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Homomorphisms and IFSM's of Quotient Modules

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Abstract

The concept of an intuitionistic fuzzy set was introduced by Krassimir T. Atanassov in 1986. As an application of this to algebra, in this article we study some properties of intuitionistic fuzzy submodules (IFSM's) of an R-Module and the characteristics of the image & the pre-image of an intuitionistic fuzzy submodule under an R-Module homomorphism. Also we introduce two methods of constructing IFSM's of a quotient module.

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1 Introduction.

As a generalization of a fuzzy set, the concept of an intuitionistic fuzzy set was introduced by K.T. Atanassov [1, 2]. Using this idea, B. Davvas etal.[5] established the intuitionistic fuzzification of the concept of submodules of an R-module. In this paper, in section 2 we give the essential preliminaries and in section 3 we study about the characteristics of the image and the pre-image of an IFSM under an R-module homomorphism. In section 4 we introduce two methods of constructing IFSM's of a quotient module M/N.

Throughout this paper, we denote by I the unit interval $[0, 1]$, by R a commutative ring with unity 1 and by M a unitary R-module. $\vee$ denotes the maximum, and $\wedge$ the minimum in the unit interval $[0, 1]$.

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2 Preliminaries

In this section we give some basic definitions and results which are used in the sequel. For knowledge regarding modules and fuzzy modules one may refer the books by Hungerford [6] and Mordeson & Malik [9] respectively.

2.1. Definition ([1]). An intuitionistic fuzzy set (in short IFS) A in a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x))/x \in X\}$ where the functions $\mu_A : X \to I$ and $\nu_A : X \to I$ denote respectively the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

For the sake of simplicity, we will denote the set of all IFS's in X as IFS(X).

2.2. Definition ([1]). Let X be a non-empty set and $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B)$ be IFS's in X. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
2. $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$
3. $A^C = (\nu_A, \mu_A)$
4. $A \cap B = \{(x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)) : x \in X\}$
5. $A \cup B = \{(x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)) : x \in X\}$

2.3. Definition ([10]). A fuzzy set μ in M is called a fuzzy submodule of M if for every $x, y \in M$ and $r \in R$, the following conditions are satisfied

1. $\mu(0) = 1$
2. $\mu(x + y) \geq \mu(x) \wedge \mu(y)$
3. $\mu(rx) \geq \mu(x)$

2.4. Definition ([5]). Let M be a module over a ring R. An IFS $A = (\mu_A, \nu_A)$ in M is called an intuitionistic fuzzy submodule (IFSM) of M if

1. $\mu_A(0) = 1$ and $\nu_A(0) = 0$
2. $\mu_A(x + y) \geq \mu_A(x) \wedge \mu_A(y)\ \forall x, y \in M$
3. $\nu_A(x + y) \leq \nu_A(x) \vee \nu_A(y)\ \forall x, y \in M$
4. $\mu_A(rx) \geq \mu_A(x)\ \forall x \in M, \forall r \in R$
5. $\nu_A(rx) \leq \nu_A(x)\ \forall x \in M, \forall r \in R$

Remark. By saying that $A = (\mu_A, \nu_A)$ is an an intuitionistic fuzzy module (IFM) we mean that $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy submodule of some R module M.

2.5. Definition ([3]). Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two IFS's in M. Then we define their sum $A + B$ as the IFS $A + B = \{(x, \mu_{A+B}(x), \nu_{A+B}(x)) : x \in M\}$ where for each $x \in M$,

$$\begin{aligned} \mu_{A+B}(x) &= \vee\{\mu_A(y) \wedge \mu_B(z) : y, z \in M, x = y + z\}, \text{ and} \\ \nu_{A+B}(x) &= \wedge\{\nu_A(y) \vee \nu_B(z) : y, z \in M, x = y + z\} \end{aligned}$$

2.6. Definition ([7]). For an IFS $A = (\mu_A, \nu_A)$ in M and for any $r \in R$, we define the IFS $rA = (\mu_{rA}, \nu_{rA})$ in M as $rA = \{(x, \mu_{rA}(x), \nu_{rA}(x)) : x \in M\}$ where for each $x \in M$, $\mu_{rA}(x) = \vee\{\mu_A(y) : y \in M, x = ry\}$, and $\nu_{rA}(x) = \wedge\{\nu_A(y) : y \in M, x = ry\}$.

