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A Result on Simple (-1,1) Rings

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Abstract

In this paper we show the identity that (x,y,[R,R]) = 0 holds good in a simple (-1,1) ring of characteristic $\neq$ 2,3. Using this a simple not associative (-1,1) ring of characteristic $\neq$ 2,3 is strongly (-1,1) ring.

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In [2] E.Kleinfeld proved that a right alternative ring of characteristic $\neq$ 2,3 which satisfies (i) S(xy,z,x) = S(y,z,x)x and (ii) S (x,y,zx) = xS(x,y,z) is either alternative or strongly (-1,1), where S(x,y,z) = (x,y,z) + (y,z,x) + (z,x,y). In 2000 K.Suvarna [3] proved that a prime (-1,1) ring of characteristic $\neq$ 2,3 is either associative or strongly (-1,1). Again Kleinfeld [1] proved that a semi prime right alternative ring with characteristic $\neq$ 2,3 under the assumption of [[a,b],a] = 0 is strongly (-1,1). In this paper without any additional assumption we prove that a simple not associative (-1,1) ring of characteristic $\neq$ 2,3 is strongly (-1,1) ring.

A (-1,1) ring R is a non-associative ring in which the following identities hold:

$$(x,y,z) + (x,z,y) = 0 \qquad \ldots\ldots\ldots\ldots(1)$$

$$(x,y,z) + (y,z,x) + (z,x,y) = 0 \qquad \ldots\ldots\ldots\ldots(2)$$

for all z,y,z in R. The associator (x,y,z) is defined by (x,y,z) = xy.z – x.yz and the commutator [x,y] = xy-yx. The ring R is called strongly (-1,1) if R satisfies identity (1) and $[R,[R,R]] = 0$. If there exists a positive integer n such that $na = 0$ for every element a of the ring R, the smallest such positive integer is called the characteristic of R. K.Suvarna proved that [Lemma1 of [3]] in a (-1,1) ring of characteristic $\neq 2,3$ every associator commutes with every element of R.

From [3] we have

$$(w,(x,y,z)) = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots.. \quad (3)$$

Replace w by an arbitrary commutator [R,R] in (3)

$$[[R,R],(x,y,z)] = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots... \quad (4)$$

Again replace w by an arbitrary associator (R,R,R) in (3)

$$[(R,R,R),(x,y,z)] = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Define $T = \{t \in R \,/\, [t,(R,R,R)] = 0 = [tR,(R,R,R)]\}$.

LEMMA*: T is an ideal of a (-1,1) ring of characteristic ≠ 2,3.*

PROOF : Let $t \in T$. Then it follows from the definition of T that $[tx,(R,R,R)] = 0$. Use of (4) shows that $[xt,(R,R,R)] = 0$. Since $tx.y = (t,x,y) + t.xy$. Commute this equation (R,R,R) and use equation (5) and the definition of T. Thus $[tx.y,(R,R,R)] = 0$, which proves that T is a right ideal of R. Now $xt.y = [xt,y] - (y,x,t) + [yx,t] + t.yx$. Commute this equation with (R,R,R) and using (4) , (5) and the definition of T shows that $[xt.y,(RRR)] = 0$. Thus $xt \in T$, then T is a left ideal and hence T is an ideal of R.

Consider the following identities which hold in arbitrary rings.

$$[xy,y] = [x,y]y + (y,x,y \qquad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

$[xy,z] = x[y,z] + [x,z]y + (x,y,z) + (z,x,y) - (x,z,y)$. In (-1,1) ring this equation becomes

$$[xy,z] = x[y,z] + [x,z]y - (y,z,x) + (x,y,z) \qquad \ldots\ldots\ldots\ldots \quad (7)$$

$$(wx,y,z) - (w,xy,z) + (w,x,yz) = w(x,y,z) + (w,x,y)z \quad \ldots\ldots\ldots. \quad (8)$$

THEOREM: *Let R be a simple not associative (-1,1) ring of characteristic ≠ 2,3 then R is strongly (-1,1) ring.*

PROOF: Since R is simple then either $T = R$ or $T = 0$. If $T = R$ satisfies (3). But R is not associative. Thus we are left with possibility that $T = 0$. Let u be an arbitrary associator . Commute (6) with with u. Then use equations (4) and (5), obtaining

$$[[x,y]y,(R,R,R)] = 0 \quad \ldots\ldots\ldots.. \quad (9)$$

Linearize (9) we get

$$[[x,y]z,(R,R,R)] = -[[x,z]y,(R,R,R)] \quad \ldots\ldots\ldots\ldots \quad (10)$$

Substitute $z = (a,b,c)$ an arbitrary associator in (10) and from (3) we have

$$[[x,y](a,b,c),(R,R,R)] = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

It follows from (11) and (4) that

$$[(a,b,c)[x,y],(R,R,R)] = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots. \quad (12)$$

Put z = [R,R] an arbitrary commutator in (8)

(wx,y,[R,R]) – (w,xy,[R,R]) + (w,x,y[R,R]) = w(x,y,[R,R]) + (w,x,y)[R,R]

Commute this equation with an arbitrary assiciator and use (5) and (12). Then [w(x,y,[R,R]),(R,R,R)] = 0. From (3) this can be written as

[(x,y,[R,R])w,(R,R,R)] = 0. ……………… (13)

Using (5) and (13) we have (x,y,[R,R]) €T. Since T = 0. Hence

[x,y,[R,R]] = 0. ………………. (14)

Replace x = (f,g) by a commutator and y = (a,b,c) an associator in (14), we have

([f,g],(a,b,c),[R,R]) = 0. ……………………… (15)

And from (1)

([f,g],[R,R],(a,b,c)) = 0. …………………….. (16)

Let x = (p,q) and y = (r,s) be arbitrary commutators and z = (R,R,R) an arbitrary associator. Substitute these in (7) gives [[p,q][r,s],(R,R,R)] = [p,q][[r,s],(R,R,R)] + [[p,q],(R,R,R)][r,s] – ([r,s],(R,R,R),[p,q]) + ([p,q],[r,s],(R,R,R)). This equation follows from (15) and (16) that

[[p,q][r,s],(R,R,R)] = 0. ……………. (17)

Put z = [R,R] an arbitrary commutator in (10) and using (17), we get

[[x,[R,R]]y,(R,R,R)] = 0. …………….. (18)

At this point (18) and (4) imply that [x,[R,R] € T , so that [R,[R,R]] = 0, since T = 0. Hence Ris strongly (-1,1) ring.

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