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Homomorphism in Bipolar Fuzzy Finite State Machines

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Abstract

In this paper we introduced homomorphism, strong homomorphism in bipolar fuzzy finite state machines and discuss their properties using bipolar-valued fuzzy set.

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Keywords: Bipolar fuzzy finite state machines, homomorphism, strong homomorphism.

1 Introduction

The theory of fuzzy set was introduced by L.A. Zadeh in 1965 [9]. The mathematical formulation of a fuzzy automaton was first proposed by W.G. Wee in 1967 [8]. E.S. Santos 1968 [7] proposed fuzzy automata as a model of pattern recognition. John N. Mordeson and D.S. Malik gave a detailed account of fuzzy automata and languages in their book 2002 [6]. Fuzzy sets are kind of useful mathematical structure to represent a collection of objects whose boundary is vague. There are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionstic fuzzy sets, interval-valued fuzzy sets etc. Bipolar-valued fuzzy sets, which are intoduced by Lee [4, 5], are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In [2], Y.B. Jun and J. Kavikumar introduced bipolar fuzzy finite state machines, a bipolar successor, a bipolar exchange property and a bipolar subsystem. In this paper we introduced homomorphism, strong homomorphism in bipolar fuzzy finite state machines with examples and discuss their properties using the notions of bipolar-valued fuzzy sets.

2 Preliminaries

In the traditional fuzzy sets, the membership degrees of elements range over the interval [0, 1]. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set. The membership degree 0 indicates that an element does not belong to the fuzzy set. The membership degrees on the interval (0, 1)indicate the partial membership to the fuzzy set. Sometimes, the memebership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set [1, 10]. In the view point of satisfaction degree, the membership degree 0 is assigned to elements which do not satisfy some property. The elements with membership degree 0 are usually regarded as having the same characteristics in the fuzzy set representation. By the way, among such elements some have irrelevant characteristics to the property corresponding to a fuzzy set and the others have contrary characteristics to the property. The traditional fuzzy set representation cannot tell apart contrary elements from irrelevant elements. Consider a fuzzy set "young" defined on the age domain [0, 100](see Figure 1). Now consider two ages 50 and 95 with membership degree 0. Although both of them do not satisfy the property "young", we say that age 95 is more apart from the property rather than age 50 [4].

Only with the membership degrees ranged on the interval [0, 1], it is difficult to express the difference of the irrelevant elements from the contrary elements in fuzzy sets. If a set representation could express this kind of difference, it would be more informative than the traditional fuzzy set representation. Based on these observations, Lee [4] introduced an extension of fuzzy sets named bipolar-valued sets.

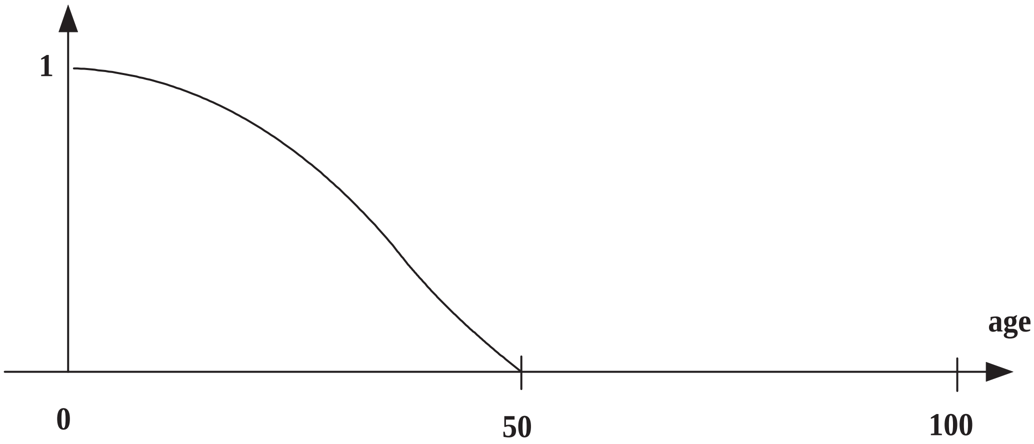

Figure 1. A fuzzy set "young"