3 Homomorphisms and IFSM's

In this section we study about the nature of the image and the pre-image of an IFSM under an R-module homomorphism.

3.1. Definition ([4]). Let X and Y be two nonempty sets and $f : X \to Y$ be a function. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be IFS's in X and Y respectively. Then the image of A under f, denoted by $f(A)$, is an IFS in Y defined as $f(A) = \{(y, \mu_{f(A)}(y), \nu_{f(A)}(y)) : y \in Y\}$, where

$$\mu_{f(A)}(y) = \begin{cases} \vee\{\mu_A(x) : x \in X, f(x) = y\} & \text{if } f^{-1}(y) \neq \phi \\ 0 \quad \text{if } f^{-1}(y) = \phi \end{cases}$$

and

$$\nu_{f(A)}(y) = \begin{cases} \wedge\{\nu_A(x) : x \in X, f(x) = y\} & \text{if } f^{-1}(y) \neq \phi \\ 1 \quad \text{if } f^{-1}(y) = \phi \end{cases}$$

The pre-image of B under f, denoted by $f^{-1}(B)$, is an IFS in X defined as
$f^{-1}(B) = \{(x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x)) : x \in X\}$, where

$$\mu_{f^{-1}(B)}(x) = \mu_B f(x) \quad \text{and} \quad \nu_{f^{-1}(B)}(x) = \nu_B f(x)$$

3.2. Definition ([6]). If M and N are modules over a ring R, a function $f : M \to N$ is said to be a homomorphism of M into N if

$$f(u + v) = f(u) + f(v) \text{ and } f(ru) = rf(u) \text{ , } \forall u, v \in M \text{ and } \forall r \in R.$$

3.3. Theorem. *Let M and N be two R-modules and f be a homomorphism of M into N. Let $r, s \in R$ and $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B)$ be two IFS's in M. Then,*

1. $f(A + B) = f(A) + f(B)$
2. $f(rA) = rf(A)$
3. $f(rA + sB) = rf(A) + sf(B)$

Proof. (1). We have $f(A + B) = \{(y, \mu_{f(A+B)}(y), \nu_{f(A+B)}(y)) : y \in N\}$ and $f(A) + f(B) = \{(y, \mu_{(f(A)+f(B))}(y), \nu_{(f(A)+f(B))}(y)) : y \in N\}$. Let $y \in N$.

If $f^{-1}(y) = \phi$ then $\mu_{f(A+B)}(y) = 0$. Also,

$\mu_{(f(A)+f(B))}(y) = \vee\{\mu_{f(A)}(y_1) \wedge \mu_{f(B)}(y_2) : y_1, y_2 \in N, y = y_1 + y_2\} = 0$, since $f^{-1}(y_1) = \phi$ or $f^{-1}(y_2) = \phi$ as $f^{-1}(y) = \phi$. Thus $\mu_{f(A+B)}(y) = \mu_{(f(A)+f(B))}(y)$ if $f^{-1}(y) = \phi$.

If $f^{-1}(y) \neq \phi$, then also we have,

$$\begin{aligned}
\mu_{f(A+B)}(y) &= \vee\{\mu_{A+B}(x) : x \in M, y = f(x)\} \\
&= \vee\{\vee\{\mu_A(x_1) \wedge \mu_B(x_2) : x_1, x_2 \in M, x = x_1 + x_2\} : x \in M, y = f(x)\} \\
&= \vee\{\vee\{\mu_A(x_1) \wedge \mu_B(x_2) : x_1, x_2 \in M, z_1 = f(x_1), z_2 = f(x_2)\} \\
&\qquad : z_1, z_1 \in N, y = z_1 + z_2\} \\
&= \vee\{(\vee\{\mu_A(x_1) : x_1 \in M, z_1 = f(x_1)\}) \wedge (\vee\{\mu_B(x_2) : x_2 \in M, z_2 = f(x_2)\}) \\
&\qquad : z_1, z_1 \in N, y = z_1 + z_2\} \\
&= \vee\{\mu_{f(A)}(z_1) \wedge \mu_{f(B)}(z_2) : z_1, z_1 \in N, y = z_1 + z_2\} \\
&= \mu_{(f(A)+f(B))}(y)
\end{aligned}$$

Thus in any case we get $\mu_{f(A+B)}(y) = \mu_{(f(A)+f(B))}(y)$.

