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More on Cartesian Products over Intuitionistic Fuzzy Sets

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Abstract. In this paper we introduce two versions of Cartesian products over Intuitionistic Fuzzy Sets and study their properties.

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1 Introduction

Atanassov [1] has defined five versions of cartesian products of two Intuitionistic Fuzzy Sets (IFSs) "$\times_1$", "$\times_2$", "$\times_3$", "$\times_4$", "$\times_5$". Also, Velin Andonov [2] introduced a sixth version "$\times_6$" of cartesian product of two IFSs. In an earlier paper by the authors, three different types of Cartesian products "$\times_7$", "$\times_8$" and "$\times_9$" are introduced. In this paper we introduce two more versions of cartesian products "$\times_{10}$" and "$\times_{11}$" of two IFSs.
Let E_1 and E_2 be two universes and let $\mathcal{A} = \{\langle x, \mu_{\mathcal{A}}(x), \nu_{\mathcal{A}}(x)\rangle | x \in E_1\}$ and $\mathcal{B} = \{\langle y, \mu_{\mathcal{B}}(y), \nu_{\mathcal{B}}(y)\rangle \; | y \in E_2\}$ be two IFSs: $\mathcal{A}$ over E_1 and $\mathcal{B}$ over E_2.

Definition 1.1. The five cartesian products of two IFSs $\mathcal{A}$ and $\mathcal{B}$ are defined

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as follows:

$$\mathcal{A} \times_7 \mathcal{B} = \{\langle\langle x, y\rangle, \min(1, \mu_{\mathcal{A}}(x) + \mu_{\mathcal{B}}(y)), \max(0, \nu_{\mathcal{A}}(x) + \nu_{\mathcal{B}}(y) - 1)\rangle \\ |x \in E_1, y \in E_2\}$$

$$\mathcal{A} \times_8 \mathcal{B} = \{\langle\langle x, y\rangle, \max(0, \mu_{\mathcal{A}}(x) + \mu_{\mathcal{B}}(y) - 1), \min(1, \nu_{\mathcal{A}}(x) + \nu_{\mathcal{B}}(y))\rangle \\ |x \in E_1, y \in E_2\}$$

$$\mathcal{A} \times_9 \mathcal{B} = \{\langle\langle x, y\rangle, \sqrt{\mu_{\mathcal{A}}(x)\mu_{\mathcal{B}}(y)}, \sqrt{\nu_{\mathcal{A}}(x)\nu_{\mathcal{B}}(y)}\rangle|x \in E_1, y \in E_2\}$$

$$\mathcal{A} \times_{10} \mathcal{B} = \{\langle\langle x, y\rangle, \frac{2\mu_{\mathcal{A}}(x)\mu_{\mathcal{B}}(y)}{\mu_{\mathcal{A}}(x) + \mu_{\mathcal{B}}(y)}, \frac{2\nu_{\mathcal{A}}(x)\nu_{\mathcal{B}}(y)}{\nu_{\mathcal{A}}(x) + \nu_{\mathcal{B}}(y)}\rangle|x \in E_1, y \in E_2\}$$

for which we will accept that
if $\mu_{\mathcal{A}}(x) = \mu_{\mathcal{B}}(y) = 0$, then $\frac{\mu_{\mathcal{A}}(x)\mu_{\mathcal{B}}(y)}{\mu_{\mathcal{A}}(x)+\mu_{\mathcal{B}}(y)} = 0$ and
if $\nu_{\mathcal{A}}(x) = \nu_{\mathcal{B}}(y) = 0$, then $\frac{\nu_{\mathcal{A}}(x)\nu_{\mathcal{B}}(y)}{\nu_{\mathcal{A}}(x)+\nu_{\mathcal{B}}(y)} = 0$.

$$\mathcal{A} \times_{11} \mathcal{B} = \{\langle\langle x, y\rangle, \frac{\mu_{\mathcal{A}}(x) + \mu_{\mathcal{B}}(y)}{2(\mu_{\mathcal{A}}(x)\mu_{\mathcal{B}}(y) + 1)}, \frac{\nu_{\mathcal{A}}(x) + \nu_{\mathcal{B}}(y)}{2(\nu_{\mathcal{A}}(x)\nu_{\mathcal{B}}(y) + 1)}\rangle|x \in E_1, y \in E_2\}$$

