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Conformal Change of Some Special Finsler Spaces with Constant Unified Main Scalar

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Abstract

In this paper we discuss the theory of conformal change of some special Finsler spaces namely C-reducible, semi C-reducible and C2-like five dimensional Finsler spaces with constant unified main scalar. We have obtained the values of main scalars and v-scalar curvature S of all the above spaces with constant unified main scalar under the assumptions that the h-connection vectors h_i, J_i, k_i and k'_i are conformally invariant.

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1 Introduction

In 1957, A. Moor introduced the concept of conformal change and conformal transformation of n-dimensional Finsler spaces [6]. Since then alot of work has been on this topic ([1], [3], [7]). The theory of five dimensional Finsler spaces was introduced in [8]. In a five-dimensional Finsler space there are seventeen main scalars $H, I, J, K, H', I', J', K', H'', I'', J'', K'', M, M', M'', N, N'$ in which the sum of H, I, K and M is LC [8] which is called unified main scalar.In a five-dimensional Finsler space there exist six v-connection vectors $u_i, v_i, w_i, u'_i, v'_i, w'_i$ and six h-connection vectors $h_i, J_i, k_i, h'_i, J'_i, k'_i$ [8] .The orthonormal frame field $(l^i, m^i, n^i, p^i, q^i)$, called the Miron frame plays an important role in five-dimensional Finsler space.

2 Scalar components in Miron frame

Let F^5 be a five-dimensional Finsler space with fundamental function $L(x, y)$. The metric tensor g_{ij} and Cartan $C-$tensor C_{ijk} of F^5 are defined by

$$g_{ij} = \frac{1}{2}\dot{\partial}_i\dot{\partial}_j L^2, \qquad C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij} = \frac{1}{4}\dot{\partial}_i\dot{\partial}_j\dot{\partial}_k L^2.$$

Throughout the paper, the symbols $\dot{\partial}_i = \frac{\partial}{\partial y^i}$ and $\partial_i = \frac{\partial}{\partial x^i}$ have been used. The frame $\left\{e^i_{(\alpha)}\right\}, \alpha = 1, 2, 3, 4, 5$ is called the Miron frame of F^5, where $e^i_{(1)} = l^i = \frac{y^i}{L}$ is called the normalized supporting element, $e^i_{(2)} = m^i = \frac{C^i}{C}$ is called the normalized torsion vector, $e^i_{(3)} = n^i, e^i_{(4)} = p^i, e^i_{(5)} = q^i$ are constructed from $g_{ij}e^i_{(\alpha)}e^j_{(\beta)} = \delta_{\alpha\beta}$.Here, C is the length of torsion vector $C_i = C_{ijk}g^{jk}$.The Greek letters $\alpha, \beta, \gamma, \delta$ vary from 1 to 5 throughout the paper. Summation convention is applied for both the Greek and Latin indices. We use the following results which have been obtained in [8] :

$$C_{222} = H, C_{233} = I, C_{244} = K, C_{333} = J, C_{344} = J', C_{444} = H', \tag{2.1}$$

$$C_{334} = I', C_{234} = K', C_{255} = M, C_{355} = J'', C_{455} = M', C_{555} = H'',$$

$$C_{335} = I'', C_{445} = K'', C_{235} = N, C_{245} = N', C_{345} = M'',$$

then, we have

$$H + I + K + M = LC, \qquad C_{223} = -\left(J + J' + J''\right), \tag{2.2}$$

$$C_{224} = -\left(H' + I' + M'\right), \qquad C_{225} = -\left(H'' + I'' + K''\right).$$

In terms of main scalars, the v-scalar curvature in a five dimensional Finsler space is

$$S = (K^2 + I^2 + M^2 - HI - HK - KI - HM - MI - MK) + 2\left(J^2 + H'^2 + H''^2\right) + 3\begin{pmatrix} I'^2 + J'^2 + M'^2 + K'^2 + N^2 + N'^2 + J''^2 \\ +I''^2 + K''^2 + M''^2 + JJ' + JJ'' + J'J'' + H'I' + M'I' + H'M' \\ I''K'' + K''H'' + I''K'' \end{pmatrix} \tag{2.3}$$