Bipolar-valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property corresponding to fuzzy set and its counter-property. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0, 1] indicate that elements somewhat satisfy the property, and the membership degrees on [-1, 0) indicate that elements somewhat satisfy the implicit counter-property [4]. Figure 2 shows a bipolar-valued fuzzy set redefined for the fuzzy set "young" of Figure 1. The negative membership degrees indicate the satisfaction extent elements to an implicit counter-property(e.g., old against the property young). This kind of bipolar-valued fuzzy set representation enables the elements with membership degree 0 (irrelevant elements) and the elements with negative membership degrees(contrary elements). The age elements 50 and 95, with membership degree 0 in the fuzzy sets of Figure 1, have 0 and a negative membership degree in the bipolar-valued fuzzy set of Figure2, respectively. Now it is manifested that 50 is an irrelevant age to the property young and 95 is more apart from the property young than 50, i.e., 95 is a contrary age to the property young [4]. Let X be the universe of discourse.

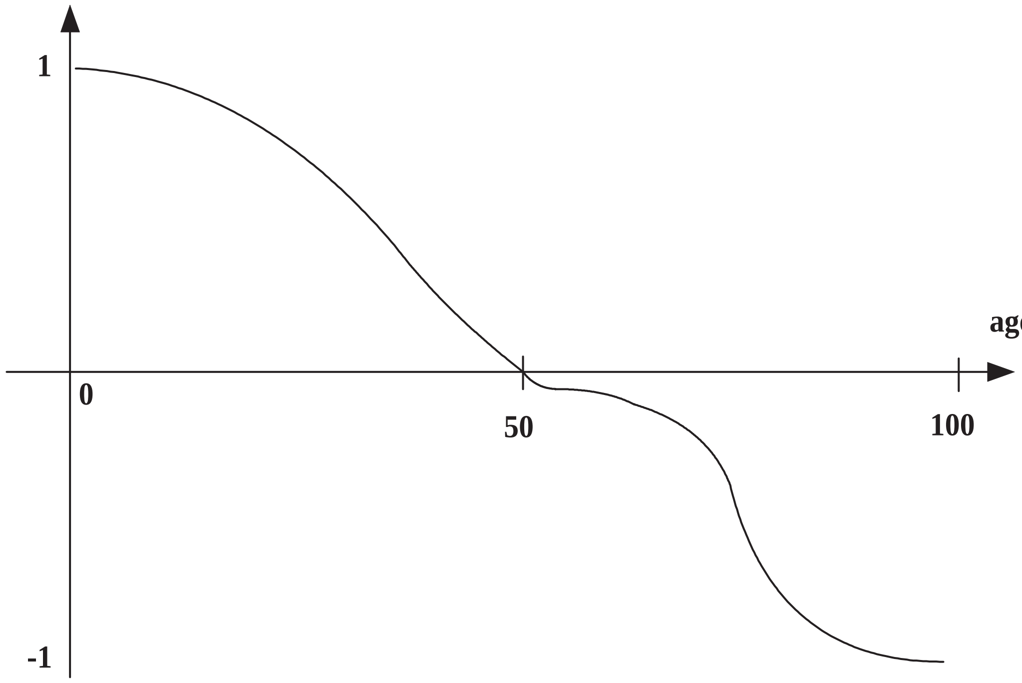

Figure 2. A bipolar fuzzy set "young"

