On Fuzzy Magnified Translation in Ternary Hemirings

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Abstract

In this paper we introduce the concept of fuzzy magnified translation of a fuzzy bi-ideal in a ternary hemiring and study some properties of it.

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1 Introduction

The concept of a fuzzy subset of a set was first introduced by L.A. Zadeh [8]. Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situation by attributing a degree to which a certain object belongs to a set. Using this concept Rosenfeld [7] established some results in fuzzy group theory. Later Kuroki [1] initiated the notion of fuzzy ideals in semigroups and Liu [4] studied them in rings. Lajos and Szasz [3] introduced the idea of bi-ideals in a ring. Kandasamy [2] and Majumder and Sarder [5] respectively explored on the idea of fuzzy translation and fuzzy magnified translation in fuzzy group theory. In this paper we introduce the concept of fuzzy magnified translation of a fuzzy bi-ideal in a ternary hemiring and study some properties of it.

2 Preliminary Notes

An algebra \((R; +, \cdot)\) is said to be a ternary semiring if \((R; +)\) is a semigroup and \((R; \cdot)\) is a ternary semigroups satisfying \(a(s + c) = asc + as\) and \((b + c)s = bsa + csa\) for all \(a, b, c, s \in R\). A ternary semiring \(R\) is said to be additively
commutative if \( a + b = b + a \) for all \( a, b \in R \). A ternary semiring \( R \) may have an identity \( 1 \), defined by \( 1sa = a1s = as1 = as \) and a zero \( 0 \), defined by \( 0 + a = a + 0 \) and \( as0 = 0sa = s0a = 0 \) for all \( a, s \in R \). A ternary semiring \( R \) is said to be a ternary hemiring if it is additively commutative with zero. In this paper we introduce the notion of fuzzy magnified translation of fuzzy bi-ideals in ternary hemirings and develop some results. Henceforth we denote a ternary hemiring by \( H \) unless otherwise stated.

We now review some definitions that are used in this paper.

**Definition 2.1** Let \( X \) be a non empty set. A fuzzy subset \( \mu \) of \( X \) is a function \( \mu : X \to [0, 1] \). Let \( \{ \mu_i \mid i \in I \} \) be fuzzy subsets of a ternary ring \( R \). The intersection of the fuzzy sets \( \mu_i \) is defined as follows:

\[
\cap \mu_i(x) = \inf_{i \in I}[\mu_i(x)], x \in X.
\]

**Definition 2.2** Let \( \mu \) be a fuzzy subset of a ternary hemiring \( H \). \( \text{Im} \mu \) is defined by

\[
\text{Im} \mu = \{ t \in [0, 1] \mid \mu(x) = t \text{ for some } x \in X \}.
\]

Let \( t \in [0, 1] \). The set \( \mu_t = \{ x \in X \mid \mu(x) \geq t \} \) is called a level subset of \( \mu \). Clearly, \( \mu_t \subseteq \mu_s \) whenever \( t \leq s \).

**Definition 2.3** A ternary subring \( S \) of a ternaryring \( R \) is called a bi-ideal of \( R \) if \( SRRRS \subseteq S \) holds where \( SRRRS \) is the additive subgroup of \( R \) generated by the set of all elements of the form \( sr_1rr_2s, s \in S \) and \( r, r_1, r_2 \in R \).

**Definition 2.4** A fuzzy subset \( \mu \) of a ternary hemiring \( H \) is called a fuzzy left(right) ideal of \( H \), if for every \( x, y, s \in H \):

(i) \( \mu \) is a fuzzy subgroup of \( (H, +) \), i.e., \( \mu(x + y) \geq \min{\mu(x), \mu(y)} \) and

(ii) \( \mu(xsy) \geq \mu(y)(\mu(xsy) \geq \mu(x)) \).

If \( \mu \) is both a fuzzy left ideal and a fuzzy right ideal of \( H \), then it is called a fuzzy ideal of \( H \).

**Definition 2.5** A non-empty fuzzy subset \( \mu \) of a ternary hemiring \( H \) (i.e. \( \mu(x) \neq 0 \) for some \( x \in H \)) is called a fuzzy bi-ideal of \( H \) if:

(i) \( \mu(x + y) \geq \min{\mu(x), \mu(y)} \),

(ii) \( \mu(xsy) \geq \min{\mu(x), \mu(s), \mu(y)} \) and

(iii) \( \mu(xstyz) \geq \min{\mu(x), \mu(z)} \) for all \( x, y, z, s, t \in H \).

