International Mathematical Forum, Vol. 7, 2012, no. 18, 877 - 885

Almost Normality and Non π-Normality of the Rational Sequence Topological Space

Sadeq Ali Saad Thabit[1] **and Hailiza Kamarulhaili**[2]

School of Mathematical Sciences, University Sains Malaysia
11800 USM, Penang, Malaysia

Abstract

The Rational Sequence Topology is one of the famous topological spaces, which is a Tychonoff and not normal. In this paper, we show that the Rational Sequence Topology is an almost normal but not a π-normal space.

Mathematics Subject Classification: 54D15; 54B10; 54D70; 54D10

Keywords: closed domain, π-closed, normal, π-normal, quasi-normal, almost normal and semi-normal

1 Introduction and Preliminary

Throughout this paper, a space X always means a topological space on which no separation axioms are assumed, unless explicitly stated. We will denote an ordered pair by $\langle x, y\rangle$, the set of positive integers by $\mathbb{N}$, the power set of A by $\mathcal{P}(A)$ and the set of real numbers by $\mathbb{R}$. For a subset A of a space X, $\overline{A}$, $\text{int}(A)$ and $X \setminus A$ denote to the closure, the interior and the complement of A in X, respectively.

Now, we need to recall the following definitions.

Definition 1.1 *A subset A of a space X is called a closed domain (resp. an open domain) if $A = \overline{\text{int}(A)}$ (resp. $A = \text{int}(\overline{A})$), [5].*

Definition 1.2 *A subset A of a space X is called a π-closed (resp. π-open) if it is a finite intersection of closed domain subsets (resp. a finite union of open domain subsets), [11].*

[1]Corresponding author's e-mail: sthabit1975@gmail.com
[2]e-mail: hailiza@cs.usm.my

Definition 1.3 *Two sets A and B of a space X are said to be separated if there exist disjoint open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$, see [1, 2, 6].*

Definition 1.4 *A space X is called a first countable if every point $x \in X$ has a countable local base $\mathcal{B} = \{B_n : n \in \mathbb{N}\}$, see [2, 6].*

Definition 1.5 *A set A of a space X is called a G_δ-set of X if it is a countable intersection of open subsets of X, see [2].*

Definition 1.6 *A topological space X is called a mildly normal, [8], (resp. quasi-normal, [11]) if any two disjoint closed domain (resp. π-closed) subsets A and B of X can be separated.*

Definition 1.7 *A space X is called an almost normal, [9], (resp. a π-normal, [4]) if any disjoint closed subsets A and B of X, one of which is closed domain (resp. π-closed), can be separated.*

Definition 1.8 *A space X is said to be a semi-normal if for any closed set A and every open set B with $A \subseteq B$, there exists an open set U such that $A \subseteq U \subseteq \operatorname{int}(\overline{U}) \subseteq B$, see [9].*

Clearly that:

$$\text{normal} \Longrightarrow \pi\text{-normal} \Longrightarrow \text{almost normal} \Longrightarrow \text{mildly normal}$$

$$\text{normal} \Longrightarrow \pi\text{-normal} \Longrightarrow \text{quasi-normal} \Longrightarrow \text{mildly normal}$$

Non of the above implications is reversible.

One of the problems that introduced by Kalantan in 2008, see [4], was "*Is there a Tychonoff space, which is an almost normal and not a π-normal?.*" We presented some characterizations and properties on π-normality in [7]. In this paper, we show that the Rational Sequence topology, which is a Tychonoff space, is an almost normal and not a π-normal. First, we need to recall the following definitions and theorems which are in [3]: Two sets A and B are said to be *equipotent* and write $A \sim B$, if there exists a one-to-one function f from A onto B. If A and B be two sets, then we write $|A| \leq |B|$ and say that the cardinality of A is less than or equal to the cardinality of B, if there exist a one-to-one function $f : A \to B$. If A be any set, then $|A| < |\mathcal{P}(A)|$, (Cantor Theorem). If X is a separable space and has an uncountable closed relatively discrete subset C, then X is not normal, (Jones' Lemma), see [1, 2, 6].