A bipolar-valued fuzzy setφ in X is an object having the form
$\varphi = \{(x,\ \varphi^-(x),\ \varphi^+(x)) \mid x\ \in\ X\}$
where $\varphi^-\ :\ X\ \rightarrow\ [-1,0]$ and $\varphi^+\ :\ X\ \rightarrow\ [0,1]$ are the mappings. The positive membership degree $\varphi^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar- valued fuzzy set $\varphi = \{(x,\ \varphi^-(x),\ \varphi^+(x))|\ x\ \in\ X\}$ and the negative membership degree $\varphi^-(x)$ denotes the satisfaction degree of x to some implicit counter-property of $\varphi = \{(x,\ \varphi^-(x),\ \varphi^+(x))|\ x\ \in\ X\}$. If $\varphi^+(x) \neq 0$ and $\varphi^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for $\varphi = \{(x,\ \varphi^-(x),\ \varphi^+(x))|\ x\ \in\ X\}$. If $\varphi^+(x) = 0$ and $\varphi^-(x) \neq 0$, it is the situation that x does not satisfy the property of $\varphi = \{(x,\ \varphi^-(x),\ \varphi^+(x))|\ x\ \in\ X\}$ but somewhat satisfies the counter-property of $\varphi = \{(x,\ \varphi^-(x),\ \varphi^+(x))|\ x\ \in\ X\}$. It is possible for an element x to be $\varphi^+(x) \neq 0$, and $\varphi^-(x) = 0$ when the membership function of the property overlaps that of its counter-property over some portion of the domain [5]. For the sake of simplicity, we shall use the symbol $\varphi = \langle\varphi^-, \varphi^+\rangle$ for the bipolar-valued fuzzy set $\varphi = \{(x,\ \varphi^-(x),\ \varphi^+(x))|\ x\ \in\ X\}$ and use the notion of bipolar fuzzy sets instead of the notion of bipolar-valued fuzzy sets.

3 Basic Definitions

Definition 3.1[10]

Let X denote a universal set. Then a fuzzy set A in X is set of ordered pairs:

$A = \{(x,\ \mu_A(x)|x\ \in\ X\}$,

$\mu_A(x)$ is called the membership function or grade of membership of x in A which maps X to the membership space $[0, 1]$.

Definition 3.2[3]

A finite fuzzy automata is a system of 5 tuples, $M = (\Sigma,\ Q,\ \pi,\ \eta,\ f_M)$ where

Q-set of states $\{q_1,\ q_2, ...,\ q_n\}$

Σ-alphabets (or) input symbols

π-$Q \rightarrow [0, 1]$ initial state designator

η-$Q \rightarrow [0, 1]$ final state designator

f_M-function from $Q \times \Sigma \times Q \rightarrow [0, 1]$

$f_M(q_i, \sigma, q_j) = \mu$ $[0 < \mu \leq 1]$ means when M is in state q_i and reads the input σ will move to the state q_j with weight function μ.

Bipolar fuzzy finite state machines

Definition 3.3[2]

A bipolar fuzzy finite state machine (bffsm, for short) is a triple $M = (Q,\ X,\ \varphi)$, where Q and X are finite nonempty sets, called the set of states and the set of input symbols, respectively and $\varphi = \langle \varphi^-, \varphi^+ \rangle$ is a bipolar fuzzy set in $Q\ \times\ X\ \times Q$.

Let X^* denote the set of all words of elements of X of finite length. Let λ denote the empty word in X^* and $|x|$ denote the length of x for every $x\ \in\ X^*$.

Definition 3.4[2]

Let $M = (Q,\ X,\ \varphi)$ be a bffsm. Define a bipolar fuzzy set $\varphi_* = \langle {\varphi_*}^+, {\varphi_*}^- \rangle$ in $Q\ \times\ X^*\ \times\ Q$ by

$$\varphi_*^-(q,\ \lambda,\ p) = \begin{cases} -1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

$$\varphi_*^+(q,\ \lambda,\ p) = \begin{cases} 1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

$\varphi_*^-(q, xa, p) = \inf_{r\ \in\ Q}[{\varphi_*}^-(q,\ x,\ r) \vee {\varphi_*}^-(r,\ a,\ p)]$

${\varphi_*}^+(q, xa, p) = \sup_{r\ \in\ Q}[{\varphi_*}^+(q,\ x,\ r) \wedge {\varphi_*}^+(r,\ a,\ p)]\ \forall\ p,\ q\ \in\ Q,\ x\ \in\ X^*$ and $a\ \in\ X$.

Result

Let $M = (Q,\ X,\ \varphi)$ be a bffsm. Then

$\varphi_*{}^-(q, xy, p) = \inf_{r\ \in\ Q}[\varphi_*{}^-(q,\ x,\ r) \vee \varphi_*{}^-(r,\ y,\ p)]$

$\varphi_*{}^+(q, xy, p) = \sup_{r\ \in\ Q}[\varphi_*{}^+(q,\ x,\ r) \wedge \varphi_*{}^+(r,\ y,\ p)]\ \forall\ p,\ q\ \in\ Q$ and $x,\ y\ \in\ X^*$.

