Shapiro-Wilk Test in Evaluation of Asymptotic Distribution on Estimators of Measure of Kurtosis and Skew

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Abstract

In this article, we evaluate the results obtained on asymptotic distribution of moments in estimator measures of skew and kurtosis in the family of skew normal-uniform distribution. With the help of simulation random sample from this distribution and then by the Shapiro-Wilk test, we verify the results.

Keywords: Kurtosis, Skew, Normal distribution, Asymptotic distribution, Normal-uniform distribution.

1 Introduction

The Shapiro-Wilk test tests the null hypothesis that a random sample, $x_1, x_2, \dots, x_n$ came from a normally distributed population. It was published in 1965 by Samuel Shapiro and Martin Wilk.[1] Let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be ordered statistics from sample. The form of Shapiro-Wilk test statistics is

$$w = \frac{(\sum_i a_i x_{(i)})^2}{\sum_i (x_i - \bar{x})^2}$$

where the values of a_i was determined in any R and SPSS statistics software.

Suppose the density function of uniform distribution has been defined on the interval $[-h, h]$, then the skew normal-uniform density function will be as follows:

$$f_X(x) = \begin{cases} 0, & \lambda x \leq -h, \\ \frac{\lambda x + h}{\sqrt{2\pi} \sigma h} \exp\left(-\frac{x^2}{2\sigma^2}\right), & -h \leq \lambda x \leq h, \\ \frac{2}{\sqrt{2\pi} \sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right), & \lambda x \geq h, \end{cases} \quad \sigma > 0, h > 0, \lambda \in R. \quad (1)$$

Nadarajah and Kotz (2003) had presented the moments for this distribution. With determining of the moments of this distribution, Abbasi (2013) presented the asymptotic distribution of central moment distribution in general.

$$\sqrt{n}(S^2 - \mu_2) \sim AN(0, \mu_4 - \mu_2^2), \quad (2)$$

$$\sqrt{n}(U_3 - \mu_3) \sim AN(0, 9\xi_3), \quad (3)$$

$$\sqrt{n}(U_4 - \mu_4) \sim AN(0, 16\xi_4). \quad (4)$$

where U_i the best unbiased estimator for i th central moment, μ_i , and $M = \frac{h}{\sigma\lambda}$,

$$\text{var} \left[(X - E(X))^2 \right] = \mu_4 - \mu_2^2 = \frac{12\sigma^8}{\sqrt{\pi}} M \{2\Phi(M) - 1\} \left(\sqrt{2} e^{-\frac{1}{2}M^2} - 3\sqrt{\pi} \{2\Phi(M) - 1\} \right) + 6\sigma^4(M)^2 \{2\Phi(M) - 1\}^2 - 3\sigma^4(M)^4 \{2\Phi(M) - 1\}^4 \quad (5)$$

and

$$\begin{aligned} \xi_3 = \sigma^6 & \left(\frac{5}{3} - \frac{2}{\pi} e^{-M^2} + \frac{\sqrt{2}}{3\sqrt{\pi}} M^{-1} e^{-\frac{1}{2}M^2} \text{erf}\left(\frac{\sqrt{2}}{2}M\right) - M^{-2} \text{erf}\left(\frac{\sqrt{2}}{2}M\right)^2 \right. \\ & - 2M^{-4} \text{erf}\left(\frac{\sqrt{2}}{2}M\right)^4 - 4M^{-6} \text{erf}\left(\frac{\sqrt{2}}{2}M\right)^6 \\ & \left. + \frac{\sqrt{2}}{3\sqrt{\pi}} M e^{-\frac{1}{2}M^2} \text{erf}\left(\frac{\sqrt{2}}{2}M\right) + \frac{14\sqrt{2}}{3\sqrt{\pi}} M^{-3} e^{-\frac{1}{2}M^2} \text{erf}\left(\frac{\sqrt{2}}{2}M\right)^3 \right) \quad (6) \end{aligned}$$