2 Main Results

First, we recall the definition of the Rational Sequence topology:

Definition 2.1 *Let $X = \mathbb{R}$. For each $x \in \mathbb{P}$, where $\mathbb{P}$ is the irrational numbers, fix a sequence $\{x_n\}_{n\in\mathbb{N}} \subset \mathbb{Q}$, such that $x_n \longrightarrow x$, where the convergency is taken in $(\mathbb{R},\mathcal{U})$, $\mathbb{R}$ with its usual topology. Let $A_n(x)$ denote the n^{th}-tail of the sequence, where $A_n(x) = \{x_j : j \geq n\}$. For each $x \in \mathbb{P}$, let $\mathcal{B}(x) = \{U_n(x) : n \in \mathbb{N}\}$, where $U_n(x) = A_n(x) \cup \{x\}$. For each $x \in \mathbb{Q}$, let $\mathcal{B}(x) = \{\{x\}\}$. Then $\{\mathcal{B}(x)\}_{x\in\mathbb{R}}$ is a neighborhood system. The unique topology on $\mathbb{R}$ generated by $\{\mathcal{B}(x)\}_{x\in\mathbb{R}}$ is called the Rational Sequence topology on $\mathbb{R}$ and denoted by $\mathcal{RS}$.*

In this space, we observe that X is a Tychonoff, first countable, not normal and separable. Any singleton $\{x\}$ is π-closed. $\mathbb{Q}$ is an open dense subset of X. Also, any subset of $\mathbb{Q}$ is an open subset of X. $\mathbb{P}$ is an uncountable closed discrete subspace of X. For more information about this space, see [10].

Now, we prove the following result.

Proposition 2.2 *The Rational sequence topology is an almost normal.*

Proof: Let A and B be any two disjoint closed sets in X such that A is closed domain. Since A is a closed domain, then $A = \overline{\text{int}(A)}$. So, A can not be in $\mathbb{P}$ (i.e $A \not\subseteq \mathbb{P}$). In fact, if $A \subseteq \mathbb{P}$, then $\overline{\text{int}(A)} = \emptyset \neq A$. Therefore, there are two cases about A, which are $A \subseteq \mathbb{Q}$ or $A \cap \mathbb{Q} \neq \emptyset \neq A \cap \mathbb{P}$. For each case about A, there are three subcases about B, which are $B \subseteq \mathbb{Q}$ or $B \subseteq \mathbb{P}$ or $B \cap \mathbb{Q} \neq \emptyset \neq B \cap \mathbb{P}$. Now, we show that A and B can be separated for each case.

Case 1. Let $A \subseteq \mathbb{Q}$.

Subcase a1. Let $B \subseteq \mathbb{Q}$.

Then, A and B are disjoint clopen (closed and open) subsets. Hence, A and B can be separated.

Subcase a2. Let $B \subseteq \mathbb{P}$.

Since $A \cap B = \emptyset$ and A is clopen. Then for each $x \in B$, we have $x \notin A$. By regularity of X, there exists an open set U_x such that $x \in U_x$ and $U_x \cap A = \emptyset$. Thus, $B \subseteq \bigcup_{x\in B} U_x$. Take $U = \bigcup_{x\in B} U_x$, which is an open set in X such that $B \subseteq U$ and $U \cap A = \emptyset$. Hence, A and B can be separated.

Subcase a3. Let $B \cap \mathbb{Q} \neq \emptyset \neq B \cap \mathbb{P}$.