Homomorphism

Definition 3.5

Let $M_1 = (Q_1,\ X_1,\ \varphi_1)$ and $M_2 = (Q_2,\ X_2,\ \varphi_2)$ be bffsms. A pair $(\alpha,\ \beta)$ of mappings $\alpha : Q_1 \to Q_2$ and $\beta : X_1 \to X_2$ is called a homomorphism written $(\alpha, \beta) : M_1 \longrightarrow M_2$ if

$\varphi_1{}^-(q, x, p) \leq \varphi_2{}^-(\alpha(q),\ \beta(x),\ \alpha(p))$

$\varphi_1{}^+(q, x, p) \leq \varphi_2{}^+(\alpha(q),\ \beta(x),\ \alpha(p))\ \forall\ q,\ p \in Q_1$ and $\forall x \in X_1$.

Example

Let $M_1 = (Q_1, X_1, \varphi_1)$and $M_2 = (Q_2, X_2, \varphi_2)$be bffsms, where $Q_1 = \{q_1, q_2, q_3, \}$ $X_1 = \{a, b\}$ $Q_2 = \{p_1, p_2\}$ $X_2 = \{a, b\}$ and φ_1 , φ_2 are defined as follows.

M1

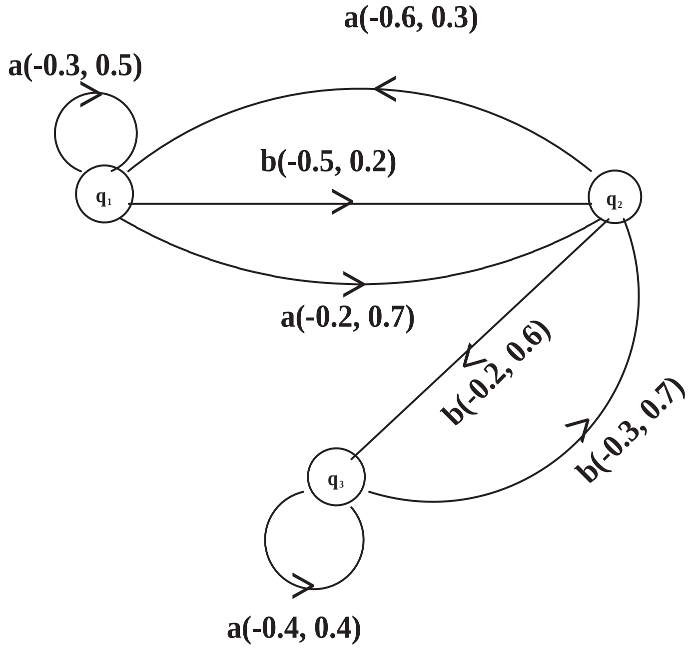

Fig- 3

Define $\alpha : Q_1 \to Q_2$ and $\beta : X_1 \to X_2$ as follows $\alpha(q_1) = \alpha(q_2) = p_1$, $\alpha(q_3) = p_2$ $\beta(a) = a$ and $\beta(b) = b$.

M2 (A Homomorphic Image of M1)

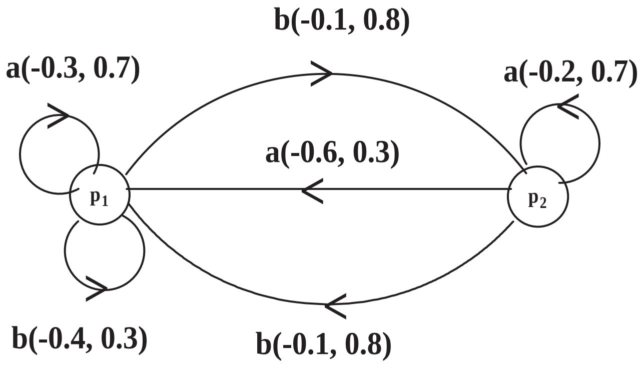

Fig- 4

Definition 3.6 Let $M_1 = (Q_1,\ X_1,\ \varphi_1)$ and $M_2 = (Q_2,\ X_2,\ \varphi_2)$ be bffsms. A pair $(\alpha,\ \beta)$ of mappings $\alpha : Q_1 \to Q_2$ and $\beta : X_1 \to X_2$ is called a strong homomorphism if