Result 1.1. *[4] $\mathcal{A} \times_7 \mathcal{B}$, $\mathcal{A} \times_8 \mathcal{B}$ and $\mathcal{A} \times_9 \mathcal{B}$ are IFS over $E_1 \times E_2$ where "$\times$" is the classical cartesian product on ordinary sets E_1 and E_2.*

Result 1.2. *$\mathcal{A} \times_{10} \mathcal{B}$ is an IFS over $E_1 \times E_2$.*

Proof. If $\mu_{\mathcal{A}}(x) + \mu_{\mathcal{B}}(y) > 0$ and $\nu_{\mathcal{A}}(x) + \nu_{\mathcal{B}}(y) > 0$ then

$$\begin{aligned} 0 &\leq \frac{2\mu_{\mathcal{A}}(x)\mu_{\mathcal{B}}(y)}{\mu_{\mathcal{A}}(x) + \mu_{\mathcal{B}}(y)} + \frac{2\nu_{\mathcal{A}}(x)\nu_{\mathcal{B}}(y)}{\nu_{\mathcal{A}}(x) + \nu_{\mathcal{B}}(y)} \\ &\leq \frac{\mu_{\mathcal{A}}(x) + \mu_{\mathcal{B}}(y)}{2} + \frac{\nu_{\mathcal{A}}(x) + \nu_{\mathcal{B}}(y)}{2} \\ &\leq 1 \end{aligned}$$

□

Result 1.3. *$\mathcal{A} \times_{11} \mathcal{B}$ is an IFS over $E_1 \times E_2$.*

Proof. For four real numbers $0 \leq a, b, c, d \leq 1$, $a + c \leq ac + 1$.
Then $\dfrac{a + c}{2(ac + 1)} \leq \dfrac{1}{2}$
(If $a+c > ac+1$, then $a+c(1-a) > 1$, a contradiction). Similarly $\frac{b+d}{2(bd+1)} \leq \frac{1}{2}$.

$$0 \leq \frac{a + c}{2(ac + 1)} + \frac{b + d}{2(bd + 1)} \leq 1.$$

From these inequations it follows that $A \times_{11} B$ is an IFS □

Definition 1.2. [1] If $\mathcal{A}$ is the IFS $\{\langle x, \mu_{\mathcal{A}}(x), \nu_{\mathcal{A}}(x)\rangle|x \in E\}$, then

$$\bar{\mathcal{A}} = \{\langle x, \nu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(x)\rangle|x \in E\}.$$

2 Properties

Proposition 2.1. *Let E_1 and E_2 be two universes. If $\mathcal{A}$ and $\mathcal{B}$ are IFSs over E_1 and C is an IFS over E_2, then the following equalities hold*

$$(\mathcal{A} \cup \mathcal{B}) \times \mathcal{C} = (\mathcal{A} \times \mathcal{C}) \cup (\mathcal{B} \times \mathcal{C}) \quad (1)$$
$$(\mathcal{A} \cap \mathcal{B}) \times \mathcal{C} = (\mathcal{A} \times \mathcal{C}) \cap (\mathcal{B} \times \mathcal{C}) \quad (2)$$
$$\mathcal{C} \times (\mathcal{A} \cup \mathcal{B}) = (\mathcal{C} \times \mathcal{A}) \cup (\mathcal{C} \times \mathcal{B}) \quad (3)$$
$$\mathcal{C} \times (\mathcal{A} \cap \mathcal{B}) = (\mathcal{C} \times \mathcal{A}) \cap (\mathcal{C} \times \mathcal{B}) \quad (4)$$

where $\times \in \{\times_9, \times_{10}\}$

Proof. We shall prove (1) for "$\times_9$" and (1) for "$\times_{10}$".

$$\mathcal{A} \cup \mathcal{B} = \{\langle x. \max(\mu_{\mathcal{A}}(x), \mu_{\mathcal{B}}(x)), \min(\nu_{\mathcal{A}}(x), \nu_{\mathcal{B}}(x))\rangle | x \in E_1\}$$
$$\mathcal{C} = \{\langle y, \mu_{\mathcal{C}}(y), \nu_{\mathcal{C}}(y)\rangle | y \in E_2\}$$

