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Common Fixed Point Theorems for Sub Compatible and Sub Sequentially Continuous Maps in 2 Metric Spaces

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Abstract. The present paper introduces the new concepts of sub compatibility and sub sequential continuity in 2 metric spaces which are weaker than occasionally weak compatibility and reciprocal continuity. We also establish a common fixed point theorem for four maps using sub compatibility and sub sequential continuity. Our results particularly extend and generalize the results of H. Bouhadjera and C. Godet-Thobie [6] and Jungck and Rhoades [19]. In general all known results on commuting, weakly commuting, compatible, weak compatible, semi compatible and occasionally weak compatible maps in 2 metric spaces are generalized in this note.

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1. Introduction

Ghaler [17] gave the concept of 2-metric space whose abstract properties were suggested by the area function in Euclidean space. A 2 metric space is one which finds its wide range of applications in the fields of military, medicine and economics. Employing various contractive conditions Iseki [22] set out the tradition of proving fixed point theorems in 2-metric spaces. Later on, Naidu and Prasad [24] contributed few fixed point theorems in 2-metric space introducing the concept of weak commutativity. Cho et al. [9] introduced the notion of semi-compatible maps in d-topological space. Various authors like Saliga [28] , Sharma et al. [27] and Popa [25] proved some interesting fixed point results using implicit real functions and semi-compatibility in d-complete topological spaces.

Recently, B. Singh and S. Jain [31, 32, 33 , 34] introduced the concept of semi-compatibility in fuzzy metric spaces, D-metric spaces, 2 metric space and proved fixed point results using implicit relations in these spaces.

Various authors have discussed and studied extensively various results on coincidence, existence and uniqueness of fixed and common fixed points by using the concept of weak commutativity, compatibility, non-compatibility and weak compatibility for single and set valued maps satisfying certain contractive conditions in 2 metric spaces and they have been applied to diverse problems.

Recently, Al-Thagafi and N. Shahzad [2] weakened the concept of compatibility by giving a new notion of occasionally weakly compatible (owc) maps which is more general among the commutativity concepts.

Most recently, H. Bouhadjera and C. Godet-Thobie [6], weakened the concept of occasionally weak compatibility and reciprocal continuity in the form of sub compatibility and sub sequential continuity respectively and proved some interesting results with these concepts in metric spaces.

The main purpose of our paper is to extend the results of [6] by introducing sub compatibility and sub sequential continuity in 2 metric spaces and prove some fixed point results related with these new concepts.

Our improvements in this paper are three-fold as;

(i) Completeness of the space removed

(ii) Minimal type contractive condition used

(iii) Weakened the concept of occasionally weak compatibility by a more general concept of sub compatibility and sub sequential continuity.

We first give some preliminaries and definitions using [17].

2. Preliminaries

Definition 2.1. Let X be non-empty set with real valued function d on $X \times X \times X$ satisfying the followings:
(i) $d(x, y, z) = 0$ if at least two of x, y, z are equal,
(ii) $d(x, y, z) = d(p(x, y, z))$ for all $x, y, z \in X$ and each permutation $p(x, y, z)$ of x, y, z,
(iii) $d(x, y, z) \le d(x, y, w) + d(x, w, z) + d(w, y, z)$ for all $x, y, z, w \in X$.
The function d is called a 2-metric on X and the pair (X, d) is called a 2-metric space.

Definition 2.2. Two self mappings A and S of a 2-metric space (X, d) are called compatible if $\lim_n d(ASx_n, SAx_n, a) = 0$ for all $a \in X$, where $\{x_n\}$ is a sequence in X such that $\lim_n Ax_n = \lim_n Sx_n = x$ *for some* x *in* X.

Definition 2.3. Two self mappings A and S of a 2-metric space (X, d) are called weakly compatible if they commute at their coincidence points i.e., if $Ax = Sx$, *then* $SAx = ASx$ for some $x \in X$.

Definition 2.4. Two self mappings $A, B : X \to X$ are said to be semi-compatible if $\lim_n d(ABx_n, Bx, a) = 0$ *for all* $a \in X$, whenever $\{x_n\}$ is a sequence in X such that $\lim_n Ax_n = \lim_n Bx_n = x$ *for some* $x \in X$.

