International Mathematical Forum, Vol. 7, 2012, no. 24, 1201 - 1207

Strongly g*s-Continuous Maps and Perfectly g*s-Continuous Maps in Topological Spaces

A. Pushpalatha

Department of Mathematics, Government Arts college
Udumalpet-642 126, Tirupur District
Tamilnadu state, India.
velu_pushpa@yahoo.co.in

K. Anitha

Department of Mathematics
Sri Subramanya college of Engineering and Technology
Palani-624 601, Dindugal District
Tamilnadu state, India.
jegapraba@gmail.com

Abstract

A. Pushpalatha and K. Anitha introduced properties of g*s-closed sets in Topological space. In this paper, we introduced T_L-space, T_A-space strongly g*s-continuous maps, perfectly g*s-continuous maps and g*s-compactness.

Mathematics Subject Classification: 54C05, 54D30

Keywords: T_L -space, T_A -space, strongly g*s-continuous maps, perfectly g*s-continuous maps,g*s-compactness

1-INTRODUCTION

Levine [3] introduced and investigated the concept of strong continuity in topological spaces. Sundaram [6] introduced strongly g-continuous maps and perfectly g-continuous maps in topological spaces. In [6],Sundaram introduced the concept of GO-compact space by using g-open covers. Pushpalatha and Anitha [5] introduced g*s-closed sets in topological spaces.

In this paper, we introduce the notion of T_L-space, T_A-space ,strongly g*s-continuous maps, perfectly g*s-continuous maps and g*s-compactness in topological spaces and obtain some of its basic properties.

2-PRELIMINARIES

Definition:2.1 A subset of a topological space (x,τ) is called
(i) Generalised closed (briefly g-closed) [1] if cl(A)⊆ U Whenever A⊆U and U is open x.
(ii) Generalised semiclosed [2] (briefly gs-closed) if scl(A)⊆ U whenever A⊆U and U is open in X.
(iii) Semi-generalised closed (briefly sg-closed) A⊆U and U is semiopen in X. Every semi closed set is sg-closed.
(iv) Weakly closed (briefly w-closed) [4] if cl(A)⊆ U whenever A⊆U and U is semi open in X.
(v) Weakly generalized closed (briefly wg-closed) [5] if cl(int A) ⊆U whenever A⊆U and U is open in X.
(vi) Generalised α-closed (briefly g α-closed) [7] if α-cl (A) ⊆ U whenever A⊆U and U is α-open in X.
(vii) α-gneralised closed (briefly αg-closed) [8] if α-cl (A)⊆ U whenever A⊆U and U is open in X.
(viii)regular w-closed (briefly rw-closed) [9] if α- cl (A)⊆ U whenever A⊆U and U is regular semiopen in X.
(xi) Strongly g-closed [6] if cl(A)⊆ U whenever A⊆U and U is g-open in X.
(Xii) g*s-closed set if scl (A)⊆ U whenever A⊆U, U is gs-open.
The class of all g*s-closed set s in a topological space (x,τ) is denoted by g*s-c (x,τ).

The complements of the above mentioned closed sets are their respective open sets.

***D*efinition 2.2 A subset A of X is called a g*s-closed set if scl(A)⊆U whenever A⊆U, U** is gs-open.The complement of g*s-closed set is g*s-open set.

Definition-2.3 A topological space X is called

(1) $T_{1/2}$ space [2] if every g-closed set of X is closed in X.
(2) T_b space [1]if every gs-closed set of X is closed in X.
(3) T_d space [1] if every gs-closed set of x is g-closed in X.
(4) T_{gs}-space [1] if every gs-closed set of X is sg-closed in X.
(5) T_p-space[7] if every strongly g-closed set of X is closed in X.
(6)T_s-space [7] if every g-closed set of X is strongly g-closed in X.

***D*efinition 2.4** A function $f: X \to Y$ is called
Strongly continuous[3] if $f^{-1}(V)$ is both open and closed in X for each subset V in Y.

Perfectly continuous [4] if f^{-1} V is both open and closed in X for each open set V in Y.
Strongly g-continuous [6] if $f^{-1}(V)$ is open in X for each g-open set V in Y.
Perfectly g-continuous [6] if $f^{-1}(V)$ is both open and closed in X for each g-open set V in Y.
g*s-continuous [1] if the inverse image of every closed set in Y is g*s-closed in X.

