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On Generalized Pre Regular Weakly (gprw)-Closed Sets in Topological Spaces

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Abstract

The aim of this paper is to introduce and study the new class of sets called *gprw*-closed sets. This new class of sets lies between the class of regular weakly closed (briefly *rw*-closed) sets and the class of generalized pre regular closed (briefly *gpr*-closed) sets. And also we study the fundamental properties of this class of sets.

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1 Introduction

Every topological space can be defined either with the help of axioms for the closed sets or the Kutatowiski closure axioms. So one can imagine that, how important the concept of closed sets is in the topological spaces. In 1970, levine [14] initiated the study of so-called generalized closed sets. By definition, a subset S of a topological space X is called generalized closed set if $cl(A) \subset U$ whenever $A \subset U$ and U is open. This notion has been studied extensively in the recent years by many topologists because generalized closed sets are not only natural generalization of closed sets. Moreover, they also suggest several new properties of topological spaces. Most of these new properties are separation axioms weaker than T_1, some of them found to be useful in computer science and digital topology. Furthermore, the study of generalized closed sets also provides new characterization of some known classes of spaces, for example, the class of extremely disconnected spaces by Caw, Ganster and Reilly [5]. In 1997 Y. Gnanambal [12] proposed the definition of generalized preregular-closed sets (briefly gpr-closed) and further notion of preregular $T_{1/2}$ space and generalized preregular continuity was introduced. And in 2007, notion of regular weakly closed set is defined by S.S. Benchalli and R.S. Wali [3] and proved that this class lie between the class of all w-closed sets given by P. Sundaram and M. Sheik John [25] and the class of all regular generalized closed sets defined by N. Palaniappan and K.C. Rao [21]. In this paper, we introduced and studies new class of sets called generalized pre regular weakly closed set (briefly $gprw$-closed) in topological space which is properly placed between the regular weakly closed sets and generalized pre regular closed sets.

2 Preliminary Notes

Definition 2.1 *A subset A of X is called generalized closed (briefly g-closed) [14] set iff $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.*

Definition 2.2 *A subset A of X is called regular open (briefly r-open) [24] set if $A = int(cl(A))$ and regular closed (briefly r-closed) [24] set if $A = cl(int(A))$.*

Definition 2.3 *A subset A of X is called pre-open set [18] if $A \subseteq int(cl(A))$ and pre-closed [18] set if $cl(int(A)) \subseteq A$.*

Definition 2.4 *A subset A of X is called semi-open set [15] if $A \subseteq cl(int(A))$ and semi-closed [15] set if $A \subseteq int(cl(A))$.*

Definition 2.5 *A subset A of X is called α-open [20] if $A \subseteq int(cl(int(A)))$ and α-closed [20] if $cl(int(cl(A))) \subseteq A$.*

Definition 2.6 *A subset A of X is called semi-preopen [2] if $A \subseteq cl(int(cl(A)))$ and semi-preclosed [2] if $int(cl(int(A))) \subseteq A$.*

Definition 2.7 *A subset A of X is called θ-closed [27] if $A = cl_\theta(A)$, where $cl_\theta(A) = \{x \in X : cl(U) \cap A \neq \phi, U \in \tau and x \in U\}$.*

Definition 2.8 *A subset A of X is called δ-closed [27] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \phi, U \in \tau and x \in U\}$.*

Definition 2.9 *A subset A of a space (X, τ) is called regular semiopen [6] if there is a regular open set U such that $U \subset A \subset cl(U)$. The family of all regular semiopen sets of X is denoted by $RSO(X)$.*

Definition 2.10 *A subset A of a space (X, τ) is said to be semi-regular open [9] if it is both semiopen and semiclosed.*

Definition 2.11 *A subset of a topological space (X, τ) is called*

1. *Semi-generalized closed (briefly sg-closed) [4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X.*

2. *Generalized semiclosed (briefly gs-closed) [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

3. *Generalized α-closed (briefly gα-closed) [16] if $\alpha - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.*

4. *Generalized semi-preclosed (briefly gsp-closed) [10] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

5. *Regular generalized closed (briefly rg-closed) [21] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.*

6. *Generalized preclosed (briefly gp-closed) [17] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

7. *Generalized pre regular closed (briefly gpr-closed) [12] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.*

8. *θ-generalized closed (briefly $\theta - g$-closed) [7] if $cl_\theta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