Now, we give some examples to show the relationship of compatible, weakly compatible and semi-compatible maps.

Example 1. Let $X = \left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{n}, ...\right\}$. Define $d : X \times X \times X \to [0, \infty)$ by

$$d(x,y,z)=\begin{cases}0 \text{ if } x,y,z \text{ are distinct and } \left\{\frac{1}{n},\frac{1}{n+1}\right\}\subset\{x,y,z\}\\ 1 \text{ other wise.}\end{cases}.$$

Then (X,d) is a 2-metric space as proved in [11].

Define $A,B: X \to X$ as

$Ax = 1-x$, $Bx=\begin{cases} x \text{ if } 0\le x\le \frac{1}{2}\\ 1 \ , \ \frac{1}{2}<x\le 1\end{cases}$. Consider a sequence $x_n = \left(\frac{1}{2}-\frac{1}{n}\right)$ for all n.

Then, $Ax_n = 1-\left(\frac{1}{2}-\frac{1}{n}\right)=\frac{1}{2}+\frac{1}{n}$ *and* $Bx_n=\frac{1}{2}-\frac{1}{n}$.

Here $Ax_n, Bx_n \to \frac{1}{2}$ *for all n.*

Also $BAx_n = B\left(\frac{1}{2}+\frac{1}{n}\right)\to 1$ *and* $ABx_n \to \frac{1}{2}$, $B\frac{1}{2}=\frac{1}{2}$ = $A\frac{1}{2}$

Now, $\lim_n d\left(ABx_n, B\frac{1}{2}, a\right)=0$ for all $a\in X$, which implies that (A , B) is semi-compatible.

But, $\lim_n d\left(BAx_n, A\frac{1}{2}, a\right)\ne 0$, which implies that (B , A) is not semi-compatible.

Thus semi-compatibility of the pair (A , B) does not imply the semi-compatibility of the pair (B,A).

Moreover, weak-compatibility need not imply the semi-compatibility. Here B and A are weak compatible as they commute at their coincident point $\frac{1}{2}$ but they are not semi-compatible.

Also, semi-compatible maps need not be compatible. Here (A , B) is semi-compatible but not compatible as,

$\lim_n d\left(ABx_n, BAx_n, a\right)=\lim_n d\left(\frac{1}{2}+\frac{1}{n},1,a\right)\ne 0$ for all a in X and t > 0.

Again, weak compatibility does not imply compatibility as the maps B and A are weak compatible but not compatible.

The next example shows that compatible maps need not be semi-compatible.

Example 2. Let $X=\left\{0,1,\frac{1}{2},\frac{1}{3},\frac{1}{4},.....,\frac{1}{n},....\right\}$. Define $d: X\times X\times X\to[0,\infty)$ by

$$d(x,y,z)=\begin{cases}0 \text{ if } x,y,z \text{ are distinct and } \left\{\frac{1}{n},\frac{1}{n+1}\right\}\subset\{x,y,z\}\\ 1 \text{ other wise.}\end{cases}.$$

Then (X,d) is a 2-metric space .

Define $A,B: X\to X$ as

$$Ax=x \text{ and } Bx=\begin{cases}x \ , \ 0\le x<\frac{1}{2}\\ 1 \ , \quad x\ge\frac{1}{2}\end{cases}.$$ Consider the sequence $x_n=\frac{1}{2}-\frac{1}{n}$.

Then $Ax_n, Bx_n\to\frac{1}{2}$, $ABx_n=A\left(\frac{1}{2}-\frac{1}{n}\right)=\left(\frac{1}{2}-\frac{1}{n}\right)\to\frac{1}{2}$

and $BAx_n=B\left(\frac{1}{2}-\frac{1}{n}\right)=\left(\frac{1}{2}-\frac{1}{n}\right)\to\frac{1}{2}$.

Thus, $\lim_n d(ABx_n, BAx_n, a)=0$, which implies that A and B are compatible.

