Fredholm Type Integral Equations with Aleph-Function and General Polynomials

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Abstract

This paper is devoted to the useful method of solving the one-dimensional integral equation of Fredholm type. The Mellin transform technique for solving a general fredholm type integral equation with the $\aleph$ - function and a generalized polynomial in the kernel is considered. By specializing the coefficients and various parameters in the generalized polynomials and $\aleph$ -function, our main theorem would readily yield several results involving simpler kernels.

Keywords: Fredholm type integral equation, Mellin transform technique & $\aleph$ - function

1. Introduction

In the last several years a large number of Fredholm type integral equations involving various polynomials or special functions as their kernels have been studied by many authors notably

Buchman [10], Higgins [11], Love ([3] and [2]), Prabhakar and Kashyap [12], Srivastava and Raina [6] and others. In the present paper, we obtain the following one dimensional Fredholm integral equation (1.1) involving the $\aleph$ -function and a generalised polynomials in the kernel can be solved by resorting to the application of Mellin transforms.

$$\int_0^\infty y^{-\alpha} S_{n_1, \dots, n_R}^{m_1, \dots, m_R} \left[E\left(\frac{x}{y}\right)^p, \dots, E\left(\frac{x}{y}\right)^p \right] \aleph_{p_i, q_i, \tau_i; r}^{m, n} \left[z \left(\frac{x}{y}\right)^q \left| \begin{matrix} (a_j, A_j)_{1, n}, \dots, [\tau_j (a_j, A_j)]_{n+1, p_i} \\ (b_j, B_j)_{1, m}, \dots, [\tau_j (b_j, B_j)]_{m+1, q_i} \end{matrix} \right. \right] f(y) dy = g(x) \quad (1.1)$$

The Aleph $\aleph$ -function occurring in (1.1) introduced by Sudland et al. [9] which is defined as a contour integral of Mellin Barnes Type:

$$\begin{aligned} \aleph[Z] &= \aleph_{p_i, q_i, \tau_i; r}^{m, n} [Z] = \aleph_{p_i, q_i, \tau_i; r}^{m, n} \left[z \left(\frac{x}{y}\right)^q \left| \begin{matrix} (a_j, A_j)_{1, n}, \dots, [\tau_j (a_j, A_j)]_{n+1, p_i} \\ (b_j, B_j)_{1, m}, \dots, [\tau_j (b_j, B_j)]_{m+1, q_i} \end{matrix} \right. \right] \\ &= \frac{1}{2\pi\omega} \int_L \Omega_{p_i, q_i, \tau_i; r}^{m, n} (s) z^{-s} ds \end{aligned} \quad (1.2)$$

for all $z \neq 0$, $\omega = \sqrt{-1}$ and

$$\Omega_{p_i, q_i, \tau_i; r}^{m, n} (s) = \frac{\prod_{j=1}^m \Gamma(b_j + B_j s) \prod_{j=1}^n \Gamma(1 - a_j - A_j s)}{\sum_{i=1}^r \tau_i \prod_{j=n+1}^{p_i} \Gamma(a_{ji} + A_{ji} s) \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - B_{ji} s)} \quad (1.3)$$

The integration path $\ell = \ell_{i\gamma\infty}$, $\gamma \in \mathbb{R}$ extends from $\gamma - i\infty$ to $\gamma + i\infty$, and is such that the poles, assumed to be simple, of $\Gamma(1 - a_j - A_j s)$, $j = \overline{1, n}$ do not coincide with the pole $\Gamma(b_j + B_j s)$, $j = \overline{1, m}$. The parameters p_i, q_i are non-negative integers satisfying $0 \leq n \leq p_i$, $1 \leq m \leq q_i$, $\tau_i > 0$ for $i = \overline{1, r}$. The parameters $A_j, B_j, A_{ji}, B_{ji} > 0$ and $a_j, b_j, a_{ji}, b_{ji} \in \mathbb{C}$. The empty product in (1.3) is interpreted as unity. The existence conditions for the defining integral (1.1) are given below:

$$\psi_l > 0, \quad |\arg(z)| < \frac{\pi}{2} \psi_l \quad l = \overline{1, r}; \quad (1.4)$$

$$\psi_l \geq 0, \quad |\arg(z)| < \frac{\pi}{2} \psi_l \quad \text{and} \quad \Re\{\zeta_l\} + 1 < 0, \quad (1.5)$$