3 Conformal Change of Cartan's connection

We consider a conformal change $L(x,y) \longrightarrow \overline{L}(x,y) = e^{\sigma(x)}L(x,y)$ of a five-dimensional Finsler space $F^5 = (M^5, L(x, y))$ with the fundamental function L(x, y), where $\sigma(x)$ is a scalar function of position x^i alone, called the conformal factor. We shall denote the Finsler space with changed fundamental function $\overline{L}(x,y)$ by $\overline{F}^5 = (M^5, \overline{L}(x, y))$) and quantities of $\overline{F}^5$ by upper line. The following change of important quantities are known ([2]).

$$\overline{l}_i = e^{\sigma}l_i, \overline{m}_i = e^{\sigma}m_i, \overline{n}_i = e^{\sigma}n_i, \overline{p}_i = e^{\sigma}p_i, \overline{q}_i = e^{\sigma}q_i, \overline{g}_{ij} = e^{2\sigma}g_{ij}, \tag{3.1}$$

$$\overline{l}^i = e^{-\sigma}l^i, \overline{m}^i = e^{-\sigma}m^i, \overline{n}^i = e^{-\sigma}n^i, \overline{p}^i = e^{-\sigma}p^i, \overline{q}^i = e^{-\sigma}q^i, \tag{3.2}$$

$$\overline{g}^{ij} = e^{-2\sigma}g^{ij},$$

$$\overline{C}_{ijk} = e^{2\sigma}C_{ijk}, \overline{C}^i_{jk} = C^i_{jk}, \overline{H} = H, \overline{I} = I, \overline{J} = J, \overline{K} = K, \tag{3.3}$$

$$\overline{M} = M, \overline{N} = N, \overline{H}' = H', \overline{I}' = I', \overline{J}' = J', \overline{K}' = K', \overline{M}' = M',$$
$$\overline{N}' = N', \overline{H}'' = H'', \overline{I}'' = I'', \overline{J}'' = J'', \overline{K}'' = K'', \overline{M}'' = M''.$$

We use the following results [9] :

3.1 Theorem

The h-connection vector h_i of F^5 is invariant under σ-conformal change if and only if

(*i*) $\sigma_2 u_2 + \sigma_3 u_3 + \sigma_4 u_4 + \sigma_5 u_5 = 0,$
(*ii*) $\sigma_6 u_2 + \sigma_7 u_3 + \sigma_8 u_4 + \sigma_9 u_5 + \sigma_{17} + \sigma_{56} = 0,$
(*iii*) $\sigma_7 u_2 + \sigma_{10} u_3 + \sigma_{11} u_4 + \sigma_{12} u_5 + \sigma_{20} + \sigma_{57} = 0,$
(*iv*) $\sigma_8 u_2 + \sigma_{11} u_3 + \sigma_{13} u_4 + \sigma_{14} u_5 + \sigma_{21} + \sigma_{58} = 0,$
(*v*) $\sigma_9 u_2 + \sigma_{12} u_3 + \sigma_{14} u_4 + \sigma_{15} u_5 + \sigma_{22} + \sigma_{59} = 0.$

3.2 Theorem

The h-connection vector J_i of F^5 is invariant under σ-conformal change if and only if

(*i*) $\sigma_2 v_2 + \sigma_3 v_3 + \sigma_4 v_4 + \sigma_5 v_5 = 0,$
(*ii*) $\sigma_6 v_2 + \sigma_7 v_3 + \sigma_8 v_4 + \sigma_9 v_5 + \sigma_{18} + \sigma_{60} = 0,$
(*iii*) $\sigma_7 v_2 + \sigma_{10} v_3 + \sigma_{11} v_4 + \sigma_{12} v_5 + \sigma_{21} + \sigma_{61} = 0,$
(*iv*) $\sigma_8 v_2 + \sigma_{11} v_3 + \sigma_{13} v_4 + \sigma_{14} v_5 + \sigma_{23} + \sigma_{62} = 0,$
(*v*) $\sigma_9 v_2 + \sigma_{12} v_3 + \sigma_{14} v_4 + \sigma_{15} v_5 + \sigma_{24} + \sigma_{63} = 0.$