Similarly we can obtain $\nu_{f(A+B)}(y) = \nu_{(f(A)+f(B))}(y)\ \forall y \in N$. Hence we have $f(A + B) = f(A) + f(B)$, which proves (1).

(2). Now,

$$f(rA) = \{(y, \mu_{f(rA)}(y), \nu_{f(rA)}(y)) : y \in N\}, \text{ and}$$
$$rf(A) = \{(y, \mu_{rf(A)}(y), \nu_{rf(A)}(y)) : y \in N\}.$$

Let $y \in N$. Then if $f^{-1}(y) = \phi$ then obviously $\mu_{f(rA)}(y) = 0$. Also,

$\mu_{rf(A)}(y) = \vee\{\mu_{f(A)}(v) : v \in N, y = rv\} = 0$, since $f^{-1}(v) = \phi$ as $f^{-1}(rv) = f^{-1}(y) = \phi$. Thus $\mu_{f(rA)}(y) = \mu_{rf(A)}(y)$ if $f^{-1}(y) = \phi$. Now if $f^{-1}(y) \neq \phi$, then

$$\begin{aligned}
\mu_{f(rA)}(y) &= \vee\{\mu_{rA}(x) : x \in M, y = f(x)\} \\
&= \vee\{\vee\{\mu_A(u) : u \in M, x = ru\} : x \in M, y = f(x)\} \\
&= \vee\{\vee\{\mu_A(u) : u \in M, v = f(u)\} : v \in N, y = rv\} \\
&= \vee\{\mu_{f(A)}(v) : v \in N, y = rv\} \\
&= \mu_{rf(A)}(y)
\end{aligned}$$

Thus we get $\mu_{f(rA)}(y) = \mu_{rf(A)}(y)\ \forall\ y \in N$. Similarly we get $\nu_{f(rA)}(y) = \nu_{rf(A)}(y)\ \forall\, y \in N$.

(3). This follows from (1) and (2).

3.4. Theorem. *Let M and N be two R-modules and f be a homomorphism of M into N. If $A = (\mu_A, \nu_A)$ is an IFSM of M, then $f(A)$ is an IFSM of N.*

Proof. We have $f(A) = \{(y, \mu_{f(A)}(y), \nu_{f(A)}(y)) : y \in Y\}$. First of all we note that

$$\begin{aligned} \mu_{f(A)}(0) &= \vee\{\mu_A(x) : x \in M, f(x) = 0\} &= \mu_A(0) = 1; \\ \nu_{f(A)}(0) &= \wedge\{\nu_A(x) : x \in M, f(x) = 0\} &= \nu_A(0) = 0 \end{aligned}$$

To prove the second condition of an IFSM, let $y_1, y_2 \in N$. If $f^{-1}(y_1) = \phi$ or $f^{-1}(y_2) = \phi$, then correspondingly $\mu_{f(A)}(y_1) = 0$ or $\mu_{f(A)}(y_2) = 0$. So in this case $\mu_{f(A)}(y_1) \wedge \mu_{f(A)}(y_2) = 0$ and so $\mu_{f(A)}(y_1 + y_2) \geq \mu_{f(A)}(y_1) \wedge \mu_{f(A)}(y_2)$.
Now if $f^{-1}(y_1) \neq \phi \neq f^{-1}(y_2)$, then