9. *δ-generalized closed (briefly $\delta - g$-closed) [11] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

10. *Weakly generalized closed (briefly wg-closed) [19] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X.*

11. *Strongly generalized closed [25] (briefly g^*-closed [26]) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.*

12. *π-generalized closed (briefly $\pi - g$-closed) [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.*

13. *Weakly closed (briefly w-closed) [22] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X.*

14. *Mildly generalized closed (briefly mildly g-closed) [23] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.*

15. *Semi weakly generalized closed (briefly swg-closed) [19] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X.*

16. *Regular weakly generalized closed (briefly rwg-closed) [19] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.*

17. *Regular weakly closed (briefly rw-closed) [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semiopen in X.*

The complements of the above mentioned closed sets are their respective open sets.

Theorem 2.12 *Every regular semiopen set in X is semiopen but not conversely.*

Theorem 2.13 *[13] If A is regular semiopen in X, then X/A is also regular semiopen.*

Theorem 2.14 *[13] In a space X, the regular closed sets, regular open sets and clopen sets are regular semiopen.*

3 The Generalized pre regular weakly ($gprw$)-closed sets

After investigating several closed sets through extensive study we now introduce and studies basic properties of new class of sets called generalized pre regular weakly closed (briefly *gprw*-closed) set in this section.

Definition 3.1 *Let (X, τ) be a topological space and A be a subset of X said to be $gprw$-closed if $pcl(A) \subset U$, whenever $A \subset U$ and U is regular semi open. The family of all $gprw$-closed set in space X is denoted by $GPRWC(X)$.*

Theorem 3.2 *Any rw-closed set of topological space (X, τ) is $gprw$-closed set, but its converse is not true.*

Proof: Let A be an arbitrary rw-closed in (X, τ) such that $A \subset U$ and U is regular semi open. By definition of rw-closed we have, $cl(A) \subset U$. Since every closed set in a topological space is pre-closed therefore $pcl(A) \subset cl(A)$, So, we can say $pcl(A) \subset cl(A) \subset U$. Hence A is $gprw$-closed set.
Converse of the above theorem is not true. it can be seen from the following example.

Example 3.3 *Let* $X = \{1, 2, 3, 4, 5\}$ *be a space with topology* $\tau = \{\phi, X, \{1, 2\}, \{3, 4\}, \{1, 2, 3, 4\}\}$. *Here* $A = \{1\}$ *is gprw-closed set but not rw-closed set.*

Theorem 3.4 *Any gprw-closed set of topological space* (X, τ) *is gpr-closed set, but its converse is not true.*

Proof: Let (X, τ) be a topological space and A be $gprw$-closed subset of X such that $A \subset U$ where U is regular open. Since every regular open set is regular semi open, therefore U is regular semi open and by definition of $gprw$-closed set we have $pcl(A) \subset U$ and as given that A contained in U. Hence A is gpr-closed set.
Converse of the above theorem is not true. It can be seen from the following example.

Example 3.5 *Let* $X = \{1, 2, 3, 4\}$ *be space with topology* $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$. *As* $A = \{1, 3\}$ *is gpr-closed but not gprw-closed in* X.

Theorem 3.6 *Every closed set of topological space* (X, τ) *is gprw-closed set. But its converse is not true.*

Proof: According [3] that every closed set is rw-closed and by theorem 3.2, every closed set is $gprw$-closed. The converse does not hold.

Example 3.7 *Let* $X = \{1, 2, 3, 4\}$ *be space with topology* $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$. *Now here* $\{1, 2\}$ *is gprw-closed in* (X, τ)*, but it is not closed.*

Theorem 3.8 *Every regular closed set of topological space* (X, τ) *is gprw-closed set. But its converse is not true.*

Proof: According [3] as every regular closed set is rw-closed and by theorem 3.2 we can say that every closed set is $gprw$-closed. The converse do not hold. converse of this theorem can be seen with the help of example 3.7 and the fact that every regular closed set is closed.

Theorem 3.9 *Every* θ*-closed set of topological space* (X, τ) *is gprw-closed set. But its converse is not true.*

Proof: According to [27] as every θ-closed set is closed and by theorem 3.6 every θ-closed set is *gprw*-closed.
Converse, As shown in example 3.7 $A = \{1, 2\}$ is *gprw*-closed. To show $A = \{1, 2\}$ is not θ-closed. Let $A = \{1, 2\}$ is θ-closed so it must be closed. But it is a contradiction, as $A = \{1, 2\}$ is not closed. Thus $A = \{1, 2\}$ is not θ-closed.