3. SEPARATION AXIOMS IN TOPOLOGICAL SPACE

In this section we introduced the concepts of T_L-space and T_A-space in topological spaces.

***D*efinition 3.1** A topological space X is called a T_L -space if every g*s-closed set of X is closed in X.

***D*efinition 3.2** A topological space X is called T_A -space if every gs-closed set of X is g*s-closed in X.

Thorem3.3 If X is T_b then it is T_L but not conversely.
Proof: Let X be a T_b-space. Since every g*s-closed set is gs-closed and X is T_b,X is T_L.
The converse need not be true as seen from the following example.

Example 3.4 Let $X = \{a, b, c\}$ and $\tau = \ \emptyset, X, \ b \ , \ c \ , \ b, c \ , \ c, a$. Then (X,τ) is T_L-space but not T_b -space. Since C(X,τ)= $\emptyset, X, \ a \ , \ b \ , \ a, c \ , \ a, b$, g*s C (X,τ)= $\emptyset, X, \ a \ , \ b \ , \ a, c \ , \ a, b$ and GSC(X,τ)= $\emptyset, X, \ a \ , \ b \ , \ c \ , \ a, b \ \{a, c\}$.

Theorem:3.5 If X is T_b, then it is $T_A$but not conversely.
Proof: Let X be a T_b-space. Let A be any gs-closed set in X. Since X is T_b,A is closed in X. Since every closed set is g*s-closed, A is g*s-closed. Hence X is $T_{A.}$
The converse of the above theorem need not be true as seen from the following example.

Examlple:3.6 Let $X = \{a, b, c\}$ with $\tau = \ \emptyset, X, \ a, b$. Then X is T_A but not T_b.
Since C (X,τ)= $\emptyset, X, , \ c$,g*s- C (X,τ)= $\emptyset, X, \ c \ , \ b, c \ , \ a, c$ and GSC (X,τ)= $\emptyset, X, \ c \ , \ b, c \ , \ a, c$.

4 -STORNGLY g*s-CONTINUOUS MAPS AND PERFECTLY g*s-CONTINUOUS MAPS IN TOPOLOGICAL SPACES

In this section, we introduced the concepts of Strongly g*s-continuous maps and perfectly g*s-continuous maps in topological spaces.

Definition:4.1 A map $f: X \to Y$ is said to be strongly g*s-continuous if the inverse image of every g*s-open set in Y is open in X.

Remark:4.2 When Y is T_L, strongly g*s- continuity coincides with continuity.

Therorem:4.3 If a map $f: X \to Y$ from a topological space X into a topological space Y is strongly g*s-continuous then it is continuous but not conversely.
Proof: Assume that f is strongly g*s-continuous. Let G be any open set set in Y. Since every open set is g*s-open, G is g*s-open in Y. Since f is strongly g*s-continuous, $f^{-1}(G)$ is open in X. Therefore f is continuous.

The converse need not be true as seen from the following example.

Example:4.4 Let $X = Y = \{a, b, c\}$ with the topologies $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{a, b\}\}$ Define a map $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = a = f(b)$ and $f(c) = b$. Then f is continuous. But f is not strongly g*s-continuous. Since $f^{-1}(\{a\}) = \{a, b\}$ is not open in X where $\{a\}$ is g*s-open in X.

Theorem4.5 A map $f: X \to Y$ from a topological space X into a topological space Y is strongly g*s-continuous if and only if the inverse image of every g*s-closed set in Y is closed in X.
Proof: Assume that f is strongly g*s-continuous. Let F be any g*s-closed set in Y. Then F^c is g*s-open set in Y. Since f is strongly g*s-continuous, $f^{-1}(F^c)$ is open in X. But $f^{-1}(F^c) = X - f^{-1}(F)$ and so $f^{-1}(F)$ is closed in X.

Conversely assume that the inverse image of every g*s-closed set in Y is closed in X. Let G be any g*s-open set in Y. Then G^c is g*s-closed set in Y. By assumption, $f^{-1}(G^c)$ is closed in X. But $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is open in X. Therefore f is strongly g*s-continuous.

Theorem:4.6 If a map $f: X \to Y$ is strongly continuous then it is strongly g*s-continuous,but not conversely.
Proof: Assume that f is strongly continuous. Let G be any g*s-open set in Y. Since f is g*s-continuous, $f^{-1}(G)$ is open in X by definition 2.3(1).Therefore f is strongly g*s-continuous.