$\varphi_2{}^{-}(\alpha(q),\ \beta(x),\ \alpha(p)) = \vee \{\varphi_1{}^{-}(q, x, t)/t \in Q_1\ \alpha(t) = \alpha(p)\}$ and

$\varphi_2{}^{+}(\alpha(q),\ \beta(x),\ \alpha(p)) = \vee \{\varphi_1{}^{+}(q, x, t)/t \in Q_1\ \alpha(t) = \alpha(p)\}\ \forall\ q, p\ \in Q_1$ and $\forall\ x \in X_1$.

Example

Let $M_1 = (Q_1, X_1, \varphi_1)$and $M_2 = (Q_2, X_2, \varphi_2)$be bffsms. Where $Q_1 = \{q_1, q_2, q_3, \}$ $X_1 = \{a, b\}$ $Q_2 = \{p_1, p_2\}$ $X_2 = \{a, b\}$ φ_1 and φ_2 are defined as follows.

M1

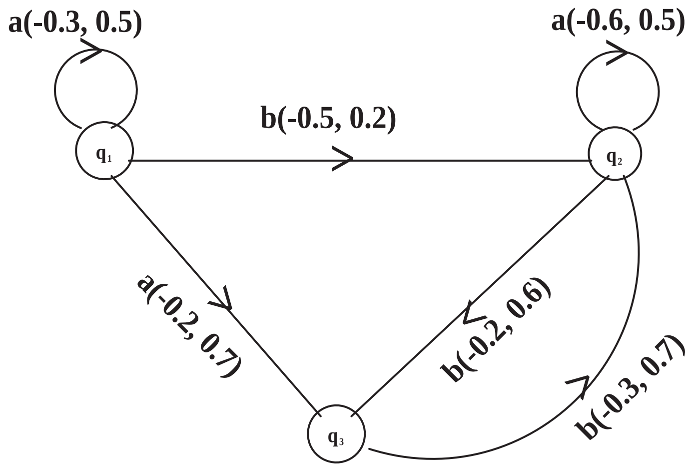

Fig- 5

Define $\alpha : Q_1 \rightarrow Q_2$ and $\beta : X_1 \rightarrow X_2$ as follows $\alpha(q_1) = \alpha(q_3) = p_1$, $\alpha(q_2) = p_2$ $\beta(a) = a$ and $\beta(b) = b$.

M2 (A Strong Homomorphic Image of M1)

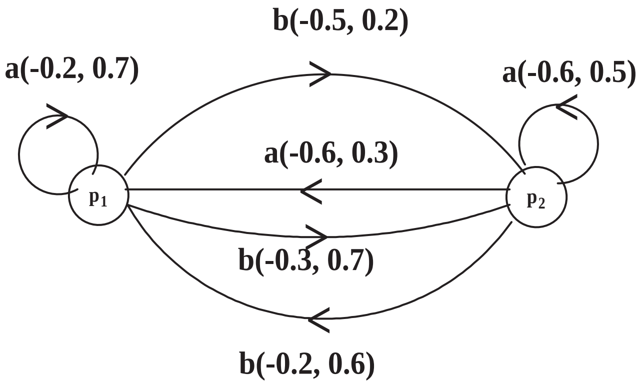

Fig- 6

Definition 3.7 Let $M_1 = (Q_1,\ X_1,\ \varphi_1)$ and $M_2 = (Q_2,\ X_2,\ \varphi_2)$ be two bffsms. Let $(\alpha, \beta) : M_1 \rightarrow M_2$ be bipolar homomorphism. Define $\beta^* : X_1^* \rightarrow X_2^*$ by $\beta^*(\lambda) = \lambda$ and $\beta^*(ua) = \beta^*(u)\beta(a)\ \forall\ u \in X_1^*,\ a \in X_1$.