Theorem 3.10 *Every δ-closed set of topological space (X, τ) is gprw-closed set but not conversely.*

Proof: According to [27] as every δ-closed set is closed and by theorem 3.6, every δ-closed set is *gprw*-closed.

Theorem 3.11 *Every π-closed set of topological space (X, τ) is gprw-closed set but not conversely.*

Proof: According to [11] as every π-closed set is closed and by theorem 3.6, every π-closed set is *gprw*-closed.

Theorem 3.12 *Every w-closed set of topological space (X, τ) is gprw-closed set. But its converse is not true.*

Proof: Proof of this theorem can be verified from [3] and 3.6. And Converse of this theorem can be seen from the example 3.7. In that example if we take a subset $\{3\}$ of X then it is *gprw*-closed. But $\{3\}$ is not w-closed set.

Theorem 3.13 *Every pre-closed set of topological space (X, τ) is gprw-closed set in a topological space (X, τ), but not conversely.*

Proof: Let a subset A of (X, τ) be pre-closed set, such that $A \subset U$, where U is regular semi open set. Since A is pre-closed therefore $pcl(A) = A$. So we got $pcl(A) \subset U$ whenever $A \subset U$, and U is regular semi open set. Hence A is *gprw*-closed in (X, τ).Converse of this theorem is not true.

Example 3.14 *Let $X = \{1, 2, 3, 4\}$ be space with topology $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$. Now if we take $A = \{1, 2, 4\}$ then it is not pre-closed set. But $A = \{1, 2, 4\}$ is gprw-closed set in (X, τ). Hence from this example we have seen that every gprw-closed set is not always pre-closed set.*

Remark 3.15 *We can see from the following examples that gprw-closed set is independent of mildly g-closed set, g^*-closed set, wg-closed set, semi closed set,α-closed set, $g\alpha$-closed set, αg-closed set, sg-closed set, gs-closed set, gsp-closed set, β-closed set, gp-closed set, swg-closed set, πg-closed set, θ-generalized closed set, δ-genralized closed set, g^*s-closed set, g-closed set, rg-closed set, rwg-closed set, gr-closed set.*

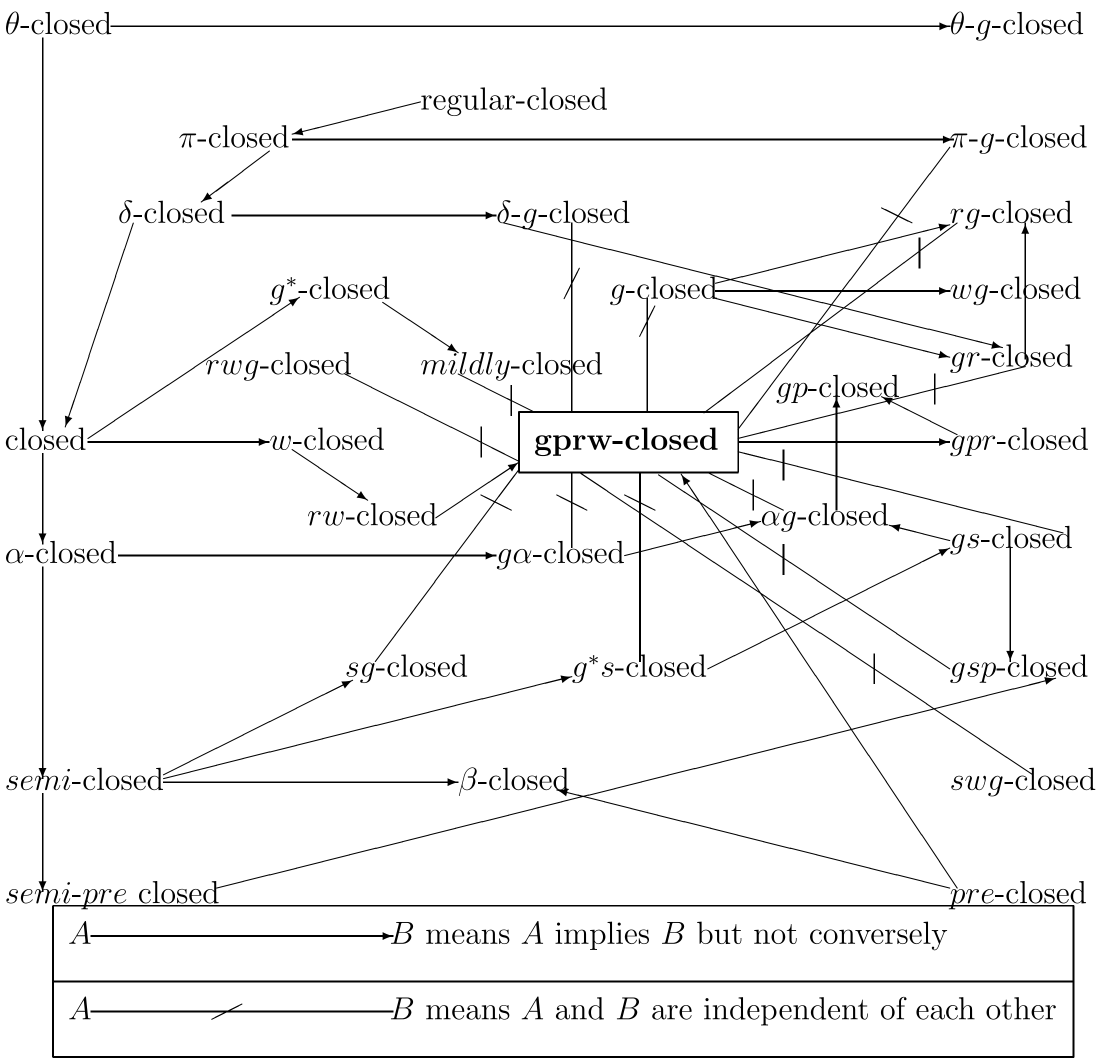
Example 3.16 *Let $X = \{1, 2, 3, 4\}$ be a space with $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$ then*

