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On Fuzzy Magnified Translation in Ternary Hemirings

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Abstract

In this paper we introduce the concept of fuzzy magnified translation of a fuzzy bi-ideal in a ternary hemiring and study some properties of it.

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1 Introduction

The concept of a fuzzy subset of a set was first introduced by L.A. Zadeh [8]. Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situation by attributing a degree to which a certain object belongs to a set. Using this concept Rosenfeld [7] established some results in fuzzy group theory. Later Kuroki [1] initiated the notion of fuzzy ideals in semigroups and Liu [4] studied them in rings. Lajos and Szasz [3] introduced the idea of bi-ideals in a ring. Kandasamy [2] and Majumder and Sarder [5] respectively explored on the idea of fuzzy translation and fuzzy magnified translation in fuzzy group theory. In this paper we introduce the concept of fuzzy magnified transla- tion of a fuzzy bi-ideal in a ternary hemiring and study some properties of it.

2 Preliminary Notes

An algebra $(R; +, .)$ is said to be a ternary semiring if $(R; +)$ is a semigroup and $(R; .)$ is a ternary semigroups satisfying $as(b + c) = asb + asc$ and $(b + c)sa = bsa + csa$ for all $a, b, c, s \in R$. A ternary semiring R is said to be additively commutative if $a+b=b+a$ for all $a,b\in R$. A ternary semiring R may have an identity 1, defined by $1sa=a1s=as1=as$ and a zero 0, defined by $0+a=a=a+0$ and $as0=0sa=s0a=0$ for all $a,s\in R$. A ternary semiring R is said to be a ternary hemiring if it is additively commutative with zero. In this paper we introduce the notion of fuzzy magnified translation of fuzzy bi-ideals in ternary hemirings and develop some results. Henceforth we denote a ternary hemiring by H unless otherwise stated.

We now review some definitions that are used in this paper.

Definition 2.1 *Let X be a non empty set. A fuzzy subset μ of X is a function $\mu : X \to [0,1]$. Let $\{\mu_i \mid i\in I\}$ be fuzzy subsets of a ternary ring R. The intersection of the fuzzy sets μ_i is defined as follows:*

$$\cap\mu_i(x)=\inf_{i\in I}[\mu_i(x)], x\in X.$$

Definition 2.2 *Let μ be a fuzzy subset of a ternary hemiring H. $Im\mu$ is defined by*

$$Im\mu=\{t\in[0,1] \mid \mu(x)=t \text{ for some } x\in X\}.$$

Let $t\in[0,1]$. The set $\mu_t=\{x\in X \mid \mu(x)\geq t\}$ is called a *level subset* of μ. Clearly, $\mu_t\subseteq\mu_s$ whenever $t\leq s$.

Definition 2.3 *A ternary subring S of a ternaryring R is called a bi-ideal of R if $SRRRS\subseteq S$ holds where $SRRRS$ is the additive subgroup of R generated by the set of all elements of the form $sr_1rr_2s, s\in S$ and $r,r_1,r_2\in R$.*

Definition 2.4 *A fuzzy subset μ of a tenary hemiring H is called a fuzzy left(right) ideal of H, if for every $x,y,s\in H$:*
(i) *μ is a fuzzy subgroup of $(H,+)$, i.e., $\mu(x+y)\geq\min\{\mu(x),\mu(y)\}$ and*
(ii) *$\mu(xsy)\geq\mu(y)(\mu(xsy)\geq\mu(x))$.*
If μ is both a fuzzy left ideal and a fuzzy right ideal of H, then it is called a fuzzy ideal of H.

Definition 2.5 *A non-empty fuzzy subset μ of a ternary hemiring H (i.e. $\mu(x)\neq 0$ for some $x\in H$) is called a fuzzy bi-ideal of H if:*
(i) *$\mu(x+y)\geq\min\{\mu(x),\mu(y)\}$,*
(ii) *$\mu(xsy)\geq\min\{\mu(x),\mu(s),\mu(y)\}$ and*
(iii) *$\mu(xsytz)\geq\min\mu(x),\mu(z)$ for all $x,y,z,s,t\in H$.*

Let μ be a fuzzy subset of a set H and $\alpha\in[0,1-\sup\{\mu(x)\mid x\in H\}]$. A mapping $\mu^T_\alpha : H\to[0,1]$ is called a *fuzzy translation* of μ if $\mu^T_\alpha(x)=\mu(x)+\alpha$ for all $x\in H$.

Let μ be a fuzzy subset of H and $\beta\in[0,1]$. A mapping $\mu^M_\beta : H\to[0,1]$ is called a *fuzzy multiplication* of μ if $\mu^M_\beta(x)=\beta\cdot\mu(x)$ for all $x\in H$.