$$\begin{aligned} \mu_{f(A)}(y_1 + y_2) &= \vee\{\mu_A(x) : x \in M, y_1 + y_2 = f(x)\} \\ &= \vee\{\mu_A(x_1 + x_2) : x_1, x_2 \in M, y_1 + y_2 = f(x_1 + x_2)\} \\ &\geq \vee\{\mu_A(x_1 + x_2) : x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\} \\ &= \vee\{\mu_A(x_1 + x_2) : x_1, x_2 \in M, y_1 = f(x_1), y_2 = f(x_2)\} \\ &\geq \vee\{\mu_A(x_1) \wedge \mu_A(x_2) : x_1, x_2 \in M, y_1 = f(x_1), y_2 = f(x_2)\} \\ &\geq (\vee\{\mu_A(x_1) : x_1 \in M, y_1 = f(x_1)\}) \wedge (\vee\{\mu_A(x_2) : x_2 \in M, y_2 = f(x_2)\}) \\ &= \mu_{f(A)}(y_1) \wedge \mu_{f(A)}(y_2) \end{aligned}$$

Thus $\mu_{f(A)}(y_1 + y_2) \geq \mu_{f(A)}(y_1) \wedge \mu_{f(A)}(y_2) \ \ \forall y_1, y_2 \ \in N$.

Similarly we can show that $\nu_{f(A)}(y_1 + y_2) \leq \nu_{f(A)}(y_1) \vee \nu_{f(A)}(y_2) \ \ \forall y_1, y_2 \ \in N$.

Now to prove the third condition of an IFSM, let $y \in N$. If $f^{-1}(y) = \phi$, then $\mu_{f(A)}(y) = 0$. So in this case $\mu_{f(A)}(ry) \geq \mu_{f(A)}(y)$.
Now if $f^{-1}(y) \neq \phi$, then

$$\begin{aligned} \mu_{f(A)}(ry) &= \vee\{\mu_A(x) : x \in M, ry = f(x)\} \\ &\geq \vee\{\mu_A(rx_1) : rx_1 \in M, ry = f(rx_1)\} \\ &\geq \vee\{\mu_A(rx_1) : x_1 \in M, x_1 \in f^{-1}(y)\} \\ &= \vee\{\mu_A(rx_1) : x_1 \in M, y = f(x_1)\} \\ &\geq \vee\{\mu_A(x_1) : x_1 \in M, y = f(x_1)\} \\ &= \mu_{f(A)}(y) \end{aligned}$$

Similarly we can show that $\nu_{f(A)}(ry) \leq \nu_{f(A)}(y) \ \ \forall y \ \in N$. Thus $f(A)$ is an IFSM of N.

3.5. Theorem. *Let M and N be two R-modules and f be a homomorphism of M into N. If $B = (\mu_B, \nu_B)$ is an IFSM of N, then $f^{-1}(B)$ is an IFSM of M.*

Proof. We have $f^{-1}(B) = \{(x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x)) : x \in M\}$ where $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ and $\nu_{f^{-1}(B)}(x) = \nu_B(f(x)) \ \forall \ x \in M$. So $\mu_{f^{-1}(B)}(0) = \mu_B(f(0)) = \mu_B(0) = 1$ and $\nu_{f^{-1}(B)}(0) = \nu_B(f(0)) = \nu_B(0) = 0$.

$$\begin{aligned} \text{Now, } \ \mu_{f^{-1}(B)}(x + y) &= \mu_B(f(x + y)) = \mu_B(f(x) + f(y)) \\ &\geq \mu_B(f(x)) \wedge \mu_B(f(y)) \\ &= \mu_{f^{-1}(B)}(x) \wedge \mu_{f^{-1}(B)}(y) \ \forall \ x, y \in M. \end{aligned}$$

Similarly we can show that $\nu_{f^{-1}(B)}(x+y) \leq \nu_{f^{-1}(B)}(x) \vee \nu_{f^{-1}(B)}(y)$, $\forall\, x, y \in M$.

Also, $\mu_{f^{-1}(B)}(rx) = \mu_B f(rx) = \mu_B(rf(x)) \geq \mu_B f(x) = \mu_{f^{-1}(B)}(x)$; and similarly $\nu_{f^{-1}(B)}(rx) \leq \nu_{f^{-1}(B)}(x) \quad \forall\ r \in R, x \in M$. Thus $f^{-1}(B)$ is an IFSM of M.