The converse need not be true as seen from the following example.

Example: 4.7 Let $X = Y = \{a, b, c\}$ the topologies $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{\emptyset, Y, \{b\}, \{a, b\}, \{b, c\}\}$ .Define a map $f: (X, \tau) \to (Y, \sigma)$ as the identity map. Then f is strongly g*s-continuous but f is not strongly continuous. For the subset $\{b\}$ of Y, $f^{-1}(\{b\}) = \{b\}$ is open in X , but not closed in X.

Theorem:4.8 If a map $f: X \to Y$ is strongly g*s-continuous and a map $g: Y \to Z$ isg*s-continuous, then the composition $g.f: X \to Z$ is continuous.

Proof: Let G be any open set in Z. Since g is g*s-continuous , $g^{-1}(G)$ is g*s-open in Y. Since f is strongly g*s-continuous, $f^{-1}(g^{-1}(G))$ is open in X. But $(g.f)^{-1}(G)$= $f^{-1}(g^{-1}(G))$. Therefore g.f is continuous.

Definition :4.9 A map $f: X \rightarrow Y$ is said to be perfectly g*s-continuous if the inverse image of every g*s-open set in Y is both open and closed in X.

Theorem:4.10 If a map $f: X \rightarrow Y$ from a topological space X into a topological space Y is perfectly g*s-continuous then it is strongly g*s-continuous but not conversely.

Proof: Assume that f is perfectly g*s-continuous . Let G be any g*s-open set in Y. Since f is perfectly g*s-continuous, $f^{-1}(G)$ is open in X. Therefore f is strongly g*s-continuous.

The converse need not be true as seen from the following example.

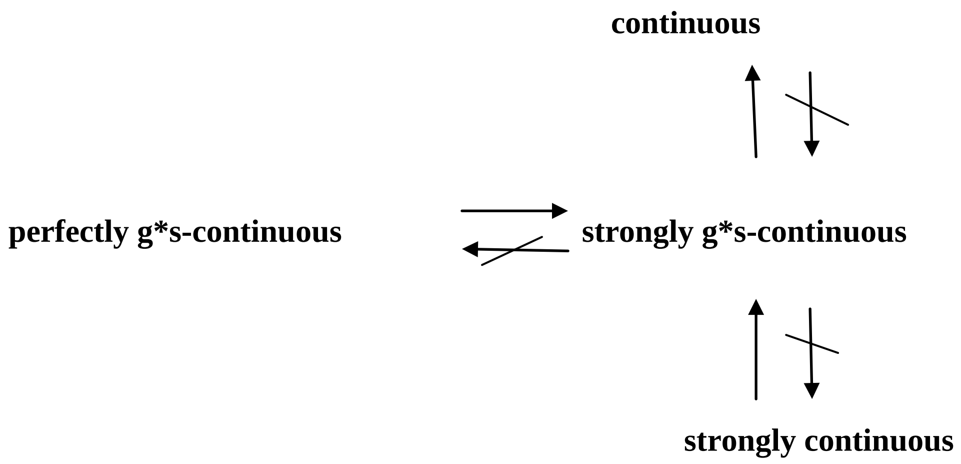
Example :4.11 Let $X = Y = \{a, b, c\}$ with the topologies $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. and $\sigma = \{\emptyset, Y, \{b\}, \{a, b\}, \{b, c\}\}$. Define a map f: $(X, \tau) \rightarrow (Y, \sigma)$ as the identity map. Then f is strongly g*s-continuous but f is not perfectly g*s-continuous. For the subsets {c} of Y, {a,c} of Y are closed sets in X but not open in X.

Theorem:4.12 A map $f: X \rightarrow Y$ from a topological space X into a topological space Y is perfectly g*s-continuous if and only if $f^{-1}(G)$ is both open and closed in X for every g*s-closed set G in Y.

Proof: Assume that f is perfectly g*s-continuous. Let F be any g*s-closed set in Y. Then F^c is g*s-open set in Y. Since f is perfectly g*s-continuous, $f^{-1}(F^c)$ is both open and closed in X. But $f^{-1}(F^c) = X - f^{-1}(F)$ and so $f^{-1}(F)$ is both open and closed in X.