Let μ be a fuzzy subset of H, $\alpha\in[0,1-\sup\{\mu(x)\mid x\in H\}]$ and $\beta\in[0,1]$. A mapping $\mu^C_{\beta\alpha} : H\to[0,1]$ is called a *fuzzy magnified translation* of μ if $\mu^C_{\beta\alpha}(x)=\beta\cdot\mu(x)+\alpha$ for all $x\in H$.

3 Main Results

In this section we present the main results of our paper.

Theorem 3.1 *Let μ be a non empty fuzzy subset of a ternary hemiring H. Then μ is a fuzzy bi-ideal of H if and only if the fuzzy magnified translation $\mu^C_{\beta\alpha}$ of μ is a fuzzy bi-ideal of H.*

Proof. Let μ be a fuzzy bi-ideal of a ternary hemiring H.
Now for all $x, y, z, s, t \in H$, we have

$$\begin{aligned} \mu^C_{\beta\alpha}(x+y) &= \beta \cdot \mu(x+y) + \alpha \\ &\geq \beta \cdot \min\{\mu(x), \mu(y)\} + \alpha \\ &= \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(y) + \alpha\} \\ &= \min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(y)\}. \end{aligned}$$

Again

$$\begin{aligned} \mu^C_{\beta\alpha}(xsy) &= \beta \cdot \mu(xsy) + \alpha \\ &\geq \beta \cdot \min\{\mu(x), \mu(s), \mu(y)\} + \alpha \\ &= \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(s) + \alpha, \beta \cdot \mu(y) + \alpha\} \\ &= \min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(s), \mu^C_{\beta\alpha}(y)\}. \end{aligned}$$

Also

$$\begin{aligned} \mu^C_{\beta\alpha}(xsytz) &= \beta \cdot \mu(xsytz) + \alpha \\ &\geq \beta \cdot \min\{\mu(x), \mu(z)\} + \alpha \\ &= \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(z) + \alpha\} \\ &= \min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(z)\}. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}$ is a fuzzy bi-ideal of H.
Conversely let $\mu^C_{\beta\alpha}$ be a fuzzy bi-ideal of H. Then for all $x, y, z, s, t \in H$,

$$\begin{aligned} \mu(x+y) &= \tfrac{1}{\beta}[\mu^C_{\beta\alpha}(x+y) - \alpha] \geq \tfrac{1}{\beta}[\min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(y)\} - \alpha] \\ &= \tfrac{1}{\beta}[\min\{\mu^C_{\beta\alpha}(x) - \alpha, \mu^C_{\beta\alpha}(y) - \alpha\}] = \min\{\tfrac{\mu^C_{\beta\alpha}(x)-\alpha}{\beta}, \tfrac{\mu^C_{\beta\alpha}(y)-\alpha}{\beta}\} \\ &= \min\{\mu(x), \mu(y)\}. \end{aligned}$$

Again

$$\begin{aligned} \mu(xsy) &= \tfrac{1}{\beta}[\mu^C_{\beta\alpha}(xsy) - \alpha] \\ &\geq \tfrac{1}{\beta}[\min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(s), \mu^C_{\beta\alpha}(y)\} - \alpha] \\ &= \tfrac{1}{\beta}[\min\{\mu^C_{\beta\alpha}(x) - \alpha, \mu^C_{\beta\alpha}(s) - \alpha, \mu^C_{\beta\alpha}(y) - \alpha\}] \\ &= \min\{\tfrac{\mu^C_{\beta\alpha}(x)-\alpha}{\beta}, \tfrac{\mu^C_{\beta\alpha}(s)-\alpha}{\beta}, \tfrac{\mu^C_{\beta\alpha}(y)-\alpha}{\beta}\} \\ &= \min\{\mu(x), \mu(s), \mu(y)\}. \end{aligned}$$

Also

$$\begin{aligned} \mu(xsytz) &= \tfrac{1}{\beta}[\mu^C_{\beta\alpha}(xsytz) - \alpha] \\ &\geq \tfrac{1}{\beta}[\min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(z)\} - \alpha] \\ &= \tfrac{1}{\beta}[\min\{\mu^C_{\beta\alpha}(x) - \alpha, \mu^C_{\beta\alpha}(z) - \alpha\}] \\ &= \min\{\tfrac{\mu^C_{\beta\alpha}(x)-\alpha}{\beta}, \tfrac{\mu^C_{\beta\alpha}(z)-\alpha}{\beta}\} \\ &= \min\{\mu(x), \mu(z)\}. \end{aligned}$$