Where

$$\psi_l = \sum_{j=1}^n A_j + \sum_{j=1}^m B_j - \tau_l \left(\sum_{j=n+1}^{p_l} A_{jl} + \sum_{j=m+1}^{q_l} B_{jl} \right) \quad (1.6)$$

$$\varsigma_l = \sum_{j=1}^m b_j - \sum_{j=1}^n a_j + \tau_l \left(\sum_{j=m+1}^{q_l} b_{jl} - \sum_{j=n+1}^{p_l} a_{jl} \right) + \frac{1}{2}(p_l - q_l) \quad l = \overline{1, r}. \quad (1.7)$$

The general polynomials of R variables given by Srivastava [4] defined and represented as :

$$S_{n_1, \dots, n_R}^{m_1, \dots, m_R} [x_1, \dots, x_R] = \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)_{m_1 s_1}}{\angle s_1} \dots \frac{(-n_R)_{m_R s_R}}{\angle s_R} \quad (1.8)$$

$$A \left[n_1, s_1; \dots; n_R, s_R \right] x_1^{s_1} \dots x_R^{s_R}$$

Where m_i is an arbitrary positive integer and coefficients $A \left[n_1, s_1; \dots; n_R, s_R \right]$ are arbitrary constants, real or complex.

Let $\int$ denote the space of all functions f which are defined on $R^+ = [0, \infty)$ and satisfy

- (i) $f \in b^\infty(R^+)$,
- (ii) $\lim_{x \rightarrow \infty} [x^\gamma f^r(x)] = 0$ for all non negative integers γ and r .
- (iii) $f(x) = O(1)$ as $x \rightarrow 0$.

For correspondence to the space of good functions defined on the whole real line $(-\infty, \infty)$ see (Lighthill) [8].

The Riemann – Liouville fractional integral (of order μ) is defined by

$$D^{-\mu} \{f(x)\} = {}_0 D_x^\mu \{f(x)\} = \frac{1}{\Gamma(\mu)} \int_0^x (x-w)^{\mu-1} f(w) dw, \quad (\operatorname{Re}(\mu) > 0; f \in \mathcal{E}), \quad (1.9)$$

where $D^\mu \{f(x)\} = \phi(x)$ is understood to mean that Φ is a locally integrable solution of $f(x) = D^{-\mu} \{\phi(x)\}$, implying that D^μ is the inverse of the fractional integral operator $D^{-\mu}$ (whenever necessary, we shall simply write $D_x^{-\mu}$ for ${}_0 D_x^{-\mu}$ for the Riemann-Liouville fractional integral operator defined by eq. (1.8) above).

$$W^{-h} \{f(x)\} = {}_x D_\infty^{-h} \{f(x)\} = \frac{1}{\Gamma(h)} \int_x^\infty (\varsigma - x)^{h-1} f(\varsigma) d\varsigma, \quad (\operatorname{Re}(h) > 0; f \in \mathcal{E}) \quad (1.10)$$

2. PRELIMINARY RESULTS

We first prove the following result which will be required in proving theorem 1 below.

Lemma 1. With the set of sufficient conditions (1.4), (1.5), (1.6) and (1.7) and let us suppose $\operatorname{Re}(\alpha) > \operatorname{Re}(\beta)$; $\operatorname{Re}[\beta + q(b_j/B_j)] > 0$, $(j = \overline{1, m})$, $q > 0$.