3.3 Theorem

The h-connection vector k_i of F^5 is invariant under σ-conformal change if and only if

(i) $\sigma_2 w_2 + \sigma_3 w_3 + \sigma_4 w_4 + \sigma_5 w_5 = 0,$
(ii) $\sigma_6 w_2 + \sigma_7 w_3 + \sigma_8 w_4 + \sigma_9 w_5 + \sigma_{19} + \sigma_{64} = 0,$
(iii) $\sigma_7 w_2 + \sigma_{10} w_3 + \sigma_{11} w_4 + \sigma_{12} w_5 + \sigma_{22} + \sigma_{65} = 0,$
(iv) $\sigma_8 w_2 + \sigma_{11} w_3 + \sigma_{13} w_4 + \sigma_{14} w_5 + \sigma_{24} + \sigma_{66} = 0,$
(v) $\sigma_9 w_2 + \sigma_{12} w_3 + \sigma_{14} w_4 + \sigma_{15} w_5 + \sigma_{25} + \sigma_{67} = 0.$

3.4 Theorem

The h-connection vector k'_i of F^5 is invariant under σ-conformal change if and only if

(i) $\sigma_2 w'_2 + \sigma_3 w'_3 + \sigma_4 w'_4 + \sigma_5 w'_5 = 0,$
(ii) $\sigma_6 w'_2 + \sigma_7 w'_3 + \sigma_8 w'_4 + \sigma_9 w'_5 + \sigma_{39} + \sigma_{76} = 0,$
(iii) $\sigma_7 w'_2 + \sigma_{10} w'_3 + \sigma_{11} w'_4 + \sigma_{12} w'_5 + \sigma_{42} + \sigma_{77} = 0,$
(iv) $\sigma_8 w'_2 + \sigma_{11} w'_3 + \sigma_{13} w'_4 + \sigma_{14} w'_5 + \sigma_{44} + \sigma_{78} = 0,$
(v) $\sigma_9 w'_2 + \sigma_{12} w'_3 + \sigma_{14} w'_4 + \sigma_{15} w'_5 + \sigma_{45} + \sigma_{79} = 0.$

4 Conformal change of some special Finsler Spaces

4.1 Conformal Change of C-reducible Finsler Space

4.1.1 Definition

A Finsler space of dimension n(≥ 2) , is called C-reducible if C_{ijk} is written in the form [4]

$$C_{ijk} := \frac{1}{n+1}\left(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j\right),$$

where $C_i = C_{ijk}g^{jk}$ is the torsion vector and h_{ij} is the angular metric tensor given by $h_{ij} = g_{ij} - l_i l_j$.

In a five- dimensional C-reducible Finsler space C_{ijk} takes the value

$$C_{ijk} := \frac{1}{6}\left(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j\right),$$

where $C_i = C_{ijk}g^{jk} = Cm_i = C\delta_{2\alpha}e_{\alpha)i} = C_{\alpha\beta\beta}e_{\alpha)i}$ and $h_{ij} = \left(\delta_{\beta\gamma} - \delta_{1\beta}\delta_{1\gamma}\right)e_{\beta)j}$ $e_{\gamma)k}$.

In terms of scalar components C_{ijk} can be written as

$$C_{\alpha\beta\gamma} = \frac{1}{6}LC\left\{\delta_{2\alpha}\left(\delta_{\beta\gamma} - \delta_{1\beta}\delta_{1\gamma}\right) + \delta_{2\beta}\left(\delta_{\gamma\alpha} - \delta_{1\gamma}\delta_{1\alpha}\right) + \delta_{2\gamma}\left(\delta_{\alpha\beta} - \delta_{1\alpha}\delta_{1\beta}\right)\right\}, \tag{4.1.1}$$

where $\alpha, \beta, \gamma = 1, ..., 5$.

