$.

Being the first approximation to the subsequent non-linear iterative evolutionary specifications, the linear approximation of essential non-linear equations of Navier-Stokes with good quality and correctly reflects distribution of pulsing fields of velocities and their correlation moments. The point of interface being the border of viscous sub-layer and zone of generation for mean filtration channel, correctly defines even the very first linear iterative stage of the calculation. While evaluating the non-linearity $U(\eta)$ the specific point $\eta=R$ is not mixed and is equal to $R = 32$.

This confirms a physical solvency of linear approximation of a zone with constant stress $\eta=0\text{-}50$.

4. Physical and Mathematical Modeling of Filtration in View of Inertia-capillary balance

The resulted energy-stable solution of mean inflow profile enables to arrange *3D* multi-phase super-simulator of a new generation to calculate the net-type models of geological objects under development with a volume of thousand millions cells using the method of parallel calculations separated by physical properties. The calculation of mean filtration fields may be done using the volumetric net of up to U_{min}~$32u_*$ irrespective of boundary conditions. While reaching the surface of critical equilibrium points it is necessary to shift to another method, namely to square-type low of wave impulse of filtration attenuation using in this case the planes cell sides to transfer the wave impulse. The regulating parameter of this innovative simulation model as per the proposed analytical solution of Navier-Stokes equation for deformed porous space is its velocity and not the pressure at all. [3,17,25].

"The Pseudo-potential method of flow" at the basis of the equations for hydrodynamics with use of pseudo-functions of inertia-capillary phase balance of related phase permeability (RFP) and petro-physical constants with interface of various scales in the areas of capillary, gravitational and boundary surfaces, is realized in a software package (*"Flora"*) [19].

The equations of three-phase 3D (black-oil) simulators used by the leading international oil companies look as follows:

$$div(\lambda_\alpha(\nabla p_\alpha - \gamma_\alpha g\nabla D)) = \frac{\partial}{\partial t}(\varphi\frac{S_\alpha}{B_\alpha}) + Q_\alpha \qquad (37)$$

$$div(\lambda_g(\nabla p_g - \gamma_g g\nabla D)) + R_g div(\lambda_o(\nabla p_o - \gamma_o g\nabla D)) = \frac{\partial}{\partial t}(\varphi(\frac{S_g}{B_g} + R_g\frac{S_o}{B_o})) + Q_g + R_g Q_o \qquad (38)$$

$$S_w+S_o+S_g=1; \quad p_o - p_{\acute{\alpha}} = -p_{co\acute{\alpha}}; \ \alpha=o,w; \ \acute{\alpha}=w,g \qquad (39)$$

where p_α - phase pressure, S_α – saturation, $\lambda_\alpha=k{\cdot}k_{r\alpha}/B_\alpha\mu_\alpha$ - mobility, R_g - solubility of gas, k - permeability; $k_{r\alpha}(S_\alpha)$ - RFP; Q_α – rate, D – depth, α - phase.

The ratio of hydrodynamic and capillary forces is expressed in dimensionless parameter of capillary number $N_c=\mu v/\sigma$, where σ - superficial tension. In case with off-stationary filtration this dependence between RFP and N_c is accepted as follows:

$$k_{ro}(N_c,S) = k^*_{ro}(N_c)(\frac{S_o - S_{ro}(N_c)}{1 - S_{ro}(N_c)})^{\varepsilon_o(N_c)} \qquad (40)$$

where $k_{ro}^*(N_c)=A_oN_c$; $C(N_c)=f(N_c)C_{min}+(1-f(N_c))C_{max}$ - any factor of function (40), $f=(1+a(N_c/N_c^{cr})^b)^{-1}$, N_c^{cr}- critical capillary numbers, A_o, a, b - constants. The equation of mass conservation (37) for phase α, e.g. for oil, in sheared layer accepts the form of the equation of pulse preservation

$$div(\frac{\tilde{\lambda}_o}{B_o}|u_o|u_o) = \frac{\partial}{\partial t}(\varphi\frac{S_o}{B_o}) + Q_o; \ \tilde{\lambda}_o = \frac{A_o\mu_o}{\sigma_o}; \quad u_o = k\frac{k_{ro}(s)}{\mu_o}(\nabla p_o - \rho_{o\alpha}g\nabla D)$$

The diffusive conductivity of displacement front $\lambda_\Sigma=\tilde{\lambda}_o\lambda^2_o|u_o|/B_o$ is proportional to velocity, reverse-proportional to superficial tension and volumetric factor and has the obvious zone heterogeneity. Conductivity of unit borders for α phase attenuates under the quadratic law at permeability reduction of k, RFP and change in compressibility $\partial/\partial x_i(1/B_\alpha)$. While the conductivity pair for the opposite phase at completely non-tight border can increase.