But $\lim_n d\left(ABx_n, B\frac{1}{2}, a\right)\ne 0$, implies that A and B are not semi-compatible.

Hence, compatibility does not imply semi-compatibility.

Definition 2.5. Two self maps f and g on a set X are said to be occasionally weakly compatible (owc) if and only if there is a point $x\in X$ which is a coincidence point of f and g at which f and g commute. i.e., there exists a point $x\in X$ such that $fx=gx$ and $fgx=gfx$.

Definition 2.6. Two self maps f and g on a 2 metric space (X,d) are said to be sub compatible if and only if there exists a sequence $\{x_n\}$ in X such that $\lim_n fx_n=\lim_n gx_n=z$, $z\in X$ and which satisfy $\lim_n d(fgx_n, gfx_n, a)=0$ for all $a\in X$.

Obviously two occasionally weakly compatible maps are sub compatible maps, however, the converse is not true in general as shown in the following example.

Example 3. Let X = $\mathbb{R}$ and $d(x,y,z)=\min[\,|x-y|,|y-z|,|z-x|\,]$, for all $x,y,z\in X$.

Then clearly (X,d) is a 2 metric space.

Define the maps $A,B:X\to X$ by setting

$$Ax=\begin{cases}x^2, x<1\\ 2x-1, x\geq 1\end{cases},\quad Bx=\begin{cases}3x-2, x<1\\ x+3, x\geq 1\end{cases}$$

Define a sequence $x_n=1-\frac{1}{n}$, then $Ax_n=\left(1-\frac{1}{n}\right)^2\to 1$, $Bx_n=3\left(1-\frac{1}{n}\right)-2=1-\frac{3}{n}\to 1$

$ABx_n=A\left(1-\frac{3}{n}\right)=\left(1-\frac{3}{n}\right)^2=1+\frac{9}{n^2}-\frac{6}{n}$ and

$BAx_n=B\left(1-\frac{1}{n}\right)^2=3\left(1-\frac{1}{n}\right)^2-2=3\left[\left(1+\left(\frac{1}{n}\right)^2-\frac{2}{n}\right)\right]-2=1+\left(\frac{1}{n}\right)^2-\frac{6}{n}$ and

$\lim_n M\left(ABx_n,BAx_n,a\right)\to 0$.

Thus, A and B are sub compatible but A and B are not owc maps as,
$A(4)=7=B(4)$ and $AB(4)=A(7)=13\neq BA(4)=10$.

It is also interesting to see the following one way implication .

Commuting $\Rightarrow$ Weakly commuting $\Rightarrow$ Compatibility $\Rightarrow$Weak compatibility$\Rightarrow$Occasionally weak compatibility $\Rightarrow$Sub compatibility.

Now, we aim at our second objective which is to introduce a new notion called sub sequential continuity in fuzzy metric spaces by weakening the concept of reciprocal continuity introduced by Pant [26].

Definition 2.7. Two self maps A and S on a 2 metric space (X,d) are called reciprocal continuous if $\lim_n ASx_n=At$ and $\lim_n SAx_n=St$ for some $t\in X$ whenever $\{x_n\}$ is a sequence in X such that $\lim_n Ax_n=\lim_n Sx_n=t\in X$.

Definition 2.8. Two self maps A and S on a 2 metric space (X,d) are said to be sub sequentially continuous if and only if there exists a sequence $\{x_n\}$ in X such that

$\lim_n Ax_n = \lim_n Sx_n = t$ for some $t \in X$ and satisfy $\lim_n ASx_n = At$ and $\lim_n SAx_n = St$.

Remark 1. If A and S are both continuous and reciprocally continuous then they are obviously sub sequentially continuous.

The next example shows that there exist sub sequentially continuous pairs of maps which are neither continuous nor reciprocally continuous.

Example 4. Let $X = R$ be the set of real numbers. We define 'd' as

$$d(x,y,z) = \begin{cases} 0, \textit{if atleast two of the three points are equal.} \\ \\ 2, \textit{other wise.} \end{cases}$$

Then clearly (X, d) is a 2 metric space.

