A Note on Alto Douro’s Wine Coopers’s $\varphi$

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Abstract

Since 2006, we are developing a project – “How about mathematics transforms my land in the ‘capital of universe’?” (6th Live Science Portuguese Project) – based on traditional activities of the Alto Douro’s Portuguese Region. One of the activities studied was the wine cooper’s traditional job of this region and we had the opportunity to see how they make the barrels. We identified and interpreted some mathematical procedures used in this traditional job. We show that the well-known mathematical golden ratio ($\varphi$) can be found in their barrels. We present, as far as we know, an unknown approximation for the value of $\varphi$ designed by us the wine coopers' phi.

Keywords: Mathematical education, wine cooper, golden ratio.

1 Introduction

In 2006, we had the opportunity to submit a project that was approved and was based on traditional activities of the Alto Douro’s Portuguese Region, in the interior northeastern of Portugal. With this project, we try to draw School and Community together and to promote the learning of Mathematics in everyday life and traditions.

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Here we present our work based on the interviews done, between April 2006 and June 2007, to two wine coopers of this region. In the study of wine cooper’s traditional job we have executed the identification of the mathematical procedures used in this job and the explanation, interpretation and register of these processes framed in the mathematical skills required for teaching and learning. We have found several interesting mathematical elements in this job such as an unknown approximation for the number pi ($\pi$) named by us “wine coopers’ pi” and described by Costa et al. in [3].

Some sequences of numbers have been studied over the years, with emphasis on studies of well-known Fibonacci sequence (and then the Lucas sequence) that is related to the golden ratio. Many papers and research work are dedicated to Fibonacci sequence, such as the work of Hoggatt, in [7] and Vorobiov, in [6], among others and the golden ratio has inspired many artists and architects at least since the Renaissance. Believing that this proportion is aesthetically pleasing, they used it to choose the dimension for their works. The more frequent use of this golden proportion is in the golden rectangle, in which the ratio of the longer side to the shorter one is the golden ratio. As Livio describes in [5] “(...) the fascination with the golden ratio is not confined just to mathematicians. (...) the golden ratio has inspired thinkers of all disciplines like no other number in the history of mathematics.”

Here we focus our attention in the golden ratio ($\phi$) that we found in a certain aesthetic criteria expressed by Alto Douro’s wine coopers. Nevertheless, unlike those artists, Alto Douro’s wine coopers do not use this proportion in a conscious way.

### 2 Wine coopers and the golden ratio

The Alto Douro’s Region has always been dependent on agriculture and therefore also on all works connected with this occupation. This region is well known for the production of Oporto wine, of olive oil, of gastronomic specialties, of pottery, among others. One of the traditional jobs related with agriculture is the cooper closely connected with the production of wine. As in Costa et al. in [3], “(...) the wine cooper is a craftsman that makes or repairs all kinds of wood (chestnut and oak) barrels mainly to collect, transport and wine making (...)”.

During our study, through the interviews we had the perception that wine coopers use intuitively the golden ratio, and now the meaning of this use is presented. In the interviews, the wine coopers gave us information about what they call “working the round”, saying that it “is present in everything that they need for execution (including the dimensions) of all pieces for make the barrels as well as in the tools used”, Costa et al. in [3]. This expression suggests the mathematical concepts involved in the barrels’ building. These barrels usually have a round shape, joining the cylindrical and truncated conical forms (Figure 1).
The wine cooper uses three key dimensions to build a barrel: the head, the length and the bilge as illustrated in Figure 2.