Conversely assume that the inverse image of every g*s-closed set in Y is both open and closed in X. Let G be any g*s-open set in Y. Then G^c is g*s-closed set in Y. By assumption $f^{-1}(G^c)$ is both open and closed in X. But $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is both open and closed in X. Therefore f is perfectly g*s-continuous.

Remark:4.13 From the above observations we have the following implications :



5. g*s-compactness

In this section, we introduced the concepts of g*s-compact in topological spaces.

Definition:5.1 A collection $\{A_i; i \in I\}$ of g*s-open sets in a topological space X is called a g*s-open cover of a subset B in X if $B \subseteq \cup_{i \in I} A_i$.

Definition:5.2 A topological space X is g*s-compact if every g*s-open cover of X has a finite subcover of X.

Definition:5.3 A subset B of a topological space X is called g*s-compact relative to X, if for every collection $\{A_i; i \in I\}$of g*s-open subsets of X such that $B \subseteq \cup_{i \in I} A_i$. there exists a finite subset I_o of I such that $B \subseteq \cup_{i \in I_o} A_i$.

Definition:5.4 A subset B of a topological space X is called g*s-compact if B is g*s-compact as the subspace of X.

Theorem:5.5 A g*s-closed subset of g*s-compact space is g*s-compact relative to X.

Proof: Let A be a g*s-closed subset of a g*s-compact space X. Then A^c is g*s-open in X. Let S be a g*s-open cover of A in X. Then, S along with A^c form a g*s-open cover of X. Since X is g*s-compact, it has a finite subcover, say$\{G_1, G_2 \ldots\ldots\ldots G_n]$. If this subcover contains A^c, we discard it. Otherwise leave the subcover as it is. Thus we have obtained a finite subcover of A and so A is g*s-compact relative to X.

Theorem:5.6 A g*s-continuous image of a g*s-compact space is compact.

Proof: Let f:x→y be a g*s-continuous map from a g*s-compact space X onto a topological space Y. Let {A_i;i∈I}be an open cover of Y. Then {$f^{-1}(A_i)$;i∈I} is a g*s-open cover of X. Since X is g*s-compact, it has a finite subcover say {$f^{-1}(A_1)$, $f^{-1}(A_2)$……… $f^{-1}(A_{n)}$}. Since f is onto ,{ A_1, A_2……….A_n} is an open cover of Y and so Y is compact.

Theorem:5.7 : If f:x→y is Strongly g*s-continuous map from a compact space X onto a topological space Y, then Y is g*s-compact.

Proof: Let {A_i;i∈I} be a g*s-open cover of Y. Then {$f^{-1}(A_i)$;i∈I} is an open cover of X. Since f is Strongly g*s-continuous. Since X is compact, it has a finite subcover say {$f^{-1}(A_1)$, $f^{-1}(A_2)$……… $f^{-1}(A_{n)}$} and since f is onto , { A_1, A_2……….A_n} is a finite subcover of Y. Therefore Y is g*s-compact.

Theorem: 5.8 If a map f:x→y is g*s-continuous [6] from a compact space X onto a topological space Y, then g*s-compact.

Proof: Since every g*s-continuous map is Strongly g*s-continuous and the result follows from theorem.5.7.

Theorem: 5.9 If a map f: x→y is a perfectly g*s-continuous from a compact space X onto a topological space Y, then Y is g*s-compact.

Proof: Since every perfectly g*s-continuous function is Strongly g*s-continuous , the result follows from theorem.

REFERENCES

[1] Devi, R., Studies On generalizations of closed maps and homeomorphisms intopological spaces, Ph.D. Thesis, Bharathiar University, Coimbatore (1994).

[2] Levine, N., Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(1970), 89-96

[3] Levine, N., Strong continuity in topological spaces, Amer. Math.Monthly, 67 (1960), 269.

[4] Noiri, T., Strong form of continuity in topological spaces, Rend. Circ. Math. Palermo, (1986), 107-113.

[5] Pushpalatha.P. and Anitha.K. g*S-closed sets in Topologicals spaces, Int. J. Contemp. Math. Sciences,Vol .6.,March-2011,no 19,917-929.

[6] Sundaram, P., Studies on Generalizations of Continuous maps in topological spaces, Ph. D., Thesis, Bharathiar University, Coimbatore. (1991).

[7] Sundram,P and Pushpalatha,A strongly generalized closed sets in topological spaces, far East J.Math.sc(FEJMS),3(4)(2001),563-575.

Received: November, 2011