Then, $B \cap \mathbb{Q}$ is an open and $B \cap \mathbb{P}$ is a closed. Thus, we have A and $B \cap \mathbb{Q}$ are disjoint open subsets. Put $U_1 = A$ and $V_1 = B \cap \mathbb{Q}$. So, we can write

$$A \subseteq U_1 \ , \ B \cap \mathbb{Q} \subseteq V_1 \ \text{ and } \ U_1 \cap V_1 = \emptyset \tag{1}$$

Since $A \cap (B \cap \mathbb{P}) = \emptyset$ and A is clopen , then by Subcase a2., there is an open set V_2 such that

$$B \cap \mathbb{P} \subseteq V_2 \ \text{ and } \ A \cap V_2 = \emptyset \tag{2}$$

From (1) and (2), we have

$$B \subseteq V_1 \cup V_2 \ \text{ and } \ A \cap (V_1 \cup V_2) = \emptyset$$

Now, put $U = A$ and $V = V_1 \cup V_2$. Then, U and V are open sets of X such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \emptyset$. Hence, A and B can be separated.
Case 2. Suppose $A \cap \mathbb{Q} \neq \emptyset \neq A \cap \mathbb{P}$.
Subcase b1. Let $B \subseteq \mathbb{Q}$.
Then, the open set $A \cap \mathbb{Q}$ is disjoint from the clopen set B. Thus, they can be separated by putting $U_1 = A \cap \mathbb{Q}$ and $V_1 = B$. So, we can write

$$A \cap \mathbb{Q} \subseteq U_1 \ , \ B \subseteq V_1 \ \text{ and } \ U_1 \cap V_1 = \emptyset \tag{3}$$

Also, the closed set $A \cap \mathbb{P}$ is disjoint from the clopen set B. Then by subcase a2., there exists an open set U_2 such that

$$A \cap \mathbb{P} \subseteq U_2 \quad \text{and } \ U_2 \cap B = \emptyset \tag{4}$$

From (3) and (4), we have

$$A \subseteq U_1 \cup U_2 \ \text{ and } \ B \cap (U_1 \cup U_2) = \emptyset$$

Put $U = U_1 \cup U_2$ and $V = B$. Thus, there exist open sets U and V of X such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \emptyset$. Hence, A and B can be separated.
Subcase b2. Let $B \subseteq \mathbb{P}$.
Then, $(A \cap \mathbb{Q}) \cap B = \emptyset$, where $A \cap \mathbb{Q}$ is open. Since A is closed domain and $\mathbb{Q}$ is an open dense subset of X, then we have $\overline{A \cap \mathbb{Q}} = A$. Now, for each $x \in B$, we have $x \notin A = \overline{A \cap \mathbb{Q}}$. Therefore, for each $x \in B$, there exists a basic open neighborhood V_x of x such that $V_x \cap (A \cap \mathbb{Q}) = \emptyset$. Now, we have $B \subseteq \bigcup_{x \in B} V_x$. Let $V = \bigcup_{x \in B} V_x$. Then, V is an open set of X such that $B \subseteq V$ and $V \cap (A \cap \mathbb{Q}) = \emptyset$. Since $A \cap \mathbb{Q}$ is an open, then we have $\overline{V} \cap (A \cap \mathbb{Q}) = \emptyset$ and $V \cap \overline{A \cap \mathbb{Q}} = \emptyset$. Therefore, there exists an open set V of X such that

$$B \subseteq V \ \ , \ \ \overline{V} \cap (A \cap \mathbb{Q}) = \emptyset \quad \text{and} \quad V \cap A = \emptyset \tag{5}$$

Claim: $A \cap \overline{V} = \emptyset$.
Suppose that $\overline{V} \cap A \neq \emptyset$. Then, there exists an element $y \in X$ such that $y \in \overline{V}$ and $y \in A$. By (5), we have $y \notin A \cap \mathbb{Q}$ and $y \notin V$. Now, since $y \in A = \overline{A \cap \mathbb{Q}}$, then for each basic open neighborhood U_y of y we have