4 Properties of Homomorphism in Bipolar Fuzzy Finite State Machines

Lemma 4.1

Let $M_1 = (Q_1,\ X_1,\ \varphi_1)$ and $M_2 = (Q_2,\ X_2,\ \varphi_2)$ be two bffsms. Let $(\alpha, \beta) : M_1 \to M_2$ be a strong homomorphism. Then $\forall\ q, r\ \in\ Q_1$ and $\forall\ x\ \in\ X_1$ if

${\varphi_2}^-(\alpha(q),\ \beta(x),\ \alpha(r)) < 0$ and ${\varphi_2}^+(\alpha(q),\ \beta(x),\ \alpha(r)) > 0$ then $\exists t\ \in\ Q_1$ such that ${\varphi_1}^-(q,\ x,\ t) < 0\ {\varphi_1}^+(q,\ x,\ t) > 0$ and $\alpha(t) = \alpha(r)$. Furthermore $\forall\ p\ \in\ Q_1$ if $\alpha(p) = \alpha(q)$ then ${\varphi_1}^-(q,\ x,\ t) \geq {\varphi_1}^-(p,\ x,\ r)$ and ${\varphi_1}^+(q,\ x,\ t) \geq {\varphi_1}^+(p,\ x,\ r)$

Proof:

Let $p, q, r\ \in\ Q_1$ and $x\ \in\ X_1$

${\varphi_2}^-(\alpha(q),\ \beta(x),\ \alpha(r)) = \vee\{{\varphi_1}^-(q, x, s)/s \in Q_1\ \alpha(s) = \alpha(r)\} < 0$ and

${\varphi_2}^+(\alpha(q),\ \beta(x),\ \alpha(r)) = \vee\{{\varphi_1}^+(q, x, s)/s \in Q_1\ \alpha(s) = \alpha(r)\} > 0$(By strong homomorphism). Since Q_1 is finite $\exists\ t\ \in\ Q_1$ such that $\alpha(t) = \alpha(r)$ and

${\varphi_1}^-(q,\ x,\ t) = \vee\{{\varphi_1}^-(q, x, s)/s \in Q_1\ \alpha(s) = \alpha(r)\} < 0$ and

${\varphi_1}^+(q,\ x,\ t)) = \vee\{{\varphi_1}^+(q, x, s)/s \in Q_1\ \alpha(s) = \alpha(r)\} > 0$.

Suppose $\alpha(p) = \alpha(q)$ then

${\varphi_1}^-(q,\ x,\ t) = {\varphi_2}^-(\alpha(q),\ \beta(x),\ \alpha(r)) = {\varphi_2}^-(\alpha(p),\ \beta(x),\ \alpha(r)) \geq {\varphi_1}^-(p,\ x,\ r)$

${\varphi_1}^+(q,\ x,\ t) = {\varphi_2}^+(\alpha(q),\ \beta(x),\ \alpha(r)) = {\varphi_2}^+(\alpha(p),\ \beta(x),\ \alpha(r)) \geq {\varphi_1}^+(p,\ x,\ r)$.

Lemma 4.2

Let $M_1 = (Q_1,\ X_1,\ \varphi_1)$ and $M_2 = (Q_2,\ X_2,\ \varphi_2)$ be two bffsms. Let $(\alpha, \beta) : M_1 \to M_2$ be a homomorphism. Define $\beta^* : X_1^* \to X_2^*$. Then $\beta^*(uv) = \beta^*(u)\beta^*(v) \forall\ u, v\ \in\ X_1^*$.

Proof:

Let $u, v\ \in\ X_1^*$ and $|v| = n$. If $n = 0$ then $v = \lambda$ and hence $\beta^*(uv) = \beta^*(u) = \beta^*(u)\beta^*(v)$. Suppose now the result is true $\forall\ y\ \in\ X_1^*$ such that $|y| = n - 1, n > 0$. Let $v = ya$ where $y\ \in\ X_1^*, a\ \in\ X_1$ and $|y| = n - 1$. Then $\beta^*(uv) = \beta^*(uya) = \beta^*(uy)\beta(a) = \beta^*(u)\beta^*(y)\beta^*(a) = \beta^*(u)\beta^*(ya) = \beta^*(u)\beta^*(v)$. Therfore $\beta^*(uv) = \beta^*(u)\beta^*(v)\ \forall u, v\ \in\ X_1^*$.