Hence μ is a fuzzy bi-ideal of H. ■

Theorem 3.2 *If μ is a fuzzy left (right, two-sided) ideal of a ternary hemiring H then the fuzzy magnified translation $\mu^C_{\beta\alpha}$ of μ is a fuzzy bi-ideal of H.*

Proof. Let μ be a fuzzy left (right, two-sided) ideal of a ternary hemiring H. Then for all $x, y, z, s, t \in H$,

$$\begin{aligned} \mu^C_{\beta\alpha}(x+y) &= \beta \cdot \mu(x+y) + \alpha \\ &\geq \beta \cdot \min\{\mu(x), \mu(y)\} + \alpha \\ &= \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(y) + \alpha\} \\ &= \min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(y)\}. \end{aligned}$$

Again

$$\begin{aligned} \mu^C_{\beta\alpha}(xsy) &= \beta \cdot \mu(xsy) + \alpha \\ &\geq \beta \cdot \min\{\mu(x), \mu(s), \mu(y)\} + \alpha \\ &= \min\{\beta \cdot \mu(x) + \alpha, \beta \cdot \mu(s) + \alpha, \beta \cdot \mu(y) + \alpha\} \\ &= \min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(s), \mu^C_{\beta\alpha}(y)\}. \end{aligned}$$

Also

$$\begin{aligned} \mu^C_{\beta\alpha}(xsytz) &= \beta \cdot \mu(xsytz) + \alpha \\ &\geq \beta \cdot \mu(x) + \alpha \\ &= \mu^C_{\beta\alpha}(z) \\ &\geq \min\{\mu^C_{\beta\alpha}(x), \mu^C_{\beta\alpha}(z)\}. \end{aligned}$$

Thus $\mu^C_{\beta\alpha}$ is a fuzzy bi-ideal of H. Similarly we can prove the other statements. ■

Theorem 3.3 *The fuzzy magnified translation of the intersection of an arbitrary collection of fuzzy bi-ideals of a ternary hemiring H is a fuzzy bi-ideal of H if it is not empty.*

Proof. Let $\mu_i \mid i \in I\}$ be an arbitrary collection of fuzzy bi-ideals of H and $\mu = \bigcap_{i \in I} \mu_i$ be not empty.

Let $x, y, z, s, t \in H$. Then

$$\begin{aligned} \mu(x+y) &= \bigcap_{i \in I} \mu_i(x+y) \\ &= \inf_{i \in I}\{\mu_i(x+y)\} \\ &\geq \inf_{i \in I}\{\min\{\mu_i(x), \mu_i(y)\}\} \\ &= \min\{\inf_{i \in I}\{\mu_i(x)\}, \inf_{i \in I}\{\mu_i(y)\}\} \\ &= \min\{\bigcap_{i \in I} \mu_i(x), \bigcap_{i \in I} \mu_i(y)\}. \end{aligned}$$

Again

$$\begin{aligned} \mu(xsy) &= \bigcap_{i \in I} \mu_i(xsy) \\ &= \inf_{i \in I}\{\mu_i(xsy)\} \\ &\geq \inf_{i \in I}\{\min\{\mu(x), \mu(s), \mu(y)\}\} \\ &= \min\{\inf_{i \in I}\{\mu_i(x)\}, \inf_{i \in I}\{\mu_i(s)\}, \inf_{i \in I}\{\mu_i(y)\}\} \\ &= \min\{\bigcap_{i \in I} \mu_i(x), \bigcap_{i \in I} \mu_i(s), \bigcap_{i \in I} \mu_i(y)\}. \end{aligned}$$

Also

$$\begin{aligned}\mu(xsytz) &= \bigcap_{i\in I}\mu_i(xsytz)\\ &= \inf_{i\in I}\{\mu_i(xsytz)\}\\ &\geq \inf_{i\in I}\{\min\{\mu_i(x),\mu_i(z)\}\}\\ &= \min\{\inf_{i\in I}\{\mu_i(x)\},\inf_{i\in I}\{\mu_i(z)\}\}\\ &= \min\{\bigcap_{i\in I}\mu_i(x),\bigcap_{i\in I}\mu_i(z)\}.\end{aligned}$$

Hence $\mu = \bigcap_{i\in I}\mu_i$ is a fuzzy bi-ideal of H. Thus Theorem 3 follows from Theorem 1. ■

The proofs of the following two theorems are straight forward and therefore are omitted.

Theorem 3.4 *A subset A of a ternary hemiring H is a bi-ideal of H if and only if its charac- teristic function χ_A is a fuzzy bi-ideal of H.*

Theorem 3.5 *Let μ be a fuzzy subset of a ternary hemiring H. μ is a fuzzy bi-ideal of H if and only if its level sets ts are bi-ideals of H for all $t \in Im\mu$.*

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