$$U_y \cap (A \cap \mathbb{Q}) \neq \emptyset \tag{6}$$

Since X is a first countable, see [10], and $y \in \overline{V} = \overline{V \cap \mathbb{Q}}$, then there exists a sequence $\{y_n : n \in \mathbb{N}\}$ of points of $V \cap \mathbb{Q} \subseteq V$ such that $y_n \longrightarrow y$. Let $D_y = \{y_n : n \in \mathbb{N}\} \cup \{y\}$. Then, D_y is an open neighborhood of y. By (6), we have $D_y \cap (A \cap \mathbb{Q}) \neq \emptyset$. Since $y \notin A \cap \mathbb{Q}$, then there exists an element y_m for some $m \in \mathbb{N}$ such that $y_m \in A \cap \mathbb{Q}$. But $y_m \in V$. Hence $V \cap (A \cap \mathbb{Q}) \neq \emptyset$, which is a contradiction as by (5), $V \cap (A \cap \mathbb{Q}) = \emptyset$. Therefore, $A \cap \overline{V} = \emptyset$.
Now, $A \cap \overline{V} = \emptyset$. This implies that $A \subseteq X \setminus \overline{V}$. Put $U = X \setminus \overline{V}$. Then, U and V are disjoint open sets of X such that $A \subseteq U$ and $B \subseteq V$. Hence, A and B can be separated.
Subcase b3. Let $B \cap \mathbb{P} \neq \emptyset \neq B \cap \mathbb{Q}$.
Since $A \cap B = \emptyset$, then $A \cap (B \cap \mathbb{P}) = \emptyset$, where $B \cap \mathbb{P}$ is closed set in X. Then by Subcase b2, there exist open sets U_1 and V_1 such that

$$A \subseteq U_1 \quad , \quad B \cap \mathbb{P} \subseteq V_1 \text{ and } U_1 \cap V_1 = \emptyset \tag{7}$$

Also, Let $V_2 = B \cap \mathbb{Q}$. Then, V_2 is an open set of X such that $A \cap V_2 = \emptyset$ and $A \cap \overline{V_2} = \emptyset$. This implies that $A \subseteq X \setminus \overline{V_2}$. Now, $A \subseteq U_1 \cap X \setminus \overline{V_2}$ and $B \subseteq V_1 \cup V_2$. Put $U = U_1 \cap X \setminus \overline{V_2}$ and $V = V_1 \cup V_2$. Then, U and V are disjoint open sets of X such that $A \subseteq U$ and $B \subseteq V$. Hence, A and B can be separated. For each case, we have shown that A and B can be separated. Therefore, X is almost normal space.

In view of the facts that every almost normal, semi-normal space is normal and that the Rational Sequence topology is not normal, we have the following corollaries.

Corollary 2.3 *The Rational Sequence topology is not semi-normal.*

Corollary 2.4 *Every π-normal, semi-normal space is normal.*

Observe that the Rational Sequence topology is an example of an almost normal, Tychonoff space but not semi-normal.

Now, we show that the Rational Sequence topology is not quasi-normal. First, we give the following lemmas.

Lemma 2.5 *In the Rational Sequence topology, every closed subset $A \subseteq \mathbb{P}$ is a G_δ-set.*

Proof: Let $A \subseteq \mathbb{P}$ and let $x \in A$. Since X is a first countable and T_1-space, then $\{x\}$ is a G_δ-set of X, see [2]. Therefore, $\{x\}$ has a decreasing sequence $\{U_n(x) : n \in \mathbb{N}\}$ of open sets of X such that $\{x\} = \bigcap_{n\in\mathbb{N}} U_n(x)$. So for each n, $x \in U_n(x)$ and $A \subseteq \bigcup_{x\in A} U_n(x)$. Therefore, $A = \bigcup_{x\in A}(\bigcap_{n\in\mathbb{N}} U_n(x)) = \bigcap_{n\in\mathbb{N}}(\bigcup_{x\in A} U_n(x))$. Put $U_n(A) = \bigcup_{x\in A} U_n(x)$. Then $\{U_n(A) : n \in \mathbb{N}\}$ is a decreasing sequence of open sets of X such that $A = \bigcap_{n\in\mathbb{N}} U_n(A)$. Hence, A is a G_δ-set.