Define $A, S : X \to X$ as;

$$Ax = \begin{cases} 2, x < 3 \\ x, x \geq 3 \end{cases} , \quad Sx = \begin{cases} 2x-4, x \leq 3 \\ 3, x > 3 \end{cases}$$

Consider a sequence $x_n = 3+\frac{1}{n}$, then $Ax_n = \left(3+\frac{1}{n}\right) \to 3$, $Sx_n = 3$

$SAx_n = S\left(3+\frac{1}{n}\right) = 3 \neq B(3) = 2$. Thus A and S are not reciprocally continuous but if we consider a sequence $x_n = 3-\frac{1}{n}$, then $Ax_n = 2$, $Sx_n = 2\left(3-\frac{1}{n}\right) - 4 = \left(2-\frac{2}{n}\right) \to 2$

$ASx_n = A\left(2-\frac{2}{n}\right) = 2 = A(2)$, $SAx_n = S(2) = 0 = S(2)$.

Therefore, A and S are sub sequentially continuous.

3. Results and Discussion

Now, we prove our main result.

Theorem 1. Let f, g, h and k be four self maps on a 2 metric space (X, d). If the pairs (f, h) and (g, k) are sub compatible and sub sequentially continuous, then

(i) f and h have a coincidence point,

(ii) g and k have a coincidence point.

Further, If

(1.1) $d(fx, gy, a) \le \phi\left[\max\begin{Bmatrix} d(hx, ky, a), d(fx, hx, a), d(gy, ky, a) \\ d(hx, gy, a), (ky, fx, a) \end{Bmatrix}\right]$ for all

$x, y, a \in X$, where $\phi : [0,1] \to [0,1]$ is a continuous function such that $\phi(s) > s$ for each $0 < s < 1$. Then f, g, h and k have a unique common fixed point in X.

Proof. Since the pairs (f, h) and (g, k) are sub compatible and sub sequentially

continuous, therefore, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_n fx_n = \lim_n hx_n = u$ for some $u \in X$ and which satisfy

$\lim_n d(fhx_n, hfx_n, a) = d(fu, hu, a) = 0$,

$\lim_n gy_n = \lim_n ky_n = v$ for some $v \in X$ and which satisfy

$\lim_n d(gky_n, kgy_n, a) = d(gv, kv, a) = 0$.

Therefore, $fu = hu$ and $gv = kv$. i.e., u is the coincidence point of f and h and v is a coincidence point of g and k.

Now, using (1.1) for $x = x_n$ and $y = y_n$, we get

$$d(fx_n, gy_n, a) \le \phi\left[\max\begin{Bmatrix} d(hx_n, ky_n, a), d(fx_n, hx_n, a), d(gy_n, ky_n, a) \\ d(hx_n, gy_n, a), d(ky_n, fx_n, a) \end{Bmatrix}\right]$$

Letting $n \to \infty$,

$$d(u,v,a) \le \phi\left[\max\left\{d(u,v,a),0,0,d(u,v,a),d(u,v,a)\right\}\right]$$

i.e., $d(u,v,a) \le \phi\left[d(u,v,a)\right] > d(u,v,a)$, a contradiction.

Hence, $u = v$.

Again using (1.1) for $x = u$, $y = y_n$, we obtain

$$d(fu,gy_n,a) \le \phi\left[\max\left\{\begin{matrix} d(hu,ky_n,a),d(fu,hu,a),d(gy_n,ky_n,a) \\ d(hu,gy_n,a),d(ky_n,fu,a) \end{matrix}\right\}\right]$$

Letting $n \to \infty$,

$$d(fu,v,a) \le \phi\left[\max\left\{d(fu,v,a),0,0,d(fu,v,a),d(fu,v,a)\right\}\right]$$

i.e., $d(fu,v,a) = \phi\left[d(fu,v,a)\right] > d(fu,v,a)$, which yields $fu = v = u$.

Therefore, $u = v$ is a common fixed point of f , g , h and k .