From (2.1) and (4.1.1) , we get

(4.1.2) $H = \frac{1}{2}LC, I = K = M = \frac{1}{6}LC$ and $H' = H'' = I' = I'' = K' = K'' = M' = M'' = J = J' = J'' = N = N' = 0.$

Hence we have the following:

4.1.2 Theorem

In a five-dimensional C-reducible Finsler space, the only non-zero main scalars are H, I, K and M, given by (4.1.2) *and the remaining main scalars vanish identically.*

Using theorem (4.1.2) in the equation (2.17) of [8] , we get

(4.1.3) $u_i = v_i = w_i = w'_i = 0, i = 1, ..., 5.$

Hence we have the following:

4.1.3 Theorem

In a five-dimensional C-reducible Finsler space with non-zero constant unified main scalar, the four v-connection vectors u_i, v_i, w_i and w'_i vanish identically.

Using the theorems (4.1.2) and (4.1.3), we see that (i) , (iv) and (v) of the theorem (3.1) , (i) , (iii) and (v) of the theorem (3.2) , and (i) , (iii) and (iv) of the theorem (3.3) are satisfied identically and the remaining parts of the theorems (3.1) , (3.2) and (3.3) reduce to

(4.1.4) $18 - (LC)^2 = 0$, provided $\sigma_2 \neq 0, \sigma_3 \neq 0, \sigma_4 \neq 0, \sigma_5 \neq 0.$

Hence we have the following:

4.1.4 Theorem

In a five-dimensional C-reducible Finsler space with constant unified main scalar, the h-connection vectors h_i, J_i and k_i are invariant under $\sigma-$ conformal change if and only if $LC = 3\sqrt{2}$, provided $\sigma_2 \neq 0$, $\sigma_3 \neq 0$, $\sigma_4 \neq 0, \sigma_5 \neq 0$.

4.1.5 Corollary

The main scalars in this case, are given by $H = \frac{3}{\sqrt{2}}$, $I = K = M = \frac{1}{\sqrt{2}}$ and $J = J' = J'' = H' = H'' = I' = I'' = K' = K'' = M' = M'' = N = N' = 0$.

Again, using the theorems (4.1.2) and (4.1.3), we see that (i) , (ii) and (iii) of the theorem (3.4) are satisfied identically and its remaining parts reduce to

(4.1.5) $36 - (LC)^2 = 0$, provided $\sigma_4 \neq 0, \sigma_5 \neq 0.$

Hence we have the following:

4.1.6 Theorem

In a five-dimensional C-reducible Finsler space with constant unified main scalar, the h-connection vectors k'_i is invariant under $\sigma-$ conformal change if and only if $LC = 6$, provided $\sigma_4 \neq 0, \sigma_5 \neq 0$.

4.1.7 Corollary

The main scalars in this case are given by $H = 3$, $I = K = M = 1$ and $J = J' = J'' = H' = H'' = I' = I'' = K' = K'' = M' = M'' = N = N' = 0$.

Since a C-reducible Finsler space is either a Randers space or a Kropina space ([7]) whose metric functions are respectively given by

$L(x, y) = \sqrt{a_{ij}(x) y^i y^j} + +b_i(x) y^i$ (Randers metric) and $L(x, y) = \frac{a_{ij}(x)y^iy^j}{b_i(x)y^i}$ (Kropina metric), therefore the theorems (4.1.4) and (4.1.6) may be restated as

4.1.8 Theorem

In a five-dimensional Randers space or Kropina space with constant unified main scalar, the h-connection vectors h_i, J_i and k_i are invariant under $\sigma-$conformal change if and only if $LC = 3\sqrt{2}$, provided $\sigma_2 \neq 0$, $\sigma_3 \neq 0$, $\sigma_4 \neq 0, \sigma_5 \neq 0$.

4.1.9 Theorem

In a five-dimensional Randers space or Kropina space with constant unified main scalar, the h-connection vectors k'_i is invariant under $\sigma-$ conformal change if and only if $LC = 6$, provided $\sigma_4 \neq 0, \sigma_5 \neq 0$.

Using the theorem (4.1.2) in the equation (2.3) , we get

$S = -\frac{1}{4}(LC)^2$ which is constant.

Hence we have the following:

4.1.10 Theorem

In a five-dimensional C-reducible Finsler space with constant unified main scalar, the v-scalar curvature S is constant.