1. *Closed sets in* (X, τ) *are* ϕ, X, $\{4\}$, $\{3,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
2. *gprw-closed sets in* (X, τ) *are* ϕ, X, $\{4\}$, $\{1,2\}$, $\{3,4\}$, $\{1,2,3\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
3. *Mildly g-closed sets in* (X, τ) *are* ϕ, X, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,2,4\}$, $\{2,3,4\}$, $\{1,3,4\}$.
4. g^**-closed sets in* (X, τ) *are* ϕ, X, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
5. *wg-closed sets in* (X, τ) *are* ϕ, X, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,3\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
6. *Semi-closed sets in* (X, τ) *are* ϕ, X, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,3\}$, $\{1,3\}$, $\{2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
7. α*-closed sets in* (X, τ) *are* ϕ, X, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
8. $g\alpha$*-closed sets in* (X, τ) *are* ϕ, X, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{2,3,4\}$, $\{1,3,4\}$.
9. αg*-closed sets in* (X, τ) *are* ϕ, X, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
10. *sg-closed sets in* (X, τ) *are* ϕ, X, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{2,3\}$, $\{1,4\}$, $\{2,4\}$, $\{1,3\}$, $\{1,3,4\}$, $\{2,3,4\}$.
11. *gs-closed sets in* (X, τ) *are* ϕ, X, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{2,3\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,3\}$, $\{1,3,4\}$, $\{1,2,4\}$, $\{2,3,4\}$.
12. *gsp-closed sets in* (X, τ) *are* ϕ, X, $\{3\}$, $\{4\}$, $\{2,3\}$, $\{3,4\}$, $\{1,4\}$, $\{1,3\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
13. β*-closed sets in* (X, τ) *are* ϕ, X, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,3\}$, $\{1,3\}$, $\{2,4\}$, $\{2,3,4\}$, $\{1,3,4\}$.
14. *gp-closed sets in* (X, τ) *are* ϕ, X, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,2,4\}$.
15. *swg-closed sets in* (X, τ) *are* ϕ, X, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
16. πg*-closed sets in* (X, τ) *are* ϕ, X, $\{3\}$, $\{4\}$, $\{1,3,4\}$, $\{2,3,4\}$, $\{1,3\}$, $\{2,3\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,2,4\}$.
17. θ*-generalized closed sets in* (X, τ) *are* ϕ, X, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.
18. δ*-generalized closed sets in* (X, τ) *are* ϕ, X, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.