4 IFSM's of Quotient Modules

If N is a submodule of R-module M, then M/N denotes the quotient module of M with respect to N and $[x]$ represents the coset $x + N$. In this section we introduce two methods of constructing IFSM's of the quotient module M/N.

Method-1

4.1. Theorem. *Let $A = (\mu_A, \nu_A)$ be an IFSM of M and N be a submodule of M. Define $Q = (\mu_Q, \nu_Q)$, an IFS in M/N as follows. For $x \in M$, $\mu_Q([x]) = \vee\{\mu_A(u) : u \in [x]\}$ and $\nu_Q([x]) = \wedge\{\nu_A(u) : u \in [x]\}$. Then $Q = (\mu_Q, \nu_Q)$ is an IFSM of M/N.*

Proof. We have, $\mu_Q([0]) = \vee\{\mu_A(u) : u \in [0]\} = \mu_A(0) = 1$ and $\nu_Q([0]) = \wedge\{\nu_A(u) : u \in [0]\} = \nu_A(0) = 0$. Now for $x, y \in M$,

$$\begin{aligned}
\mu_Q([x]+[y]) &= \vee\{\mu_A(u) : u \in [x]+[y]\} \\
&= \vee\{\mu_A(u_1+u_2) : u_1+u_2 \in [x]+[y]\} \\
&\geq \vee\{\mu_A(u_1+u_2) : u_1 \in [x], u_2 \in [y]\} \\
&\geq \vee\{\mu_A(u_1) \wedge \mu_A(u_2) : u_1 \in [x], u_2 \in [y]\} \\
&= (\vee\{\mu_A(u_1) : u_1 \in [x]\}) \wedge (\vee\{\mu_A(u_2) : u_2 \in [y]\}) \\
&= \mu_Q([x]) \wedge \mu_Q([y])
\end{aligned}$$

Similarly we get $\nu_Q([x]+[y]) \leq \nu_Q([x]) \vee \nu_Q([y])\ \forall\, x, y \in M$.

Now for any $r \in R$, $x \in M$,

$$\begin{aligned}
\mu_Q(r[x]) &= \mu_Q([rx]) = \vee\{\mu_A(u) : u \in [rx]\} \\
&= \vee\{\mu_A(rx+v) : v \in N\} \\
&\geq \vee\{\mu_A(rx+rv_1) : v_1 \in N\} \\
&= \vee\{\mu_A(r(x+v_1)) : v_1 \in N\} \\
&\geq \vee\{\mu_A(x+v_1) : v_1 \in N\} \\
&= \vee\{\mu_A(u_1) : u_1 \in [x]\} \\
&= \mu_Q([x])
\end{aligned}$$

Similarly we get $\nu_Q(r[x]) \leq \nu_Q([x])\ \forall\ r \in R, x \in M$. Thus $Q = (\mu_Q, \nu_Q)$ is an IFSM of M/N.

Remark. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be IFSM's of M such that $A \subseteq B$. Then A^* and B^* are submodules of M such that $A^* \subseteq B^*$ (see [8]). Therefore

A^* is submodule of B^*, and clearly $B|_{B^*}$ is an IFSM of B^*. Now define $Q = (\mu_Q, \nu_Q)$, an IFS in B^*/A^*, where for $x \in B^*$, $\mu_Q([x]) = \vee\{\mu_B(u) : u \in [x]\}$, $\nu_Q([x]) = \wedge\{\mu_B(u) : u \in [x]\}$, $[x]$ denoting the coset $x + A^*$. Then by above theorem $Q = (\mu_Q, \nu_Q)$ is an IFSM of B^*/A^* and it is called the quotient of B w.r.to A, written as B/A.