Let

$$\begin{aligned} & W^{\beta-\alpha} \left\{ y^{-\alpha} S_{n_1, \dots, n_R}^{m_1, \dots, m_R} \left[E\left(\frac{x}{y}\right)^p, \dots, E\left(\frac{x}{y}\right)^p \right] \mathfrak{K}_{p_i, q_i, \tau_i; r}^{m, n} \left[z\left(\frac{x}{y}\right)^q \left| \begin{matrix} (a_j, A_j)_{1, n}, \dots, [\tau_j(a_j, A_j)]_{n+1, pi} \\ (b_j, B_j)_{1, m}, \dots, [\tau_j(b_j, B_j)]_{m+1, qi} \end{matrix} \right. \right] \right\} \\ &= y^{-\beta} \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)m_1 s_1}{\angle s_1} \dots \frac{(-n_R)m_R s_R}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] E^{s_1 + \dots + s_R} \left(\frac{x}{y}\right)^{ps_1 + \dots + ps_R} \\ & \mathfrak{K}_{p_i+1, q_i+1, \tau_i; r}^{m, n+1} \left[z\left(\frac{x}{y}\right)^q \left| \begin{matrix} [1-\alpha-p(s_1+\dots+s_R); q], (a_j, A_j)_{1, n}, \dots, [\tau_j(a_j, A_j)]_{n+1, pi} \\ (b_j, B_j)_{1, m}, \dots, [\tau_j(b_j, B_j)]_{m+1, qi}, [1-\beta-p(s_1+\dots+s_R); q] \end{matrix} \right. \right] \end{aligned} \quad (2.1)$$

Proof: To prove Lemma 1, we first use the definition of Weyl fractional integral given in (1.10) express the $\mathfrak{K}$ -function and a generalized polynomial, then we change the order of summations and integration (which is justified under the stated conditions), evaluate the t-integral and reinterpreting the resulting Mellin-Barnes contour integral in terms of the $\mathfrak{K}$ -function, we easily arrive at the desired result.

Theorem 1 - Under the sufficient conditions (i), (ii), (iii) and (iv) of Lemma

$$\begin{aligned} & \int_0^\infty y^{-\beta} \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)m_1 s_1}{\angle s_1} \dots \frac{(-n_R)m_R s_R}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] \\ & E^{s_1 + \dots + s_R} \left(\frac{x}{y}\right)^{ps_1 + \dots + ps_R} dy \\ &= \mathfrak{K}_{p_i+1, q_i+1, \tau_i; r}^{m, n+1} \left[z\left(\frac{x}{y}\right)^q \left| \begin{matrix} [1-\beta-p(s_1+\dots+s_R); q], (a_j, A_j)_{1, n}, \dots, [\tau_j(a_j, A_j)]_{n+1, pi} \\ (b_j, B_j)_{1, m}, \dots, [\tau_j(b_j, B_j)]_{m+1, qi}, [1-\alpha-p(s_1+\dots+s_R); q] \end{matrix} \right. \right] f(y) dy \\ & \int_0^\infty y^{-\alpha} S_{n_1, \dots, n_R}^{m_1, \dots, m_R} \left[E\left(\frac{x}{y}\right)^p, \dots, E\left(\frac{x}{y}\right)^p \right] \mathfrak{K}_{p_i, q_i, \tau_i; r}^{m, n} \left[z\left(\frac{x}{y}\right)^q \right] D^{\beta-\alpha} \{f(y)\} dy. \end{aligned} \quad (2.2)$$

Provided further $f \in \int$, and $x > 0$.

Proof: Let η denote the first member of the assertion (2.2). Then using Lemma 1 and applying (1.10), we have

$$\eta(y) = \int_0^\infty \frac{f(y)}{\Gamma(\alpha-\beta)} \left\{ \int_y^\infty (\zeta-y)^{\alpha-\beta-1} \zeta^{-\alpha} S_{n_1, \dots, n_R}^{m_1, \dots, m_R} \left[E\left(\frac{x}{y}\right)^p, \dots, E\left(\frac{x}{y}\right)^p \right] \mathfrak{K}_{p_i, q_i, \tau_i; r}^{m, n} \left[z\left(\frac{x}{y}\right)^q \right] d\zeta \right\} dy$$

(2.3)

$$= \int_0^\infty \zeta^{-\alpha} S_{n_1, \dots, n_R}^{m_1, \dots, m_R} \left[E\left(\frac{x}{y}\right)^p, \dots, E\left(\frac{x}{y}\right)^p \right] \mathfrak{K}_{p_i, q_i, \tau_i; r}^{m, n} \left[z\left(\frac{x}{y}\right)^q \right] \left(\int_0^\zeta \frac{(\zeta-y)^{\alpha-\beta-1}}{\Gamma(\alpha-\beta)} f(y) dy \right) d\zeta. \quad (2.4)$$

Assuming the inversion of the order of integration to be permissible just as in the proof of Lemma 1.