Let \( \mu \) be a fuzzy subset of a set \( H \) and \( \alpha \in [0, 1 - \sup\{\mu(x) \mid x \in H\}] \). A mapping \( \mu^T_\alpha : H \to [0, 1] \) is called a fuzzy translation of \( \mu \) if \( \mu^T_\alpha(x) = \mu(x) + \alpha \) for all \( x \in H \).

Let \( \mu \) be a fuzzy subset of \( H \) and \( \beta \in [0, 1] \). A mapping \( \mu^M_\beta : H \to [0, 1] \) is called a fuzzy multiplication of \( \mu \) if \( \mu^M_\beta(x) = \beta \cdot \mu(x) \) for all \( x \in H \).

Let \( \mu \) be a fuzzy subset of \( H \), \( \alpha \in [0, 1 - \sup\{\mu(x) \mid x \in H\}] \) and \( \beta \in [0, 1] \). A mapping \( \mu^C_{\beta\alpha} : H \to [0, 1] \) is called a fuzzy magnified translation of \( \mu \) if \( \mu^C_{\beta\alpha}(x) = \beta \cdot \mu(x) + \alpha \) for all \( x \in H \).
3 Main Results

In this section we present the main results of our paper.

**Theorem 3.1** Let \( \mu \) be a non empty fuzzy subset of a ternary hemiring \( H \). Then \( \mu \) is a fuzzy bi-ideal of \( H \) if and only if the fuzzy magnified translation \( \mu_{\beta \alpha}^C \) of \( \mu \) is a fuzzy bi-ideal of \( H \).

**Proof.** Let \( \mu \) be a fuzzy bi-ideal of a ternary hemiring \( H \).

Now for all \( x, y, z, s, t \in H \), we have

\[
\mu_{\beta \alpha}^C(x + y) = \beta \cdot \mu(x + y) + \alpha \geq \beta \cdot \min\{\mu(x), \mu(y)\} + \alpha = \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(y) + \alpha\} = \min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(y)\}.
\]

Again

\[
\mu_{\beta \alpha}^C(xsy) = \beta \cdot \mu(xsy) + \alpha \geq \beta \cdot \min\{\mu(x), \mu(s), \mu(y)\} + \alpha = \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(s) + \alpha, \beta \cdot \mu(y) + \alpha\} = \min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(s), \mu_{\beta \alpha}^C(y)\}.
\]

Also

\[
\mu_{\beta \alpha}^C(xsyzt) = \beta \cdot \mu(xsyzt) + \alpha \geq \beta \cdot \min\{\mu(x), \mu(z)\} + \alpha = \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(z) + \alpha\} = \min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(z)\}.
\]

Thus \( \mu_{\beta \alpha}^C \) is a fuzzy bi-ideal of \( H \).

Conversely let \( \mu_{\beta \alpha}^C \) be a fuzzy bi-ideal of \( H \). Then for all \( x, y, z, s, t \in H \),

\[
\mu(x + y) = \frac{1}{\beta}[\mu_{\beta \alpha}^C(x + y) - \alpha] \geq \frac{1}{\beta}[\min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(y)\} - \alpha] = \frac{1}{\beta}[\min\{\mu_{\beta \alpha}^C(x) - \alpha, \mu_{\beta \alpha}^C(y) - \alpha\}] = \min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(y)\}.
\]

Again

\[
\mu(xsy) = \frac{1}{\beta}[\mu_{\beta \alpha}^C(xsy) - \alpha] \geq \frac{1}{\beta}[\min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(s), \mu_{\beta \alpha}^C(y)\} - \alpha] = \frac{1}{\beta}[\min\{\mu_{\beta \alpha}^C(x) - \alpha, \mu_{\beta \alpha}^C(s) - \alpha, \mu_{\beta \alpha}^C(y) - \alpha\}] = \min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(s), \mu_{\beta \alpha}^C(y)\}.
\]

Also

\[
\mu(xsyzt) = \frac{1}{\beta}[\mu_{\beta \alpha}^C(xsyzt) - \alpha] \geq \frac{1}{\beta}[\min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(z)\} - \alpha] = \frac{1}{\beta}[\min\{\mu_{\beta \alpha}^C(x) - \alpha, \mu_{\beta \alpha}^C(z) - \alpha\}] = \min\{\mu_{\beta \alpha}^C(x), \mu_{\beta \alpha}^C(z)\}.
\]

Hence \( \mu \) is a fuzzy bi-ideal of \( H \).
Theorem 3.2 If \( \mu \) is a fuzzy left (right, two-sided) ideal of a ternary hemiring \( H \) then the fuzzy magnified translation \( \mu^{C}_{\beta \alpha} \) of \( \mu \) is a fuzzy bi-ideal of \( H \).