Lemma 2.6 *In the Rational Sequence topology, the set $\mathbb{P}$ is a π-closed subset of X.*

Proof: By the Lemma 2.5, we have $\mathbb{P}$ is a G_δ-set of X. Then, there exists a decreasing sequence $\{U_n : n \in \mathbb{N}\}$ of open sets of X such that

$$\mathbb{P} = \bigcap_{n\in\mathbb{N}} U_n \subseteq \bigcap_{n\in\mathbb{N}} \overline{U_n}$$

First, we show that $\bigcap_{n\in\mathbb{N}} \overline{U_n} \subseteq \mathbb{P}$. For that, let $y \in \bigcap_{n\in\mathbb{N}} \overline{U_n}$. This implies that $y \in \overline{U_n}$ for each $n \in \mathbb{N}$. Then either $y \in \mathbb{Q}$ or $y \in \mathbb{P}$. If $y \in \mathbb{Q}$, then $\{y\}$ is a basic open neighborhood of y and so $\{y\} \cap U_n \neq \emptyset$, for each $n \in \mathbb{N}$. Then, $y \in U_n$ for each $n \in \mathbb{N}$. Therefore, $y \in \bigcap_{n\in\mathbb{N}} U_n = \mathbb{P}$. So $y \in \mathbb{P}$, which is a contradiction as $y \in \mathbb{Q}$. Hence, $y \notin \mathbb{Q}$ and therefore $y \in \mathbb{P}$. Since y was arbitrary, then we have

$$\bigcap_{n\in\mathbb{N}} \overline{U_n} \subseteq \mathbb{P} = \bigcap_{n\in\mathbb{N}} U_n$$

Therefore, we have $\mathbb{P} = \bigcap_{n\in\mathbb{N}} U_n = \bigcap_{n\in\mathbb{N}} \overline{U_n}$.

Now, for each $n \in \mathbb{N}$, let $A_n = U_{4n-3} \setminus \overline{U_{4n-2}}$ and $B_n = U_{4n-1} \setminus \overline{U_{4n}}$. Then A_n and B_n are disjoint open sets of X for each n. Furthermore, $\overline{A_n} \cap \overline{B_n} = \emptyset$ for each $n \in \mathbb{N}$. Now, let $A = \bigcup_{n\in\mathbb{N}} A_n$ and $B = \bigcup_{n\in\mathbb{N}} B_n$. Then, A and B are open sets of X such that

$$\overline{A} = \overline{\bigcup_{n\in\mathbb{N}} A_n} = (\bigcup_{n\in\mathbb{N}} \overline{A_n}) \bigcup \mathbb{P} = (\bigcup_{n\in\mathbb{N}} A_n) \bigcup \mathbb{P}$$

and

$$\overline{B} = \overline{\bigcup_{n\in\mathbb{N}} B_n} = (\bigcup_{n\in\mathbb{N}} \overline{B_n}) \bigcup \mathbb{P} = (\bigcup_{n\in\mathbb{N}} B_n) \bigcup \mathbb{P}$$

Then, $\overline{A}$ and $\overline{B}$ are closed domain sets of X and $\overline{A} \cap \overline{B} = \mathbb{P}$. Hence, $\mathbb{P}$ is π-closed. Therefore, $\mathbb{P}$ is uncountable π-closed discrete subspace of X.

Lemma 2.7 *In the Rational sequence topology, for any closed subset $A \subseteq \mathbb{P}$, there exists an open set U of X such that $A = \overline{U} \cap \mathbb{P}$.*