Now, by definition (1.9), (2.4) gives

$$\eta = \int_0^\infty \zeta^{-\alpha} S_{n_1, \dots, n_R}^{m_1, \dots, m_R} \left[E\left(\frac{x}{y}\right)^p, \dots, E\left(\frac{x}{y}\right)^p \right] \mathfrak{K}_{p_i, q_i, \tau_i; r}^{m, n} \left[z\left(\frac{x}{y}\right)^q \right] D^{\beta-\alpha} \{f(\zeta)\} d\zeta.$$

Which is the second member of (2.2).

3. SOLUTION OF A FREDHOLM TYPE INTEGRAL EQUATION

To obtain the solution of a fredholm type integral equation (1.1), we use the Mellin Transform technique.

Theorem 2. If $f \in \mathcal{E}$, $D^{\alpha-\beta} \{f(y)\}$ exists, $q > 0$, $x > 0$, $\psi_l > 0$, $|\arg(z)| < \frac{\pi}{2} \psi_l$ $l = \overline{1, r}$;

$\psi_l \geq 0$, $|\arg(z)| < \frac{\pi}{2} \psi_l$ and $\Re\{\zeta_l\} + 1 < 0$, (ψ_i given in (1.6) and ζ_i is given in (1.7))

$\operatorname{Re}(\alpha) > \operatorname{Re}(\beta) > 0$, and

m_i is an arbitrary positive integer and coefficients $A[n_1, s_1; \dots; n_R, s_R]$ are arbitrary constants, real or complex, then the solution of the integral equation

$$\int_0^\infty y^{-\alpha} S_{n_1, \dots, n_R}^{m_1, \dots, m_R} \left[E\left(\frac{x}{y}\right)^p, \dots, E\left(\frac{x}{y}\right)^p \right] \mathfrak{K}_{p_i, q_i, \tau_i; r}^{m, n} \left[z\left(\frac{x}{y}\right)^q \right] \left[\begin{matrix} (a_j, A_j)_{1, n}, \dots, [\tau_j(a_j, A_j)]_{n+1, pi} \\ (b_j, B_j)_{1, m}, \dots, [\tau_j(b_j, B_j)]_{m+1, qi} \end{matrix} \right] f(y) dy = g(x) \quad (3.1)$$

is given by

$$f(y) = \frac{q}{2\pi w} \lim_{\rho \rightarrow \infty} \int_{\sigma - \rho w}^{\sigma + \rho w} y^{\beta - k_1 - 1} \left\{ \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)_{m_1 s_1}}{\angle s_1} \dots \frac{(-n_R)_{m_R s_R}}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] \right\}^{-1} \\ \left\{ E^{s_1 + \dots + s_R} Z^{\frac{-p(s_1 + \dots + s_R) - k_1}{q}} \Omega \left(\frac{(-p(s_1 + \dots + s_R) - k_1)}{q} \right) \right\}^{-1} \phi(k_1) dk_1. \quad (3.2)$$

provided further that $\max\{ \operatorname{Re} [(a_l - 1)/\Phi] \} < \operatorname{Re} [(p(s_1 + \dots + s_R) + k_1)/q] < \min \{ \operatorname{Re} (b_j/B_j) \}$,

($j=1, \dots, m$) and ($l = 1, \dots, n$)

Proof: On replacing f by $D^{\alpha-\beta}\{f\}$ in (3.1) and applying (2.1), we have

$$g(x) = \int_0^\infty y^{-\beta} \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)_{m_1 s_1}}{\angle s_1} \dots \frac{(-n_R)_{m_R s_R}}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] \\ E^{s_1 + \dots + s_R} \left(\frac{x}{y} \right)^{ps_1 + \dots + ps_R} \\ \times \frac{m, n+1}{p_i+1, q_i+1, \tau_i; r} \left[z \left(\frac{x}{y} \right)^q \left| \begin{matrix} [1-\beta-p(s_1+\dots+s_R); q], (a_j, A_j)_{1,n}, \dots, [\tau_j(a_j, A_j)]_{n+1, pi} \\ (b_j, B_j)_{1,m}, \dots, [\tau_j(b_j, B_j)]_{m+1, qi}, [1-\alpha-p(s_1+\dots+s_R); q] \end{matrix} \right. D^{\alpha-\beta}\{f(y)\} dy \right] \quad (3.3)$$