and

$$\begin{aligned}
\xi_4 = \sigma^8 & \left(6 + \frac{27\sqrt{2}}{8\pi} M^{-1} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right) e^{-\frac{1}{2}M^2} + \frac{\sqrt{2}}{2\pi} M \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right) e^{-\frac{1}{2}M^2} \right. \\
& + \frac{297}{4\pi^{\frac{3}{2}}} M^{-3} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right)^3 - 33h^{-2} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right)^2 \\
& - \frac{14}{\pi} \sigma^{-2} M^{-2} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right)^2 e^{-M^2} + \frac{9}{2} \sigma^2 M^{-2} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right)^2 \\
& + 9\sigma^{-2} h^{-6} M^{-2} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right) + \frac{\sqrt{2}}{2\sqrt{\pi}} M^3 \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right) e^{M^2} \\
& + \frac{27\sqrt{2}}{\sqrt{\pi}} h^4 M^{-1} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right) e^{-\frac{1}{2}M^2} + \frac{21\sqrt{2}}{4\sqrt{\pi^3}} M^{-5} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right)^5 e^{-\frac{1}{2}M^2} \\
& - \frac{135\sqrt{2}}{16\sqrt{\pi}} M^{-5} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right)^6 \\
& - 45 M^{-4} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right)^4 - \frac{9}{16} M^{-8} \operatorname{erf}\left(\frac{\sqrt{2}}{2} M\right)^8 - \frac{5}{\pi} e^{M^2} \\
& \left. - \frac{1}{\pi} \sigma^{-1} M^2 e^{M^2} \right).
\end{aligned}$$

In the above statements, the three parameters of normal, uniform, and skew are dependent. It is clear that their differences are effective in the variance decrease or increase of skew and kurtosis measures. In this article, according to simulation and verification of (2) and (3) distributions which result in kurtosis and skew are evaluated by the Shapiro-Wilk test.

2Shapiro-Wilk test for Asymptotic Distribution

According to the central moments, kurtosis and skew measures are obtained as:

$$\widehat{\alpha}_3 = \frac{U_3}{S^3}, \quad (8)$$

$$\widehat{\alpha}_4 = \frac{U_4}{S^4}. \quad (9)$$

In probability theory, Slutsky's theorem extends some properties of algebraic operations on convergent sequences of real numbers to sequences of random variables.[5] Now, employing the Slutsky's theorem and considering this point that S^2 tends to μ_2 in probability, determines distribution of estimators in (8) and (9). For special cases, the distribution of (8) and (9) have also been shown in the form of numbers.

With the help of R software, first data are simulated from normal-uniform

distribution. Then, we obtained the sample variance, sample skew measure and sample kurtosis measure. By repeating this method, a random sample of random variables has been produced.

Table 1. The p-value of Shapiro-Wilk test for skew and kurtosis measures

<i>parameters</i>	<i>Skew measure</i>	<i>Kurtosis measure</i>
$\lambda = 8$	<i>mean</i> = 0.975	<i>mean</i> = 3.214
$\sigma = 10$	<i>var</i> = 0.574	<i>var</i> = 1.67
$h = 5$	$W = 0.9084$	$W = 0.945$
$n = 50$	<i>pvalue</i> = 0.1287	<i>pvalue</i> = 0.4464
$\lambda = 8$	<i>mean</i> = 0.8913	<i>mean</i> = 3.1711
$\sigma = 10$	<i>var</i> = 0.8613	<i>var</i> = 3.32
$h = 5$	$W = 0.7706$	$W = 0.7740$
$n = 25$	<i>pvalue</i> = 0.00127	<i>pvalue</i> = 0.00143
$\lambda = 1$	<i>mean</i> = -0.04614	<i>mean</i> = 2.746
$\sigma = 2$	<i>var</i> = 0.288	<i>var</i> = 2.1819
$h = 10$	$W = -0.964$	$W = 0.827$
$n = 25$	<i>pvalue</i> = 0.741	<i>pvalue</i> = 0.00793

Table (1) shows Shapiro-Wilk statistics test differences in the behavior of different recognition of distribution of limit distribution for various parameters and sample size in fifteen repetitions. Skew parameter can change our evaluation on limit distribution with respect to the parameter obtained from uniform distribution.

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