Theorem 4.1. *Let $M_1 = (Q_1,\ X_1,\ \varphi_1)$ and $M_2 = (Q_2,\ X_2,\ \varphi_2)$ be two bffsms. Let $(\alpha, \beta) : M_1 \to M_2$ be a homomorphism. Then*

${\varphi_{1*}}^-(q,\ x,\ p) \leq {\varphi_2}^-(\alpha(q),\ \beta^*(x),\ \alpha(p))$ *and*

${\varphi_{1*}}^+(q,\ x,\ p) \leq {\varphi_2}^+(\alpha(q),\ \beta^*(x),\ \alpha(p))\ \forall q, p\ \in\ Q_1$ *and* $x\ \in\ X_1^*$.

Proof.

Let $q, p\ \in\ Q_1$ and $x\ \in\ X_1^*$. We prove the result by induction on $|x| = n$. If $n = 0$ then $x = \lambda$ and $\beta^*(x) = \beta^*(\lambda) = \lambda$

$\varphi_{1*}{}^{-}(q,\ \lambda,\ p) = -1 = \varphi_2{}^{-}(\alpha(q),\ \beta^*(\lambda),\ \alpha(p))$ if $q = p$
$\varphi_{1*}{}^{-}(q,\ \lambda,\ p) = 0 = \varphi_2{}^{-}(\alpha(q),\ \beta^*(\lambda),\ \alpha(p))$ if $q \neq p$
$\varphi_{1*}{}^{+}(q,\ \lambda,\ p) = 1 = \varphi_2{}^{+}(\alpha(q),\ \beta^*(\lambda),\ \alpha(p))$ if $q = p$
$\varphi_{1*}{}^{+}(q,\ \lambda,\ p) = 0 = \varphi_2{}^{+}(\alpha(q),\ \beta^*(\lambda),\ \alpha(p))$ if $q \neq p$.
Suppose now the result is true $\forall\ y \in X^*$ such that $|y| = n-1, n > 0$.
Let $x = ya$ where $y \in X_1^*$, $a \in X_1$ and $|y| = n-1$.

$$\begin{aligned}
\varphi_{1*}{}^{-}(q,\ x,\ p) &= \varphi_{1*}{}^{-}(q,\ ya,\ p) \\
&= \wedge_{r\in Q_1}\{\varphi_{1*}{}^{-}(q,\ y,\ r) \vee \varphi_{1*}{}^{-}(r,\ a,\ p)\} \\
&\leq \wedge_{r\in Q_1}\left\{\varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(y),\ \alpha(r)) \vee \varphi_2^{-}(\alpha(r),\ \beta(a),\ \alpha(p)\right\} \\
&\qquad \text{(By homomorphism)} \\
&\leq \wedge_{r'\in Q_2}\left\{\varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(y),\ r') \vee \varphi_2^{-}(r',\ \beta(a),\ \alpha(p))\right\} \\
&= \varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(y)\beta(a),\ \alpha(p)) \\
&= \varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(ya),\ \alpha(p)) \\
&= \varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(x),\ \alpha(p))
\end{aligned}$$

$\varphi_{1*}{}^{-}(q,\ x,\ p) \leq \varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(x),\ \alpha(p))$

Now,

$$\begin{aligned}
\varphi_{1*}{}^{+}(q,\ x,\ p) &= \varphi_{1*}{}^{+}(q,\ ya,\ p) \\
&= \vee_{r\in Q_1}\{\varphi_{1*}{}^{+}(q,\ y,\ r) \wedge \varphi_1{}^{+}(r,\ a,\ p)\} \\
&\leq \vee_{r\in Q_1}\left\{\varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(y),\ \alpha(r) \wedge \varphi_1^{+}(\alpha(r),\ \beta(a),\ \alpha(p)\right\} \\
&\qquad \text{(By homomorphism)} \\
&\leq \vee_{r'\in Q_2}\left\{\varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(y),\ r') \wedge \varphi_1^{+}(r',\ \beta(a),\ \alpha(p))\right\} \\
&= \varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(y)\beta(a),\ \alpha(p)) \\
&= \varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(ya),\ \alpha(p)) \\
&= \varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(x),\ \alpha(p))
\end{aligned}$$