Multiplying both the sides of (3.3) by $x^h - 1$ and integrating with respect to x from 0 to ∞ , we have

$$\begin{aligned}
\phi(s_1) &= \int_0^\infty x^{h-1} g(x) dx = \int_0^\infty y^{-\beta} D^{\alpha-\beta} \{f(y)\} \left(\int_0^\infty x^{h-1} \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)_{m_1} s_1}{\angle s_1} \dots \frac{(-n_R)_{m_R} s_R}{\angle s_R} \right. \\
&\quad \left. A[n_1, s_1; \dots; n_R, s_R] E^{s_1 + \dots + s_R} \left(\frac{x}{y} \right)^{p s_1 + \dots + p s_R} \right. \\
&\quad \left. \mathfrak{K}_{p_i+1, q_i+1, \tau_i; r} \left[z \left(\frac{x}{y} \right)^q \middle| \begin{matrix} [1-\beta-p(s_1+\dots+s_R); q], (a_j, A_j)_{1,n}, \dots, [\tau_j(a_j, A_j)]_{n+1, pi} \\ (b_j, B_j)_{1,m}, \dots, [\tau_j(b_j, B_j)]_{m+1, qi}, [1-\alpha-p(s_1+\dots+s_R); q] \end{matrix} \right] \right. \\
&\quad \left. dx \right) dy \tag{3.4}
\end{aligned}$$

where we have assumed the absolute (and uniform) convergence of the integrals involved, with a view to justifying the inversion of the order of integration.

Now evaluate the inner integral in (3.4) by a simple change of variables in the familiar results

(c.f., for example, [5] and [7]), eq. (3.4) reduces to

$$\begin{aligned}
\phi(s_1) &= \int_0^\infty y^{-\beta} D^{\alpha-\beta} \{f(y)\} \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)_{m_1} s_1}{\angle s_1} \dots \frac{(-n_R)_{m_R} s_R}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] \\
&\quad E^{s_1 + \dots + s_R} \frac{1}{q} y^{-p(s_1 + \dots + s_R)} Z^{\frac{-p(s_1 + \dots + s_R) - k_1}{q}} \Omega \left(\frac{(-p(s_1 + \dots + s_R) - k_1)}{q} \right)
\end{aligned}$$

$$\frac{\Gamma(\beta - k_1)}{\Gamma(\alpha - k_1)} y^{p(s_1 + \dots + s_R) + k_1} dy, \quad (3.5)$$

where $\Phi(s_1)$ is given by (3.4).

$$\begin{aligned} \phi(s_1) = & \frac{1}{q} \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)m_1 s_1}{\angle s_1} \dots \frac{(-n_R)m_R s_R}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] \\ & E^{s_1 + \dots + s_R} Z^{\frac{-p(s_1 + \dots + s_R) - k_1}{q}} \Omega \left(\frac{(-p(s_1 + \dots + s_R) - k_1)}{q} \right) \frac{\Gamma(\beta - k_1)}{\Gamma(\alpha - k_1)} \\ & \int_0^\infty y^{k_1 - \beta} D^{\alpha - \beta} \{f(y)\} dy \end{aligned} \quad (3.6)$$

Inverting (3.6) by applying the Mellin Inversion theorem [1], we get

$$\begin{aligned} D^{\alpha - \beta} \{f(y)\} = & \frac{q}{2\pi w} \lim_{\rho \rightarrow \infty} \int_{\sigma - \rho w}^{\sigma + \rho w} \left\{ \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)m_1 s_1}{\angle s_1} \dots \frac{(-n_R)m_R s_R}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] \right\}^{-1} \\ & \left\{ E^{s_1 + \dots + s_R} Z^{\frac{-p(s_1 + \dots + s_R) - k_1}{q}} \frac{\Gamma(\beta - k_1)}{\Gamma(\alpha - k_1)} \Omega \left(\frac{(-p(s_1 + \dots + s_R) - k_1)}{q} \right) \right\}^{-1} y^{\beta - k_1 - 1} \phi(k_1) dk_1. \end{aligned} \quad (3.7)$$

