A Decomposition of Complete Bipartite 4-Uniform Hypergraphs $K_{5,5}^4$ into Loose Cycles of Length 5, 10

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Abstract

A k -uniform hypergraph H is a pair (V, ε) , where $V = \{v_1, v_2, \dots, v_n\}$ is a set of n vertices and ε is a family of k -subset of V called hyperedges. We consider the problem of constructing a decompositions for complete bipartite uniform hypergraphs into loose cycle.

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1 Introduction

A decomposition of a graph $G = (V, E)$ is a partition of the edge-set E ; a Hamiltonian decomposition of G is a decomposition into Hamiltonian cycles. The problem of constructing Hamiltonian decompositions is a long-standing and well-studied one in graph theory; in particular, for the complete graph K_n , it was solved in the 1890s by Walecki [1]. Walecki showed that K_n has a Hamiltonian decomposition if and only if n is odd, while if n is even K_n has a decomposition into Hamiltonian cycles and a perfect matching. As with many

problems in graph theory, it seems natural to attempt a generalisation to hypergraphs. Indeed, the notion of Hamiltonicity was first generalised to uniform hypergraphs by Berge in his 1970 book [2]. His definition of a Hamiltonian cycle in a hypergraph $H = (V, E)$ is a sequence $(v_0, e_1, v_1, e_2, \dots, v_{n-1}, e_n, v_0)$, where $\{v_0, \dots, v_n\} = V$, and $e_1, \dots, e_n$ are distinct elements of E , such that the hyperedge e_i contains both v_{i+1} and $v_i \pmod n$. The study of decompositions of complete 3-uniform hypergraphs into cycles of this type was begun by Bermond et al in the 1970s [3] and was completed by Verrall in 1994[6]. A k -uniform hypergraph H is a pair (V, ε) , where $V = \{v_1, v_2, \dots, v_n\}$ is a set of n vertices and ε is a family of k -subset of V called hyperedges. If ε consists of all k -subsets of V , then H is a complete k -uniform hypergraph on n vertices and is denoted by K_n^k . At the same time we may refer a vertex $v_i \in V$ to v_{i+n} . A cycle of length l of H is a sequence of the form

$$(v_1, e_1, v_2, e_2, \dots, v_l, e_l, v_1),$$

where $v_1, v_2, \dots, v_l$ are distinct vertices, and $e_1, e_2, \dots, e_l$ are k -edges of H , satisfying

(i) $v_i, v_{i+1} \in e_i, 1 \leq i \leq l$, where addition on the subscripts is modulo n , and

(ii) $e_i \neq e_j$ for $i \neq j$. This cycle is known as a Berge cycle, having been introduced by Berge in [2]. A cycle of length l decomposition of H is a partition of the hyperedges of H into cycles of length l .

A set of cycles of length l of complete k -uniform hypergraph K_n^k , say $C_1, C_2, \dots, C_m$, is called cycles of length l decomposition if $\bigcup_{i=1}^m \varepsilon(C_i) = \varepsilon(K_n^k)$ and $\varepsilon(C_i) \cap \varepsilon(C_j) = \emptyset$ for $i \neq j$. In this paper, we consider the decomposition of bipartite complete 4-uniform hypergraph $K_{5,5}^4$.

2 Main results

Definition 2.1 Let H be a hypergraph under V , if $V = V_1 \cup V_2, V_1 \cap V_2 = \emptyset$, meanwhile, for any $e \in \varepsilon, e \cap V_1 \neq \emptyset$ and $e \cap V_2 \neq \emptyset$, and for any $e \in \varepsilon, |e| = k$, then we call H complete bipartite k -uniform hypergraph.

We discuss the problem of decomposition of bipartite complete 4-uniform hypergraph $K_{5,5}^4$.

Let $V_1 = \{1, 2, 3, 4, 5\}$ $V_2 = \{a, b, c, d, e\}$ be the set of $K_{5,5}^4$, we can decompose it into loose cycle of length 5, 10, there are 30 5-cycles and 5 10-cycles, we can arrange 200 hyperedges as followed:

$$\begin{aligned} &12ab, 23bc, 34cd, 45de, 15ae \\ &13ac, 35ce, 52eb, 24bd, 41da \\ &12bc, 23cd, 34de, 45ea, 51ab \end{aligned}$$

12ea,23ab,34bc,45cd,51de
 12cd,23de,34ea,45ab,51bc
 12de,23ea,34ab,45bc,51cd
 13ce,35eb,52bd,24da,41ac
 13eb,35bd,52da,24ac,41ce
 13bd,35da,52ac,24ce,41eb
 13da,35ac,52ce,24eb,41bd
 123a,345a,512a,234a,451a
 123b,345b,512b,234b,451b
 123c,345c,512c,234c,451c
 123d,345d,512d,234d,451d
 123e,345e,512e,234e,451e
 135a,524a,413a,352a,241a
 135b,524b,413b,352b,241b
 135c,524c,413c,352c,241c
 135d,524d,413d,352d,241d
 135e,524e,413e,352e,241e
 abc1,cde1,eab1,bcd1,dea1
 abc2,cde2,eab2,bcd2,dea2
 abc3,cde3,eab3,bcd3,dea3
 abc4,cde4,eab4,bcd4,dea4
 abc5,cde5,eab5,bcd5,dea5
 ace1,ebd1,dac1,ceb1,bda1
 ace2,ebd2,dac2,ceb2,bda2
 ace3,ebd3,dac3,ceb3,bda3
 ace4,ebd4,dac4,ceb4,bda4
 ace5,ebd5,dac5,ceb5,bda5
 12ac,24cd,45da,52ab,23bd,35de,51eb,13bc,34ce,41ea
 12ad,25de,51ec,14cd,45db,53bc,34ca,42ab,23be,31ea
 12be,25ea,51ad,14de,45ec,53cd,34db,42bc,23ca,31ab
 12bd,24de,45eb,52bc,23ce,35ea,51ac,13cd,34da,41ab
 12ce,24ea,45ac,52cd,23da,35ab,51bd,13de,34eb,41bc

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