International Mathematical Forum, Vol. 7, 2012, no. 23, 1135 - 1142

Analysis of Hybrid Dynamical Systems Using SCBZ

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Abstract: In this paper, the hybrid dynamical system is considered along with the SCBZ property. The system of equations being solved using Differential Transform Method. The results are compared with an inventory news boy problem with the SCBZ property.

Mathematics Subject Classification: 90B05, 34H38

Keywords: Hybrid dynamical system, SCBZ property, News boy problem, Differential Transform method

1. INTRODUCTION

Natural science has a historical significance and dates far back to Isaac Newton in the seventeenth century. He devised revolutionary equations relating to force and motion and was the co-inventor of calculus. This ground-breaking work was the launching pad of dynamical systems. His laws of force and motion are also the birth of Mathematical modeling of divine spark in this world. Since calculus by and large abhors inconsistencies, most established studies of dynamical systems and Mathematical modeling focussed on elucidation that were constant and unwavering

Chaos is a terminology in Mathematics that exemplifies intricate pattern in pivotal dynamical systems, which has short-term inexorableness but is nevertheless volatile and unpredictable in the longer duration. Extensive studies on chaos in the 1980s reaffirmed that chaos is omnipresent not only in Mathematical models but also in real-world systems.

Any dynamical systems can be modeled by any of the following systems described. A linear system is a mathematical model of a system based on the use of linear operator. A non-linear system is a system which is not linear, that is a system which does not satisfy the superposition principle or whose output is not directly proportional to its input. The third effective system is a Hybrid system, which is a dynamic system that exhibits both continuous and discrete dynamic behavior-(i.e) a system that can be described by a differential equation and / difference equation.

A hybrid system has the benefit of encompassing a larger class of systems with its structure, allowing for more flexibility in modeling dynamic phenomena. The hybrid dynamical systems are ubiquitous, they have not yet been formulated by a common mathematical description owing to their great diversity [6].

As an example, well-organized general formulations from the view point of deterministic dynamics [4] is summarized as follows:

$$\frac{dx}{dt} = F_{i(t)}(x(t), u(t), \mu) - - - - - - - - - \quad (1.1)$$
$$i(t) = G(i(t_-), x(t_-), u(t), \mu) - - - - - - - (1.2)$$
$$x(t) = R(i(t_-), x(t_-), u(t), \mu) - - - - - - - \quad (1.3)$$
$$y(t) = O(i(t), x(t), u(t), \mu) - - - - - - - - \quad (1.4)$$

where $x(t) \in R^n$ is the continuous state at time $t \in R$; $i(t) \in \{1, \ldots, N\}$ is the discrete state at time t; $F_{i(t)}$ is the vector-valued smooth function specified by $i(t)$; $u(t) \in R^m$ is the external input; $\mu \in R^l$ is the system (bifurcation) parameters; G is a map of discrete-state transitions from $i(t_-)$ to $i(t)$ with $i(t_-) \equiv \lim_{t \to t-0} i(t)$; R is a reset map of continuous states accompanying a discrete-state transition; $y(t) \in R^k$ is the output; and O is the output function.

If $F_{i(t)}$, G, R and O in equations (1.1) – (1.4) do not depend on external input $u(t)$, then the hybrid system is autonomous; otherwise, it is non-autonomous. An important class of autonomous hybrid dynamical systems is a piecewise smooth hybrid system [4] defined as follows:

$$\frac{dx}{dt} = F_i(x, \mu) \, for \; x \in S_i - - - - - - - - - (1.5)$$
$$x \to R_{ji}(x, \mu) \, if \; x \in \textstyle\sum_{ji} = \bar{S_j} \cap \bar{S_i} - - - - - (1.6)$$

where F_i is a smooth vector field specified by i. Each region $S_i \subset R^n$ has a non-empty interior. R_{ji} is a reset map on the intersection $\sum_{ji}$.When the reset map is the identity map $R_{ji}(x) = x$, the hybrid system (1.5) and (1.6) is called 'a piecewise smooth flow' or 'a piecewise smooth ordinary differential equation (ODE)', defined as follows:

$$\frac{dx}{dt} = F_i(x, \mu) \, for \; x \in S_i - - - - - (1.7)$$

Examples of a one-dimensional piecewise smooth ODE with the degrees of smoothness 1 and 2 at the origin are, respectively, given as follows:

$$\frac{dx}{dt} = -sgn(x) - - - - - - - - (1.8)$$

$$and \quad \frac{dx}{dt} = |x| - - - - - - - - -(1.9)$$

1.1. SETTING THE CLOCK BACK TO ZERO PROPERTY

In stochastic processes we can consider a sequence of random variables that are associated with probability distribution. The probability density function of a random variable *x* is denoted as f(x). The corresponding distribution function is denoted as *F(X)* ,and *S(x)= 1 – F(X)* is called the survivor function and it gives the probability that a random variable X greater than x that is *P(X>x).* For every probability distribution there are correspondingly one or more parameters.

For example, if a random variable X is distributed as exponential with parameter θ then we write it as $X \sim f(x,\theta) = \theta e^{-\theta x}$. There is a property called the *Lack of Memory Property* (LMP), and exponential distribution good for this property.

A slight modification of this property has been suggested by Raja Rao and Talwaker [9]. This property is called the *Setting the Clock Back to Zero property* (SCBZ). According to this property, the probability distribution of the random variable X undergoes a change of parameter after a particular value of X denoted as $\mathbf{x_0}$, known as truncation point. So, the p.d.f. of X is *f (x,θ)*

$$f(x,\theta) = \begin{cases} f(x,\theta_1) \ if \ \ x \le \ x_0 \\ f(x,\theta_2) if \ x > \ x_0 \end{cases} - - - - - - - - -(1.10)$$

$$where \ \ f(x,\theta_1) = \theta_1 e^{-\theta_1 x},$$

$$f(x,\theta_2) = \ e^{x_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 x}$$

This property is indicated by a condition denoted as follows $\frac{S(x + x_0, \theta_1, \theta_2)}{S(x_0, \theta_1)} = S(x, \theta_2)$. where S(x, **θ)** is the survivor function. Comparing equations (1.8) and (1.10), we arrive a conclusion that both equations (1.8) and (1.10) have two states which are peace wise linear by considering a truncation point. In equation (1.8) zero is considered as truncation point and yields two state solution. In equations (1.10) the point x_0 is considered as truncation point and the probability density function $f(x,\theta)$ is split in to two states $f(x,\theta_1)$ and $f(x,\theta_2)$ based on x_0. Hence both (1.8) and (1.10) have similarity of splitting into two states by the condition of truncation point. We conclude that equations (1.8) and (1.10) are similar. Basically the hybrid dynamical system is a exponential system. When difference of any one of the variable in the system is calculate. That will give exponential difference which is nothing but basic property of SCBZ.

1.2. HYBRID DYNAMICAL SYSTEMS

An relevance of hybrid dynamical systems to biology and medicine, particularly, cancer and its treatment are considered in [7, 11].

Osborne *et al.* [7] emphasizes on cancer dynamics of a colorectal crypt in colorectal cancer. Since cancer evolves from a mutation in the formative stages generating a single abnormal cell, there prevails a crisis of trivial numbers of cells, which is diverse from that of small copy numbers of proteins and mRNAs discussed in [10]. On account of the trivial number of cells, the study cell-based 'hybrid' models in which cells are referred as detached with dynamical changes such as division, proliferation and movement encoded either through rules or as the result of dynamics occurring on other scales. A comparative and contrasting study can be carried using two different cell-based models with a homogenized continuum model.

When the of cancer cells increases prodigiously resulting in tumour growth, another hybrid model can be used by describing cells as discrete in some parts and as a continuum in others (see also [1]). Hybrid dynamical systems are usually described with continuous and discrete-state variables. Representations of time and space, however, are also quite important; the above representation brings hybrid qualities of space into sharp relief. The hybrid properties of time have also been discussed in previous studies [3].

Thus, the mathematical modeling of hybrid dynamical systems should also be considered from the perspective of time and space; that is, the continuous and discrete structure of time and space. Tanaka *et al.* [11] further extend the deterministic hybrid models to the stochastic one.