4.2 Conformal Change of semi-C-reducible Finsler space

Next we consider a five-dimensional semi-C-reducible Finsler space with constant coefficients p and q and with constant unified main scalar.

4.2.1 Definition

A Finsler space of dimension n(≥ 2) , is called semi-C-reducible if C_{ijk} is written in the form [5]

$$C_{ijk} := \frac{p}{n+1}\left(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j\right) + \frac{q}{C^2}C_iC_jC_k,$$

where $p+q=1$, $C_i = C_{ijk}g^{jk}$ is the torsion vector and h_{ij} is the angular metric tensor given by $h_{ij} = g_{ij} - l_il_j$.

In a five- dimensional semi-C-reducible Finsler space C_{ijk} takes the value

$$C_{ijk} := \frac{p}{6}\left(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j\right) + \frac{q}{C^2}C_iC_jC_k,$$

where $C_i = C_{ijk}g^{jk} = Cm_i = C\delta_{2\alpha}e_{\alpha)i} = C_{\alpha\beta\beta}e_{\alpha)i}$ and $h_{ij} = \left(\delta_{\beta\gamma} - \delta_{1\beta}\delta_{1\gamma}\right)e_{\beta)j}$ $e_{\gamma)k}$.

In terms of scalar components C_{ijk} can be written as

(4.2.1) $$C_{\alpha\beta\gamma} = \tfrac{1}{6}pLC\left\{\delta_{2\alpha}\left(\delta_{\beta\gamma} - \delta_{1\beta}\delta_{1\gamma}\right) + \delta_{2\beta}\left(\delta_{\gamma\alpha} - \delta_{1\gamma}\delta_{1\alpha}\right) + \delta_{2\gamma}\left(\delta_{\alpha\beta} - \delta_{1\alpha}\delta_{1\beta}\right)\right\}$$
$$+qLC\delta_{2\alpha}\delta_{2\beta}\delta_{2\gamma},$$

where $\alpha, \beta, \gamma = 1, ..., 5$.

From (2.1) and (4.2.1) , we get

(4.2.2) $$H = \tfrac{1}{2}\left(p+2q\right)LC, I = K = M = \tfrac{1}{6}pLC \text{ and } H' = H'' = I' = I'' = K' = K'' = M' = M'' = J = J' = J'' = N = N' = 0.$$

Hence we have the following:

4.2.2 Theorem

In a five-dimensional semi-C-reducible Finsler space with constant coefficients, the only non-zero main scalars are H, I, K and M, given by (4.2.2) *and the remaining main scalars vanish identically.*

Using theorem (4.2.2) in the equation (2.3) , we get (4.1.3) .Hence we have the following:

4.2.3 Theorem

In a five-dimensional semi-C-reducible Finsler space with constant coefficients and non-zero constant unified main scalar, the four v-connection vectors u_i, v_i, w_i and w'_i vanish identically.

Using the theorems (4.2.2) and (4.2.3), we see that (i) , (iv) and (v) of the theorem (3.1) , (i) , (iii) and (v) of the theorem (3.2) , and (i) , (iii) and (iv) of the theorem (3.3) are satisfied identically and the remaining parts of the theorems (3.1) , (3.2) and (3.3) reduce to

(4.2.3) $$18 - p\left(2p+3q\right)\left(LC\right)^2 = 0, \text{ provided } \sigma_2 \neq 0, \sigma_3 \neq 0, \sigma_4 \neq 0, \sigma_5 \neq 0.$$

Since p+q=1, we have the following:

4.2.4 Theorem

In a five-dimensional semi-C-reducible Finsler space with constant coefficients and constant unified main scalar, the h-connection vectors h_i, J_i and k_i are invariant under $\sigma-$ conformal change if and only if $LC = \frac{3\sqrt{2}}{\sqrt{\{p(3-p)\}}}$, provided $\sigma_2 \neq 0,\ \sigma_3 \neq 0,\ \sigma_4 \neq 0, \sigma_5 \neq 0$.

4.2.5 Corollary

The main scalars in this case are given by $H = \frac{3(2-p)}{\sqrt{\{2p(3-p)\}}}$, $I = K = M = \sqrt{\left\{\frac{p}{2(3-p)}\right\}}$ and $J = J' = J'' = H' = H'' = I' = I'' = K' = K'' = M' = M'' = N = N' = 0$.