From the interviews, we also noticed that wine coopers intuitively consider the barrel as (an approximation of) a cylinder. Accordingly, the diameter corresponds to the head of the barrel and the high corresponds to its length. In order to construct a barrel with a fixed capacity the bilge is not very important to them as they referred in the interviews. However, the bilge gains relevance when the aesthetic appearance of the barrel is considered by the wine coopers. In a certain phase of the construction of a barrel, the staves are heated, traditionally with fire, firstly to get flexibility and then bent in order to shape the barrel with the wanted form. The final form/aesthetics of the barrel is also a concern of wine coopers. They said that they want their works to be beautiful and elegant. That is why, at the same time, the
dimension of the bilge has to be considered when they “imagine” the final barrel as an elegant piece of work. What is behind these aesthetical intuitive criteria of wine cooperers? We answered this question analyzing the building process used by wine cooperers. The wine cooperers want to build a barrel with a certain volume. Through their explanations about its dimensions some proportions emerged to make an elegant barrel:

Wine cooper: In order to build a barrel with 550 liters, we give 2, 80 meters and 70% of length in order to be proportional. Related with bilge we give 22% (usually, between 20% and 25%) (...) It depends of various factors...

Interviewer: Why do you use those percentages?
Wine cooper: It depends...For example, my father used 20% for the bilge and I use 22% ... but I think that the barrel looks more elegant with my percentage...

Let us consider the general case of a barrel with the following dimensions (all in the same unit): head (diameter of the cylinder) \( d \) and length (high of the cylinder) \( h \) and \( b \) for the bilge. From the interviews and following the wine cooperers’ instructions, we can conclude that, given the length \( h \), they consider 70% of length for the dimension of the head. For the dimension of the bilge they use about 20% of the dimension of the head, \( d \). More precisely, if \( h \) is the length, then the other dimensions considered by the wine cooperers are the following:

\[
d = 0,7 \ h
\]

and

\[
b = d + 0,20 \ d = 1,20 \ d = 1,20 \times (0,70 \ h) = 0,84 \ h
\]

Now we consider a rectangle with the following two dimensions (Figure 3): one equal to the bilge \( b \) and the other equal \( \frac{1}{2} h \).

![](image)

Figure 3 – Rectangle with the referred dimensions, sketches by the authors

Hence, if we compare those two dimensions and using (1) we can conclude that

\[
\begin{align*}
\frac{b}{\frac{1}{2}h} &= \frac{b}{0,50h} = \frac{0,84h}{0,50h} = 1,68
\end{align*}
\]

From this computations, the proportion founded between the dimensions of the rectangle considered is, as far as we know, an unknown approximation for the golden ratio \( \phi = \frac{1 + \sqrt{5}}{2} \approx 1,618033989 \), value well known (for example, see [6]...
and [7], among others). Therefore, using these procedures the *wine cooper’s golden ratio* approximation is given by \( \varphi \approx 1,68 \).

Such golden rectangle may be imagined in the bilge of the barrel as we show in Figure 4.

![Figure 4 – Location of the Golden Rectangle in the barrel, photos and sketches by the authors](image)

In Table 1, we organize the data obtained by doing the same computations, but now using the other percentages mentioned by the wine cooperers. We verified that **20%** is the percentage that gives the best approximation for the golden ratio.

<table>
<thead>
<tr>
<th>Head – ( d )</th>
<th>Percentage</th>
<th>Bilge – ( b )</th>
<th>Ratio ( \frac{b}{\frac{1}{2}h} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 7h</td>
<td>20%</td>
<td>1, 20d</td>
<td>1, 680</td>
</tr>
<tr>
<td>0, 7h</td>
<td>22%</td>
<td>1, 22d</td>
<td>1, 708</td>
</tr>
<tr>
<td>0, 7h</td>
<td>25%</td>
<td>1, 25d</td>
<td>1, 750</td>
</tr>
</tbody>
</table>

### 3 Final Remarks

In this study, we presented a different approximation for the golden ratio \( \varphi \approx 1,68 \). For us, it is an attractive result because, since ancient times, the golden ratio \( \varphi \) inspired works in mathematics such as in [1] and in many other fields (e.g., [4]). Furthermore, we realize that the study of this traditional job has many mathematical concepts that could and should be detailed and used to promote classroom and school activities enhancing everyday life and traditions. Since we do our work in a Portuguese Region with specific social and cultural characteristics, we think that this is a promising way to emphasize the value of
their region to and with students, teachers and schools throughout mathematics classroom. This project enabled us to develop this work and we also hope to give a contribution to preserve some cultural and heritage elements in Alto Douro's Portuguese Region.

References


Received: November 11, 2013