It is known that near the firm surface the fluids form the boundary layers with 20 - 50 nm thickness, where there is specific, pre-crystalline location of molecules.

This gives specific mechanical, rheological, thermo-dynamic and optical properties of fluid boundary layers and geophysical characteristics of facial borders. Inside this boundary layer the viscosity ceases to be the fluid material characteristics and depends on the size and energy of activation. The geological information content of block self-organizing of mega-scales for the cycles of geological ages and cycles of development also increases.

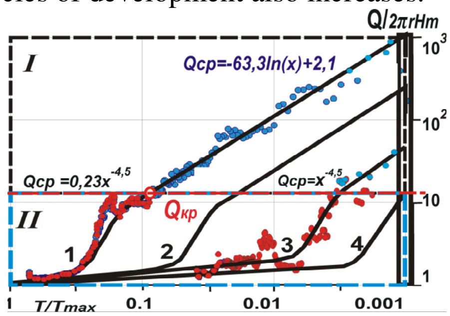

Fig. 6. Rates (Arps's curves) for low-viscous and (1) and high-viscous oil: 150 (2), 350 (3), 600 mPa s (4): I – porous-fractured reservoir, II – diffusive/erosive matrix

The experimental measurements [17] confirm substantiation of multi-scale model of filtration. The theoretical and laboratory dependences for the increase in residual oil-saturation at filtration velocity reduction will be coordinated and are proved to be true by numerous studies [5].

While simulating the profile of inflow at block-type segmental porous space with complexly organized borders, the interface of boundary conditions by pressure and rates at segments on the basis of RFP should be coordinated depending on velocities. The interface of dynamic phase velocity results in formation of energetically non-uniform segments: choke-type, channel-type, radial and convective/diffusional inflows, elastic/ gravitational, concentration-type counter-flow impregnation, capillary jamming and suppression of filtration.

Method of RFP dynamic positioning is used at three-dimensional reservoir simulation with the purpose to study the phenomena of self-organizing in high-permeable breakthrough channels and stagnant, non-drained areas in zones with super-low velocity of filtration, defining energy multi-scaled figure depending on filtration stage and period of development (Fig. 6).

The multi-scale phenomena of geological search, exploration and development are diagnosed, supervised and managed by a system of electro-magnetic, hydro-dynamic, geo-physical gauges, instantaneous energy principles of thermal, electric and magnet-resonance show [24]. Adjusted for critical energy speed of choke-type water and gas breakthroughs, the diffusive filtration, dynamic counter-flow valves weaken or completely block the inflow displacement agent into water-cut sectors, thus equalizing the inflow profile and preventing negative consequences of interface structural arrangement and give the rise in skin-factor for zone heterogeneity.

5. Conclusions

• Basing upon the solution of fundamental Navier-Stokes equations we have resolved the regional task of viscous well inflow. It is illustrated that the viscous well drain consists of various-in-scale energy levels: channel-type, porous-type and dissipative/diffusive type.

• We have developed a method to simulate the interface contacts, mass share of which is small in the general mass of blocks, and have defined the role of weak mass and superficial forces: capillary-type, gravitational-type, geothermal-type, electromagnetic-type, etc., in arranging the macroscopic fields.

• The superficial waves of seismic emission layer make such perturbation to a stationary part of mean macroscopic well inflow that dynamic sizes in them cannot be presented as a simple superposition having mean value and fluctuating frequency fields with waves.

• At macroscopic velocities of drain the viscous layers of fluid are starting to move while at super-small drift velocities the shifted layers of porous structure are becoming mobile that results in swelling borders, decompressing of roof, plugging and growing stratification of blocks, reduction of well drainage radius.

• In free viscous-elastic layer of displacement front there always exist the extending bending wave, that results in flow perturbance, excessive pressure with asymmetric increase in RFP of displacing phase, fractality of displacement front, speedy movement of channel-type filtration.

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Received: November 1, 2013