Proof: Let A be any non-empty closed subset of X such that $A \subseteq \mathbb{P}$. Then, for each $x \in A$, we have $x \in \mathbb{P}$. Thus, there exists a sequence $A(x) = \{x_n : n \in \mathbb{N}\} \subset \mathbb{Q}$ such that $x_n \longrightarrow x$ for each $x \in A$. Suppose $U(x) = A(x) \cup \{x\}$ be a basic open neighborhood of x. Then, we have

$$A \subseteq \bigcup_{x\in A} U(x) = (\bigcup_{x\in A} A(x)) \bigcup A$$

Now, let $U = \bigcup_{x\in A} U(x)$ and $W = \bigcup_{x\in A} A(x)$.Then, U and W are open sets of X such that $W \subseteq \mathbb{Q}$, $A \subseteq \overline{W}$ and $U = W \cup A$. Clearly that $A \subseteq U \subseteq \overline{U}$. Therefore, we have

$$A \subseteq \overline{U} \cap \mathbb{P}$$

Now, we need to show that $\overline{U} \cap \mathbb{P} \subseteq A$.
Let $y \in \overline{U} \cap \mathbb{P}$. Then $y \in \overline{U}$ and $y \in \mathbb{P}$. Since $y \in \mathbb{P}$, then there exists a sequence $\{y_n : n \in \mathbb{N}\} \subset \mathbb{Q}$ such that $y_n \longrightarrow y$. Now, for each $n \in \mathbb{N}$, let $V_n = \{y_m : m \geq n\} \cup \{y\}$. Then, V_n is a basic open neighborhood of y for each $n \in \mathbb{N}$. Thus, V_n is clopen for each n, see [10]. We observe that $V_1 \supseteq V_2 \supseteq V_3 \supseteq ... \supseteq V_n \supseteq ....$ Also, $\{y\} = \bigcap_{n\in\mathbb{N}} V_n = \bigcap_{n\in\mathbb{N}} \overline{V_n}$. Since $y \in \overline{U}$, then we have $V_n \cap U \neq \emptyset$ for each $n \in \mathbb{N}$.
Claim: $y \in U$.
Suppose that $y \notin U$, then $y \notin W$ and $y \notin A$. Since $y \notin A$, $V_n \setminus \{y\} \cap U \neq \emptyset$, $V_n \setminus \{y\} \subset \mathbb{Q}$ and $U = W \cup A$, then we have $V_n \cap W \neq \emptyset$ for each $n \in \mathbb{N}$. Therefore, there is an element $z \in W$ such that $z \in V_n = \overline{V_n}$ for each $n \in \mathbb{N}$. This implies that $z \in \bigcap_{n\in\mathbb{N}} \overline{V_n} = \{y\}$. Hence $z = y$. Therefore, $y \in W$, which is a contradiction as $y \notin W$. Therefore, $y \in U$.
Now, since $y \in U$, $U = W \cup A$ and $y \notin W$, then we have $y \in A$. Since y was arbitrary, then we have $\overline{U} \cap \mathbb{P} \subseteq A$. Therefore, $A = \overline{U} \cap \mathbb{P}$. Hence for any closed set $A \subseteq \mathbb{P}$, there exists an open set U of X such that $A = \overline{U} \cap \mathbb{P}$.

Since $\mathbb{P}$ and $\overline{U}$ are π-closed and the intersection of two π-closed sets is π-closed, then we have the following corollary:

Corollary 2.8 *In the Rational sequence topology, any closed set $A \subseteq \mathbb{P}$ is π-closed subset of X.*

The following result is analogous to the Jones' Lemma for normal spaces, see [2, 6].

Theorem 2.9 *If X is an infinite, separable space with a dense subset D and has an uncountable closed relatively discrete subset C such that $|\mathcal{P}(D)| \leq |C|$ and every subset of C is π-closed subset of X, then X is not quasi-normal.*