For uniqueness, let $w \ne u$ be another fixed point of f , g , h and k . Then from (1.1), we have

$$d(fu,gw,a) \le \phi\left[\max\left\{\begin{matrix} d(hu,kw,a),d(fu,hu,a),d(gw,kw,a) \\ d(hu,gw,a),d(kw,fu,a) \end{matrix}\right\}\right]$$

$$= \phi\left[\max\left\{d(fu,gw,a),0,0,d(fu,gw,a),d(fu,gw,a)\right\}\right]$$

$= \phi\left[d(fu,gw,a)\right] > d(fu,gw,a)$ which yields $w = u$ and hence the theorem.

If we put $f = g$ and $h = k$, we get the following result.

Corollary 1. Let f and h be self maps on a 2 metric space (X,d) such that the pairs (f,h) is sub compatible and sub sequentially continuous, then

(i) f and h have a coincidence point,

Further, If

(1.1) $d(fx, fy, a) \le \phi\left[\max\left\{\begin{matrix} d(hx, hy, a), d(fx, hx, a), d(fy, hy, a) \\ d(hx, fy, a), d(hy, fx, a) \end{matrix}\right\}\right]$ for all $x, y, a \in X$,

where $\phi : [0,1] \to [0,1]$ is a continuous function such that $\phi(s) > s$ for each $0 < s < 1$. Then f and h have a unique common fixed point in X.

If we put $h = k$, we get the following result.

Corollary 2. Let f, g and h be self maps on a 2 metric space (X, d). Suppose that the pairs (f, h) and (g, h) are sub compatible and sub sequentially continuous, then

(i) f and h have a coincidence point,

(ii) g and h have a coincidence point,

Further, If

(1.1) $d(fx, gy, a) \le \phi\left[\max\left\{\begin{matrix} d(hx, hy, a), d(fx, hx, a), d(gy, hy, a) \\ d(hx, gy, a), d(hy, fx, a) \end{matrix}\right\}\right]$ for all $x, y, a \in X$

where $\phi : [0,1] \to [0,1]$ is a continuous function such that $\phi(s) > s$ for each $0 < s < 1$. Then f , g and h have a unique common fixed point in X.

Now, we furnish our theorem with example.

Example 5. . Let $X = R$ be the set of real numbers. We define 'd' as

$$d(x, y, z) = \begin{cases} 0, \text{ if atleast two of the three points are equal.} \\ \\ 2, \text{ other wise.} \end{cases}$$

Then clearly (X, d) is a 2 metric space.

Define the maps f , g , h and $k : X \to X$ as

$$f(x)=\begin{Bmatrix} x, x\le 1 \\ 3x+1, x>1 \end{Bmatrix} \quad , \quad h(x)=\begin{Bmatrix} 2x-1, x\le 1 \\ 5x-1, x>1 \end{Bmatrix}$$

$$g(x)=\begin{Bmatrix} 3-2x, x\le 1 \\ 3, x>1 \end{Bmatrix} \quad , \quad k(x)=\begin{Bmatrix} 2, x<1 \\ 3x-2, x\ge 1 \end{Bmatrix}$$

Consider the sequences $\{x_n\}=\{y_n\}=1-\frac{1}{n}$.

Then, clearly fx_n , gx_n , hx_n and $kx_n \to 1$.

$fh(x_n)=f\left(1-\frac{1}{n}\right)=\left(1-\frac{1}{n}\right)\to 1=f(1)$ and $hf(x_n)=h\left(1-\frac{1}{n}\right)=\left(1-\frac{1}{n}\right)\to 1=h(1)$

Thus (f,h) is sub compatible and sub sequentially continuous.

Again,

$$gk(x_n)=g\left(1-\frac{1}{n}\right)=3-2\left(1-\frac{1}{n}\right)=\left(1+\frac{2}{n}\right)\to 1=g(1)$$

$$kg(x_n)=g\left(1+\frac{1}{n}\right)=3\left(1+\frac{1}{n}\right)-2=\left(1+\frac{3}{n}\right)\to 1=k(1),$$

which shows that (g,k) is sub compatible and sub sequentially continuous.

Also the condition (1.1) of our theorem is satisfied and '1' is unique common fixed point of f , g , h and k .

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