$\varphi_{1*}{}^{+}(q,\ x,\ p) \leq \varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(x),\ \alpha(p))$

Theorem 4.2. *Let $M_1 = (Q_1,\ X_2,\ \varphi_1)$ and $M_2 = (Q_2,\ X_2,\ \varphi_2)$ be two bffsms. Let $(\alpha, \beta) : M_1 \to M_2$ be a homomorphism. Then α is one-one if and only if* $\varphi_{1*}{}^{-}(q,\ x,\ p) = \varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(x),\ \alpha(p))$
$\varphi_{1*}{}^{+}(q,\ x,\ p) = \varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(x),\ \alpha(p))\ \forall\ q, p \in Q_1$ *and* $x \in X_1^*$.

Proof.

Suppose α is one- one. Let $p, q \in Q_1$ and $x \in X_1^*$.
Let $|x| = n$. We prove the result by induction on n.
Let $n = 0$ then $x = \lambda$ and $\beta^*(\lambda) = \lambda$.
Now $\alpha(q) = \alpha(p)$ if and only if $q = p$.
Hence $\varphi_{1*}{}^{-}(q,\ x,\ p) = -1$ if and only if $\varphi_{2*}{}^{-}(\alpha(q),\ \beta(\lambda),\ \alpha(p)) = -1$
$\varphi_{1*}{}^{+}(q,\ x,\ p) = 1$ if and only if $\varphi_{2*}{}^{+}(\alpha(q),\ \beta(\lambda),\ \alpha(p)) = 1$(By Strong homomorphism).

Suppose the result is true $\forall\ y\ \in\ X_1^*,\ |y| = n-1, n > 0$.
Let $x = ya, |y| = n-1, y\ \in\ X_1^*,\ a\ \in\ X_1$. Then

$$\begin{aligned}
\varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(x),\ \alpha(p)) &= \varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(ya),\ \alpha(p)) \\
&= \varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(y)\beta(a),\ \alpha(p)) \\
&= \wedge_{r\in Q_1}\{\varphi_{2*}{}^{-}(\alpha(q),\ \beta^*(y),\ \alpha(r) \vee \varphi_2{}^{-}(\alpha(r),\ \beta(a),\ \alpha(p))\} \\
&= \wedge_{r\in Q_1}\{\varphi_{1*}{}^{-}(q,\ y,\ r) \vee \varphi_1{}^{-}(r,\ a,\ p)\} \\
&= \varphi_{1*}{}^{-}(q,\ ya,\ p) \\
&= \varphi_{1*}{}^{-}(q,\ x,\ p)
\end{aligned}$$

Now,

$$\begin{aligned}
\varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(x),\ \alpha(p)) &= \varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(ya),\ \alpha(p)) \\
&= \varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(y)\beta(a),\ \alpha(p)) \\
&= \vee_{r\in Q_1}\{\varphi_{2*}{}^{+}(\alpha(q),\ \beta^*(y),\ \alpha(r) \wedge \varphi_2{}^{+}(\alpha(r),\ \beta(a),\ \alpha(p))\} \\
&= \vee_{r\in Q_1}\{\varphi_{1*}{}^{+}(q,\ y,\ r) \wedge \varphi_1{}^{+}(r,\ a,\ p)\} \\
&= \varphi_{1*}{}^{+}(q,\ ya,\ p) \\
&= \varphi_{1*}{}^{+}(q,\ x,\ p).
\end{aligned}$$

Conversly,

Let $q, p\ \in\ Q_1$ and let$\alpha(q) = \alpha(p)$. Then
$\varphi_{2*}{}^{-}(\alpha(q),\ \lambda,\ \alpha(p)) = \varphi_{1*}{}^{-}(q, \lambda, p)$ Hence $q = p$ and
$\varphi_{2*}{}^{+}(\alpha(q),\ \lambda,\ \alpha(p)) = \varphi_{1*}{}^{+}(q, \lambda, p)$ Hence $q = p$. Hrnce α is one-one.

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