Proof: We need to show that X is not quasi-normal.
For that, suppose X is quasi-normal. For each $A \in \mathcal{P}(C)$, where $A \neq \emptyset$ and by the Corollay 2.8, we have A and $C \setminus A$ are π-closed subsets and $A \cap C \setminus A = \emptyset$. Since X is quasi-normal, then there exist two disjoint open subsets U_A and V_A of X such that $A \subseteq U_A$ and $C \setminus A \subseteq V_A$. Let $D_A = U_A \cap D$, then $D_A \neq \emptyset$. Now, suppose that A, $B \in \mathcal{P}(C)$ and $A \neq B$. We may assume that $A \setminus B \neq \emptyset$. Let U_A and V_A are two disjoint open sets of X such that

$$A \subseteq U_A, \;\; C \setminus A \subseteq V_A$$

and let U_B and V_B are two disjoint open sets of X such that

$$B \subseteq U_B, \;\; C \setminus B \subseteq V_B$$

Since $A \setminus B \neq \emptyset$, then $A \cap C \setminus B \neq \emptyset$. Thus, we have $U_A \cap V_B \neq \emptyset$, which is an open set of X. Since D is dense, then $U_A \cap V_B \cap D \neq \emptyset$. Now, we have

$$U_A \cap V_B \cap D \subseteq U_A \cap D = D_A$$

and

$$U_A \cap V_B \cap D \nsubseteq U_B \cap D = D_B$$

Therefore, $D_A \neq D_B$. Thus, for any two distinct subsets A, $B \in \mathcal{P}(C)$, there exist two distinct subsets D_A and D_B of D. Then, we have

$$|\mathcal{P}(C)| \leq |\mathcal{P}(D)| \tag{8}$$

Since $C \not\sim \mathcal{P}(C)$ (i.e. $|C| < |\mathcal{P}(C)|$), then by (8), we have $C \not\sim \mathcal{P}(D)$ and $|C| < |\mathcal{P}(D)|$, which is not the case $|\mathcal{P}(D)| \leq |C|$. Hence, X is not quasi-normal.

The Rational sequence topology satisfies the conditions of the Theorem 2.9 and since every π-normal space is quasi-normal, we have the following corollaries:

Corollary 2.10 *The Rational sequence topology is not quasi-normal.*

Corollary 2.11 *The Rational sequence topology is not π-normal.*

We have observed that the Rational sequence topology is an example of a Tychonoff space which is an almost normal but not a π-normal.

3 Conclusion

We proved that the Rational Sequence topology is an almost normal but not a π-normal. We observed that it is not semi-normal and not quasi-normal. So, this space is an example of a Tychonoff space which is an almost normal but not a π-normal (resp. not a quasi-normal). Also, we presented some properties about it.

ACKNOWLEDGEMENTS. The authors would like to thank University Sains Malaysia, Research University Grant and USM Fellowship for funding and supporting this research.

References

[1] J. Dugundji, Topology, *Allyn and Bacon, Inc,* 1966.

[2] R. Engelking, General Topology, (*PWN,* Warszawa, 1977).

[3] W. Just and M. Weese, Discovering Modern Set Theory I, *American Mathematical Society,* Graduate Studies in Mathematics, Volume **8**, 1995.

[4] L. Kalantan, π-normal topological spaces, *Filomat,* volume **22-1** (2008), pp. 173-181.

[5] C. Kuratowski, Topology I, 4th. ed., in French,*Hafner,* New york, 1958.

[6] C. Patty, foundation of topology, *PWS-KENT Publishing Company,* Boston, 1993.

[7] Sadeq Ali Saad Thabit and Hailiza Kamarulhaili, On π-Normality, Weak Regularity and the Product of Topological Spaces, *European Journal of Scientific Research,* ISSN 1450-216X, Vol.**51**, No.**1** (2011), pp. 29-39.

[8] M. K. Singal and A. R. Singal, Mildly normal spaces, *Kyungpook Math. J.,* volume **13-1** (1973), pp 27-31.

[9] M. K. Singal and S. Arya, Almost normal and Almost completely regular spaces, *Glasnik Mate.,* vol. **5(25)** (1970), pp 141-152.

[10] Lynn A. Steen and J. Arthur Seebach, Counterexamples in Topology, *Dover Publications, INC,* New York, 1970.

[11] V. Zaitsev, On certain classes of topological spaces and their bicompactifications, *Dokl. Akad. Nauk SSSR,* **178**(1968), 778-779.

Received: October, 2011