This article is arranged as follows. In section -2, a dynamical hybrid system is introduced and analyzed. The Dynamical output described in section-3, with comparison in to SCBZ property analysis. Section - 4, gives some example results of SCBZ problem. In section-5, conclusion is given.

2- SYSTEM ANALYSIS

A famous example of hybrid dynamical system [2] which can be described as follows:

$$\frac{dx}{dt} = \alpha\big(y - x - h(x)\big) - - - - - - -(2.1)$$

$$\frac{dy}{dt} = x - y + z - - - - - - - - (2.2)$$

$$\frac{dz}{dt} = -\beta y - - - - - - - - -(2.3)$$

where x, y, and z are continuous state variables, 'α' and 'β' are positive parameters and $h(x)$ is a piecewise smooth function of x. The cover image of this Theme Issue is composed of some strange attractors obtained from equations (2.1)–(2.3) with $h(x)$ that is more complicated than the original. Which can also represented by SCBZ property as explained in section -1.1 Consider $\alpha = 10, \beta = 16.82, h(x) = -0.55x(|x+1| - |x-1|)$. Solve the system of equations (2.1), (2.2) & (2.3) by Differential Transform Method,

The power series obtained is given bellow

$$x(t) = x(0) + x(1)t + x(2)t^2 + x(3)t^3 + x(4)t^4 + \cdots \ldots \ldots - - - (2.4)$$
$$y(t) = y(0) + y(1)t + y(2)t^2 + y(3)t^3 + y(4)t^4 + \cdots \ldots \ldots - - - (2.5)$$
$$z(t) = z(0) + z(1)t + z(2)t^2 + z(3)t^3 + z(4)t^4 + \cdots \ldots .. - - - - (2.6)$$

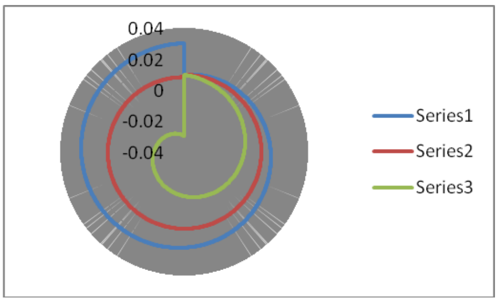
For the initial values $x(0) = 0.01, y(0) = 0.01$ & $z(0) = 0.01$, in equations (2.1),(2.2)& (2.3)by using differential transform method, the values obtained for x , y and z are given in the following table

Iteration	x	y	z
1	0.01	0.01	0.01
2	0.049	0.005	-0.0841
3	0.0201	-0.0201	-0.0421
4	-0.1025	-0.0010	0.1690
5	0.0053	0.0338	0.0084
6	0.1685	-0.0101	-0.2843
7	-0.0674	-0.0529	0.0850
8	-0.2578	0.0353	0.4449
9	0.2023	0.0759	-0.2969
10	0.3593	-0.0853	-0.63783