Proof. Let \( \mu \) be a fuzzy left (right, two-sided) ideal of a ternary hemiring \( H \). Then for all \( x, y, z, s, t \in H \),
\[
\mu^{C}_{\beta \alpha}(x + y) = \beta \cdot \mu(x + y) + \alpha \\
\geq \beta \cdot \min\{\mu(x), \mu(y)\} + \alpha \\
= \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(y) + \alpha\} \\
= \min\{\mu^{C}_{\beta \alpha}(x), \mu^{C}_{\beta \alpha}(y)\}.
\]

Again
\[
\mu^{C}_{\beta \alpha}(xsy) = \beta \cdot \mu(xisy) + \alpha \\
\geq \beta \cdot \min\{\mu(x), \mu(s), \mu(y)\} + \alpha \\
= \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(s) + \alpha, \beta \cdot \mu(y) + \alpha\} \\
= \min\{\mu^{C}_{\beta \alpha}(x), \mu^{C}_{\beta \alpha}(y), \mu^{C}_{\beta \alpha}(z)\}.
\]

Also
\[
\mu^{C}_{\beta \alpha}(xsytz) = \beta \cdot \mu(xisyt) + \alpha \\
\geq \beta \cdot \mu(x) + \alpha \\
= \mu^{C}_{\beta \alpha}(z) \\
\geq \min\{\mu^{C}_{\beta \alpha}(x), \mu^{C}_{\beta \alpha}(y), \mu^{C}_{\beta \alpha}(z)\}.
\]
Thus \( \mu^{C}_{\beta \alpha} \) is a fuzzy bi-ideal of \( H \). Similarly we can prove the other statements.

Theorem 3.3 The fuzzy magnified translation of the intersection of an arbitrary collection of fuzzy bi-ideals of a ternary hemiring \( H \) is a fuzzy bi-ideal of \( H \) if it is not empty.

Proof. Let \( \mu_{i} \mid i \in I \) be an arbitrary collection of fuzzy bi-ideals of \( H \) and \( \mu = \bigcap_{i \in I} \mu_{i} \) be not empty.
Let \( x, y, z, s, t \in H \). Then
\[
\mu(x + y) = \bigcap_{i \in I} \mu_{i}(x + y) \\
= \inf_{i \in I}\{\mu_{i}(x + y)\} \\
\geq \inf_{i \in I}\{\min\{\mu_{i}(x), \mu_{i}(y)\}\} \\
= \min\{\inf_{i \in I}\{\mu_{i}(x)\}, \inf_{i \in I}\{\mu_{i}(y)\}\} \\
= \min\{\bigcap_{i \in I} \mu_{i}(x), \bigcap_{i \in I} \mu_{i}(y)\}.
\]

Again
\[
\mu(xsy) = \bigcap_{i \in I} \mu_{i}(xsy) \\
= \inf_{i \in I}\{\mu_{i}(xsy)\} \\
\geq \inf_{i \in I}\{\min\{\mu_{i}(x), \mu_{i}(s), \mu_{i}(y)\}\} \\
= \min\{\inf_{i \in I}\{\mu_{i}(x)\}, \inf_{i \in I}\{\mu_{i}(s)\}, \inf_{i \in I}\{\mu_{i}(y)\}\} \\
= \min\{\bigcap_{i \in I} \mu_{i}(x), \bigcap_{i \in I} \mu_{i}(s), \bigcap_{i \in I} \mu_{i}(y)\}.
\]
Also
\[
\mu(xsytz) = \bigcap_{i \in I} \mu_i(xsytz)
\]
\[
= \inf_{i \in I} \{\mu_i(xsytz)\}
\]
\[
\geq \inf_{i \in I} \{\min\{\mu_i(x), \mu_i(z)\}\}
\]
\[
= \min\{\inf_{i \in I} \{\mu_i(x)\}, \inf_{i \in I} \{\mu_i(z)\}\}
\]
\[
= \min\{\bigcap_{i \in I} \mu_i(x), \bigcap_{i \in I} \mu_i(z)\}.
\]

Hence \(\mu = \bigcap_{i \in I} \mu_i\) is a fuzzy bi-ideal of \(H\). Thus Theorem 3 follows from Theorem 1.

The proofs of the following two theorems are straight forward and therefore are omitted.

**Theorem 3.4** A subset \(A\) of a ternary hemiring \(H\) is a bi-ideal of \(H\) if and only if its characteristic function \(\chi_A\) is a fuzzy bi-ideal of \(H\).

**Theorem 3.5** Let \(\mu\) be a fuzzy subset of a ternary hemiring \(H\). \(\mu\) is a fuzzy bi-ideal of \(H\) if and only if its level sets \(ts\) are bi-ideals of \(H\) for all \(t \in \text{Im}\mu\).

**References**


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