Proof (1) *for* $\times_9$. Let $\times$ be the cartesian product $\times_9$

$$(\mathcal{A} \cup \mathcal{B}) \times \mathcal{C} = \{\langle x, y\rangle, \sqrt{\max(\mu_{\mathcal{A}}(x), \mu_{\mathcal{B}}(x))\mu_{\mathcal{C}}(y)}, \sqrt{\min(\nu_{\mathcal{A}}(x), \nu_{\mathcal{B}}(x))\nu_{\mathcal{C}}(y)}\rangle | x \in E_1, y \in E_2\}$$

$$(\mathcal{A} \times \mathcal{C}) \cup (\mathcal{B} \times \mathcal{C}) = \{\langle\langle x, y\rangle, \max(\sqrt{\mu_{\mathcal{A}}(x), \mu_{\mathcal{C}}(y)}, \sqrt{\mu_{\mathcal{B}}(x), \mu_{\mathcal{C}}(y)}), \min(\sqrt{\nu_{\mathcal{A}}(x), \nu_{\mathcal{C}}(y)}, \sqrt{\nu_{\mathcal{B}}(x), \nu_{\mathcal{C}}(y)})\rangle | x \in E_1,\ y \in E_2\}.$$

Assume that $\mu_{\mathcal{A}}(x) > \mu_{\mathcal{B}}(x)$

$$\mu_{\mathcal{A}}(x)\mu_{\mathcal{C}}(y) > \mu_{\mathcal{B}}(x)\mu_{\mathcal{C}}(y)$$
$$\sqrt{\mu_{\mathcal{A}}(x)\mu_{\mathcal{C}}(y)} > \sqrt{\mu_{\mathcal{B}}(x)\mu_{\mathcal{C}}(y)}$$
$$\mu_{(\mathcal{A}\cup\mathcal{B})\times\mathcal{C}}(x, y) = \sqrt{\mu_{\mathcal{A}}(x)\mu_{\mathcal{C}}(y)} = \mu_{(\mathcal{A}\times\mathcal{C})\cup(\mathcal{B}\times\mathcal{C})}(x, y).$$

Similarly we can prove that

$$\nu_{(\mathcal{A}\cup\mathcal{B})\times\mathcal{C}}(x, y) = \nu_{(\mathcal{A}\times\mathcal{C})\cup(\mathcal{B}\times\mathcal{C})}.$$

The proof of (2) for $\times_9$ is similar.
Proof (1) *for* $\times_{10}$. Let $\times$ be the cartesian product $\times_{10}$.
Assume $\mu_A(x) > \mu_B(x)$.

$$\mu_{(\mathcal{A}\cup\mathcal{B})\times\mathcal{C}}(x, y) = \frac{2\max(\mu_{\mathcal{A}}(x), \mu_{\mathcal{B}}(x))\mu_{\mathcal{C}}(y)}{\max(\mu_{\mathcal{A}}(x), \mu_{\mathcal{B}}(x) + \mu_{\mathcal{C}}(y)} = \frac{2\mu_{\mathcal{A}}(x)\mu_{\mathcal{C}}(y)}{\mu_{\mathcal{A}}(x) + \mu_{\mathcal{C}}(y)}$$

$$\mu_{(\mathcal{A}\times\mathcal{C})}(x,y) = \frac{2\mu_{\mathcal{A}}(x)\mu_{\mathcal{C}}(y)}{\mu_{\mathcal{A}}(x)+\mu_{\mathcal{C}}(y)}$$
$$\mu_{(\mathcal{B}\times\mathcal{C})}(x,y) = \frac{2\mu_{\mathcal{B}}(x)\mu_{\mathcal{C}}(y)}{\mu_{\mathcal{B}}(x)+\mu_{\mathcal{C}}(y)}$$
$$\mu_{(\mathcal{A}\times\mathcal{C})\cup(\mathcal{B}\times\mathcal{C})}(x,y) = \max\left(\frac{2\mu_{\mathcal{A}}(x)\mu_{\mathcal{C}}(y)}{\mu_{\mathcal{A}}(x)+\mu_{\mathcal{C}}(y)}, \frac{2\mu_{\mathcal{B}}(x)\mu_{\mathcal{C}}(y)}{\mu_{\mathcal{B}}(x)+\mu_{\mathcal{C}}(y)}\right)$$
$$= \frac{2\mu_{\mathcal{A}}(x)\mu_{\mathcal{C}}(y)}{\mu_{\mathcal{A}}(x)+\mu_{\mathcal{C}}(y)} \quad \text{since if } \mu_{\mathcal{A}}(x) > \mu_{\mathcal{B}}(x),$$
$$\frac{2}{\frac{1}{\mu_{\mathcal{A}}(x)}+\frac{1}{\mu_{\mathcal{C}}(y)}} > \frac{2}{\frac{1}{\mu_{\mathcal{B}}(x)}+\frac{1}{\mu_{\mathcal{C}}(y)}}$$