19. g^*s-closed sets in (X, τ) are ϕ, X, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{2,3\}$, $\{3,4\}$, $\{1,4\}$, $\{1,3\}$, $\{2,4\}$, $\{2,3,4\}$, $\{1,3,4\}$.

20. g-closed sets in (X, τ) are ϕ, X, $\{4\}$, $\{3,4\}$, $\{1,4\}$, $\{2,4\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$.

Remark 3.17 *Here we can see following implication from the above discussion and known results.*



Remark 3.18 *The intersection of two gprw-closed set of topological space (X, τ) is generally not gprw-closed set.*

Example 3.19 *Let $X = \{1, 2, 3\}$ be a topological space with topology $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}\}$. Here $A = \{1, 2\}$ and $B = \{2, 3\}$ both are gprw-closed set but $A \cap B = \{2\}$ is not gprw-closed set.*

Remark 3.20 *The union of two gprw-closed set of topological space (X, τ) is not gprw-closed set.*

Example 3.21 *Let $X = \{1, 2, 3, 4, 5\}$ be topological space with topology $\tau = \{\phi, X, \{1, 2\}, \{3, 4\}, \{1, 2, 3, 4\}\}$. Here $A = \{3\}$ and $B = \{4\}$ are both gprw-closed but $A \cup B = \{3, 4\}$ is not gprw-closed.*

Theorem 3.22 *If A and B are two gprw-closed sets in a topological space X such that either $A \subset B$ or $B \subset A$ then both intersection and union of two gprw-closed sets is gprw-closed.*

Proof: If A contain in B or B contain in A, then $A \cup B = B$ or $A \cup B = A$ respectively. This show that $A \cup B$ is *gprw*-closed as A and B are *gpre*-closed set. Similarly $A \cap B$ is also *gpre*-closed set.

Remark 3.23 *Difference of two gprw-closed sets is not gprw-closed set. Let a topological space $X = \{1, 2, 3\}$ with topology $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}\}$. Here $A = \{2, 3\}$ and $B = \{3\}$ are gprw-closed but $A - B = \{2\}$ is not.*

Theorem 3.24 *Let (X, τ) be a topological space and if a subset A of X is gprw-closed in X, then $pcl(A) \setminus A$ does not contain any non empty regular semi open set in X.*

Proof: Suppose U be a non-empty regular semi open set in X such that $U \subset pcl(A) \setminus A$. Now as $U \subset X \setminus A$ so $A \subset X \setminus U$. By [3] $X \setminus U$ will be regular semi open. Hence by definition of *gprw*-closed, $pcl(A) \subset X \setminus U$. Now we can say that $U \subset X \setminus pcl(A)$. And as we know that $U \subset pcl(A)$, therefore $U \subset (pcl(A) \cap X \setminus pcl(A)) = \phi$, this shows that U is empty set, which is a contradiction. Hence $pcl(A) \setminus A$ does not contain any nonempty regular semi open set in X.
The converse of the above theorem need not to be true, that means if $pcl(A) \setminus A$ contain no nonempty regular semi open subset in X then A need not to be *gprw*-closed.

Example 3.25 *Let $X = \{1, 2, 3\}$ be space with topology $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}\}$. Now consider a subset $A = \{1\}$ of X then $pcl(A) \setminus A = (X \cap \{1, 3\}) \setminus \{1\} = \{3\}$, does not contain any nonempty regular semi open. But $A = \{1\}$ is not an gprw-closed set in X.*

Corollary 3.26 *If a subset A of topological space X is an gprw-closed set in X then $pcl(A) \setminus A$ does not contain any nonempty regular open set in X. But converse is not true.*

Proof: Proof of this corollary follows from theorem 3.24 and the fact that every regular open set is regular semi open.

Theorem 3.27 *Let (X, τ) be the topological space then for $x \in X$, the set $X \setminus \{x\}$ is gprw-closed or regular semi open.*

Proof: If $X \setminus \{x\}$ is *gprw*-closed or regular semi open set then we are done. Now suppose $X \setminus \{x\}$ is not regular semi open then X is only regular semi open set containing $X \setminus \{x\}$ and also $pcl(X \setminus \{x\})$ is contained in X as it is the biggest set containing all its subsets. Hence $X \setminus \{x\}$ is *gprw*-closed set in X.

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