Again, using the theorems (4.2.2) and (4.2.3), we see that $(i)\,,(ii)$ and (iii) of the theorem (3.4) are satisfied identically and its remaining parts reduce to

$$36 - (pLC)^2 = 0, \text{ provided } \sigma_4 \neq 0, \sigma_5 \neq 0. \tag{4.2.4}$$

Hence we have the following:

4.2.6 Theorem

In a five-dimensional semi-C-reducible Finsler space with constant coefficients and constant unified main scalar, the h-connection vector k'$_i$ is invariant under $\sigma-$ conformal change if and only if $LC = \frac{6}{p}$, provided $\sigma_4 \neq 0, \sigma_5 \neq 0$.

4.2.7 Corollary

The main scalars in this case are given by $H = \frac{3(p+2q)}{p} = \frac{3(2-p)}{p}$, $I = K = M = 1$ and $J = J' = J'' = H' = H'' = I' = I'' = K' = K'' = M' = M'' = N = N' = 0$.

Using the theorem (4.2.2) in the equation (2.3), we get

$$S = -\tfrac{1}{4}p\,(p+2q)\,(LC)^2 = -\tfrac{1}{4}p\,(2-p)\,(LC)^2, \text{ which is constant.}$$

Hence we have the following:

4.2.8 Theorem

In a five-dimensional semi-C-reducible Finsler space with constant coefficients and constant unified main scalar, the v-scalar curvature S is constant.

4.3 Conformal Change of C2-like Finsler space

Next we consider a five-dimensional C2-like Finsler space with constant unified main scalar.

4.3.1 Definition

A Finsler space of dimension n, is called C2-like if C_{ijk} is written in the form ([2] , [5])

$$C_{ijk} := \frac{1}{C^2} C_i C_j C_k,$$

where $C_i = C_{ijk} g^{jk}$ is the torsion vector.

In terms of scalar components C_{ijk} can be written as

$$C_{\alpha\beta\gamma} = LC\delta_{2\alpha}\delta_{2\beta}\delta_{2\gamma}, \tag{4.3.1}$$

where $\alpha, \beta, \gamma = 1, ..., 5$.

From (2.1) and (4.3.1) , we get

$$H = LC, I = K = M = H' = H'' = I' = I'' = K' = K'' = M' = M'' = J = J' = J'' = N = N' = 0. \tag{4.3.2}$$

Hence we have the following:

4.3.2 Theorem

In a five-dimensional C2-like Finsler space the only non-zero main scalar is H and the remaining main scalars vanish identically,given by (4.3.2).

Using theorem (4.3.2) in the equation (2.17) of [8] , we get

$$u_i = v_i = w_i = 0, i = 1, ..., 5. \tag{4.3.3}$$

Hence we have the following:

4.3.3 Theorem

In a five-dimensional C2-like Finsler space with non-zero constant unified main scalar, the three v-connection vectors u_i, v_i, w_i vanish identically.

Using the theorems (4.3.2) and (4.3.3), we see that (i) , (iv) and (v) of the theorem (3.1) , (i) , (iii) and (v) of the theorem (3.2) , and (i) , (iii) and (iv) of the theorem (3.3) are satisfied identically and the remaining parts of the theorems (3.1) , (3.2) and (3.3) give

$$\sigma_2 = 0, \sigma_3 = 0, \sigma_4 = 0, \sigma_5 = 0. \tag{4.3.4}$$

Hence we have the following:

4.3.4 Theorem

In a five-dimensional C2-like Finsler space with constant unified main scalar, the h-connection vectors h_i,J_i and k_i are invariant under $\sigma-$ conformal change if and only if $\sigma_2 = 0$, $\sigma_3 = 0$, $\sigma_4 = 0, \sigma_5 = 0$.

Using the theorem (4.3.2) in the equation (2.3) , we get

$$S = 0.$$

Hence we have the following:

4.3.5 Theorem

In a five-dimensional C2-like Finsler space with constant unified main scalar, the v-scalar curvature S vanishes identically.

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