Operating upon both sides by $D^{\beta - \alpha}$, (3.7) gives us

$$\{f(y)\} = \frac{q}{2\pi w} D^{\beta-\alpha} \lim_{\rho \rightarrow \infty} \int_{\sigma-\rho w}^{\sigma+\rho w} \left\{ \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)m_1 s_1}{\angle s_1} \dots \frac{(-n_R)m_R s_R}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] \right\}^{-1} \\ \left\{ E^{s_1 + \dots + s_R} Z^{\frac{-p(s_1 + \dots + s_R) - k_1}{q}} \frac{\Gamma(\beta - k_1)}{\Gamma(\alpha - k_1)} \Omega \left(\frac{(-p(s_1 + \dots + s_R) - k_1)}{q} \right) \right\}^{-1} y^{\beta - k_1 - 1} \phi(k_1) dk_1. \quad (3.8)$$

Which finally yields

$$f(y) = \frac{q}{2\pi w} \lim_{\rho \rightarrow \infty} \int_{\sigma-\rho w}^{\sigma+\rho w} y^{\beta - k_1 - 1} \left\{ \sum_{s_1=0}^{\left[\frac{n_1}{m_1} \right]} \dots \sum_{s_R=0}^{\left[\frac{n_R}{m_R} \right]} \frac{(-n_1)m_1 s_1}{\angle s_1} \dots \frac{(-n_R)m_R s_R}{\angle s_R} A[n_1, s_1; \dots; n_R, s_R] \right\}^{-1} \\ \left\{ E^{s_1 + \dots + s_R} Z^{\frac{-p(s_1 + \dots + s_R) - k_1}{q}} \Omega \left(\frac{(-p(s_1 + \dots + s_R) - k_1)}{q} \right) \right\}^{-1} \phi(k_1) dk_1. \quad (3.8)$$

As the solution of the integral equation (3.1)

4. Special Case

1. In theorem 1, if we take $R=1$, $m_1=2$ and $A_{n_1, s_1} = (-1)^{s_1}$, then we have the very interesting theorem, i.e.

Theorem 3- Suppose that the conditions corresponding to Theorem 2 are satisfied. Then

$$\int_0^\infty y^{-\alpha} E_1^{n_1/2} \left(\frac{x}{y} \right)^{\frac{n_1 s_1}{2}} H_{n_1} \left[\frac{1}{2\sqrt{E_1 \left(\frac{x}{y} \right)^{s_1}}} \right]$$

$$\mathfrak{K}_{p_i+1, q_i+1, \tau_i; r}^{m, n+1} \left[z \left(\frac{x}{y} \right)^q \middle| \begin{matrix} [1-\beta-p(s_1+\dots+s_R); q], (a_j, A_j)_{1, n}, \dots, [\tau_j(a_j, A_j)]_{n+1, p_i} \\ (b_j, B_j)_{1, m}, \dots, [\tau_j(b_j, B_j)]_{m+1, q_i}, [1-\alpha-p(s_1+\dots+s_R); q] \end{matrix} \right]$$

$$f(y) dy = g(x)$$

is given by

$$f(y) = \frac{q}{2\pi w} \lim_{\rho \rightarrow \infty} \int_{\sigma - \rho w}^{\sigma + \rho w} y^{\beta - k_1 - 1} \left\{ \sum_{s_1=0}^{\left[\frac{n_1}{2} \right]} \frac{(-n_1)_{2s_1}}{\angle s_1} (-1)^{s_1} \right\}^{-1}$$

$$\left\{ E^{s_1} Z^{\frac{-p(s_1+\dots+s_R)-k_1}{q}} \Omega \left(\frac{(-p(s_1+\dots+s_R)-k_1)}{q} \right) \right\}^{-1} \phi(k_1) dk_1. \quad (4.1)$$

Provided integral exists.

2. If we take $n_1 = \dots = n_R \rightarrow 0$, $\tau_1 = \dots = \tau_R = 1$ and $r = 1$, Theorem 2 is seen to correspond to a result given by Srivastava and Raina [6]

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