Similar is the case for non-membership grade also. The proof of (2) for $\times_{10}$ is similar.

(3) and (4) are direct corollaries of (1) and (2) □

Remark 2.1. $\mu_{\mathcal{A}\times\mathcal{C}}(x,y) = \mu_{\mathcal{C}\times\mathcal{A}}(y,x)$ and $\nu_{\mathcal{A}\times\mathcal{C}}(x,y) = \nu_{\mathcal{C}\times\mathcal{A}}(y,x)$ where $\times$ is any of $\times_7$, $\times_8$, $\times_9$, $\times_{10}$, $\times_{11}$.

Proof.

$$\mu_{\mathcal{C}\times_{11}\mathcal{A}}(y,x) = \frac{\mu_{\mathcal{C}}(y)+\mu_{\mathcal{A}}(x)}{2(\mu_{\mathcal{C}}(y)\mu_{\mathcal{A}}(x)+1)} = \mu_{\mathcal{A}\times_{11}\mathcal{C}}(x,y)$$

Similar is the case for non membership grade also. Similar result holds for $\times_7, \times_8$, $\times_9$ and $\times_{10}$. □

Proposition 2.2. *[4] If $\mathcal{A}$ is an IFS over E_1 and $\mathcal{B}$ is an IFS over E_2, then*

$$\overline{\bar{\mathcal{A}} \times_7 \bar{\mathcal{B}}} = \mathcal{A} \times_8 \mathcal{B}$$
$$\overline{\bar{\mathcal{A}} \times_8 \bar{\mathcal{B}}} = \mathcal{A} \times_7 \mathcal{B}$$
$$\overline{\bar{\mathcal{A}} \times_9 \bar{\mathcal{B}}} = \mathcal{A} \times_9 \mathcal{B}$$

Proposition 2.3. *Formulas similar to DeMorgan's law hold for the cartesian products $\times_{10}$ and $\times_{11}$. If $\mathcal{A}$ is an IFS over E_1 and $\mathcal{B}$ is an IFS over E_2, then*

$$\overline{\bar{\mathcal{A}} \times_{10} \bar{\mathcal{B}}} = \mathcal{A} \times_{10} \mathcal{B}$$
$$\overline{\bar{\mathcal{A}} \times_{11} \bar{\mathcal{B}}} = \mathcal{A} \times_{11} \mathcal{B}$$

Therefore, the cartesian products $\times_{10}$ and $\times_{11}$ are autodual.

Proof.

$$\bar{\mathcal{A}} = \{\langle x, \nu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(x)\rangle | x \in E_1\}$$
$$\bar{\mathcal{B}} = \{\langle y, \nu_{\mathcal{B}}(y), \mu_{\mathcal{B}}(y)\rangle | y \in E_2\}$$

$$\overline{\bar{\mathcal{A}} \times_{10} \bar{\mathcal{B}}} = \{\langle\langle x, y\rangle, \frac{2\mu_{\mathcal{A}}(x)\mu_{\mathcal{B}}(y)}{\mu_{\mathcal{A}}(x) + \mu_{\mathcal{B}}(y)}, \frac{2\nu_{\mathcal{A}}(x)\nu_{\mathcal{B}}(y)}{\nu_{\mathcal{A}}(x) + \nu_{\mathcal{B}}(y)}\rangle$$
$$|x \in E_1, y \in E_2\}$$
$$= \mathcal{A} \times_{10} \mathcal{B}.$$

Similarly the other result can be proved. □

3 Conclusion

In this paper we have introduced two versions of cartesian products of two IFSs and we have proved some of their properties. We have proved some properties of other versions of cartesian products also.

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