Table 2.1-Iteration values of x, y, z

3. OUT PUT OF THE DYNAMICAL SYSTEM

Using these values in the above power series, the following graph is obtained



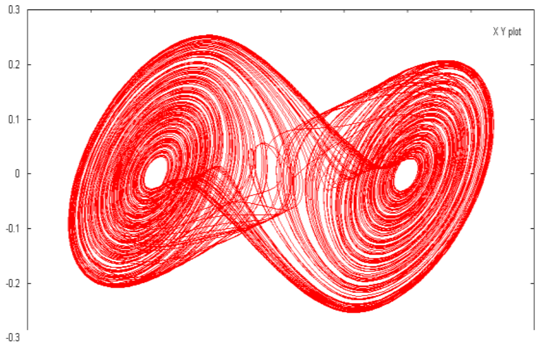


Fig. 3.2. Strange attractors obtained numerical with MATLAB SIMULINK

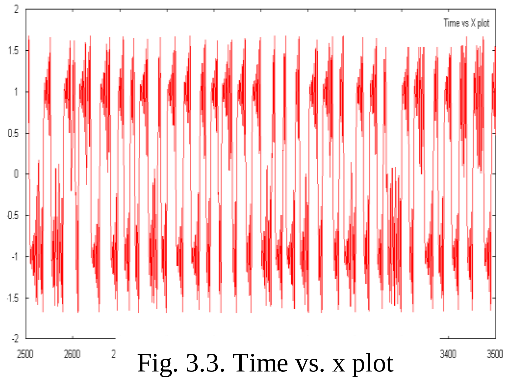
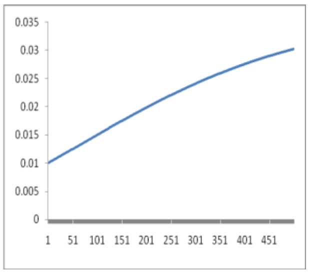
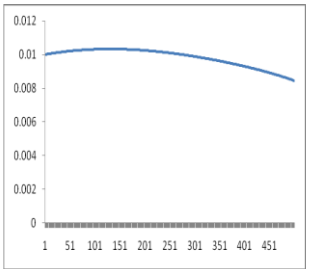


Fig. 3.3. Time vs. x plot

in the value of 'z' are of the system





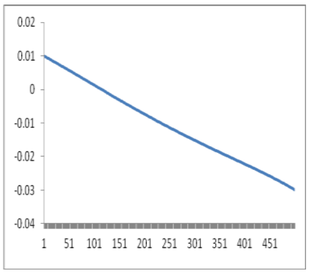


Fig. 3.2. Graph for x and d

Fig. 3.3Graph for y and d

Fig. 3.4 Graph for z and d

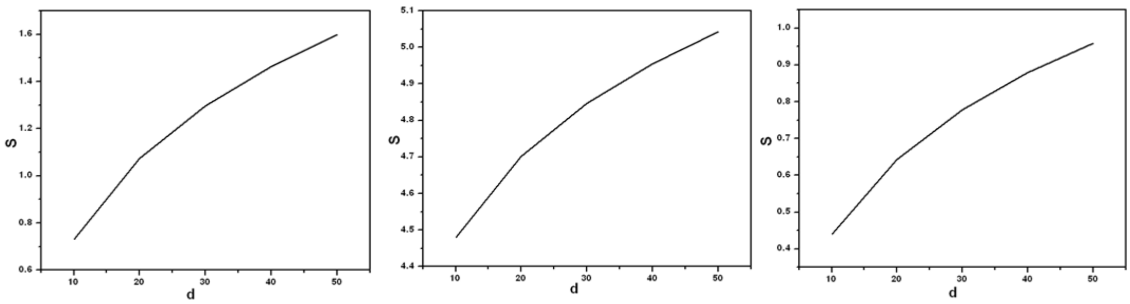
variable is calculated as reference and are plotted as shown below.

4. NEWS BOY PROBLEM WITH SCBZ

The Newsboy/Newsvendor problem is under the category of finite inventory process. There is a onetime supply of the newspapers per day and the demand is probabilistic. The classical Newsboy problem is to find order quantity for the products by which the expected profit is maximized in a single period probabilistic demand framework. Stochastic initial inventory may also be applied to the situation. Where decision about the order quantity must be made before the start of the period and the available inventory at the decision time may decrease stochastically due to the several factors.

The Newsboy problem is one in which the situation is such that the newspaper should be sold the same day itself. Every unit of the newspaper sold gives some profit, but at the same time, if it is not sold at the same day it has only negligible value. Hence excess supply will result in loss, which is equivalent to the inventory holding cost. If the supply is not adequate then shortage occurs which in turn will result in shortage loss. Under these conditions how many units of the newspaper to be ordered is the prime problem of interest. It is interesting to note that the conventional inventory holding cost is introduced in a terms of a new cost called the salvage loss.

In order to review the papers, particularly consider the paper News Boy inventory model with demand satisfying SCBZ Property [8]. For this model, SCBZ property is applied in particular inventory model, obtain the numerical results as shown given bellow.



The output of the Dynamical Hybrid system is analyzed with comparative study of SCBZ property. In this comparison, we conclude that the graphs corresponding to the solution of Hybrid dynamical system and the result of inventory news boy problem using SCBZ property are similar. Finally, we conclude that SCBZ property can be applied to solve any Hybrid dynamical systems.

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Received: October, 2011