Method-2

4.2. Theorem. *Let R be an integral domain, M be divisible module over R (i.e. $0 \neq r \in R, x \in M \Rightarrow x = ry$ for some $y \in M$) and N be a prime submodule of M (i.e. N is a submodule of M and $rx \in N, r \in R, x \in M \Rightarrow$ either $r = 0$ or $x \in N$). Let $A = (\mu_A, \nu_A)$ be an IFSM of M. Define $Q = (\mu_Q, \nu_Q)$, an IFS in M/N as follows. For $x \in M$,*

$$\mu_Q([x]) = \begin{cases} 1 \text{ if } [x] = N \\ \wedge\{\mu_A(u) : u \in [x]\} \text{ otherwise;} \end{cases} \quad \nu_Q([x]) = \begin{cases} 0 \text{ if } [x] = N \\ \vee\{\nu_A(u) : u \in [x]\} \text{ otherwise} \end{cases}$$

Then $Q = (\mu_Q, \nu_Q)$ is an IFSM of M/N.

Proof. Since $0 \in N$, $\mu_Q([0]) = 1$ and $\nu_Q([0]) = 0$. For $r \in R, x \in M$ consider $\mu_Q(r[x])$ and $\nu_Q(r[x])$. If $r[x] = N$, then $\mu_Q(r[x]) = 1 \geq \mu_Q([x])$ and $\nu_Q(r[x]) = 0 \leq \nu_Q([x])$. Now $r[x] \neq N \Rightarrow rx \notin N \Rightarrow r \neq 0, x \notin N$, and in this case

$$\begin{aligned}
\mu_Q(r[x]) &= \mu_Q([rx]) \\
&= \wedge\{\mu_A(u) : u \in [rx]\} \\
&= \wedge\{\mu_A(rx + v) : v \in N\} \\
&= \wedge\{\mu_A(rx + rz) : z \in N\} \text{ (since } M \text{ is divisible module and } N \text{ is a prime} \\
&\qquad \text{submodule of } M\text{, we get } N = \{rz : z \in N\}) \\
&= \wedge\{\mu_A(r(x + z)) : z \in N\} \\
&\geq \wedge\{\mu_A(x + z) : z \in N\} \\
&= \wedge\{\mu_A(w) : w \in [x]\} \\
&= \mu_Q([x])
\end{aligned}$$

Similarly we get $\nu_Q(r[x]) \leq \nu_Q([x])$.

Now for $x, y \in M$, consider $\mu_Q([x] + [y])$ and $\nu_Q([x] + [y])$. If $[x] = N$ or $[y] = N$ or $[x] + [y] = N$, then clearly $\mu_Q([x] + [y]) \geq \mu_Q([x]) \wedge \mu_Q([y])$ and $\nu_Q([x] + [y]) \leq \nu_Q([x]) \vee \nu_Q([y])$. Now suppose that $[x] \neq N$ or $[y] \neq N$ or $[x] + [y] \neq N$. Then

$$\begin{aligned}
\mu_Q([x] + [y]) &= \wedge\{\mu_A(u) : u \in [x] + [y]\} \\
&= \wedge\{\mu_A((x + u_1) + (y + u_2)) : u_1, u_2 \in N\} \\
&\geq \wedge\{\mu_A(x + u_1) \wedge \mu_A(y + u_2) : u_1, u_2 \in N\} \\
&\geq (\wedge\{\mu_A(x + u_1) : u_1 \in N\}) \wedge (\wedge\{\mu_A(y + u_2) : u_2 \in N\}) \\
&= (\wedge\{\mu_A(v_1) : v_1 \in [x]\}) \wedge (\wedge\{\mu_A(v_2) : v_2 \in [y]\}) \\
&= \mu_Q([x]) \wedge \mu_Q([y])
\end{aligned}$$

Similarly we can obtain $\nu_Q([x]+[y]) \leq \nu_Q([x]) \vee \nu_Q([y])\ \forall\ x, y \in M$. Hence $Q = (\mu_Q, \nu_Q)$ is an IFSM of M/N. This completes the proof.

Now we know that if R is a field, then it is an integral domain. In this case every nonzero element in R has a multiplicative inverse in R and M becomes a vector space over R. So it is easy to see that M is a divisible module and every submodule(subspace) of M is a prime submodule. Hence we have the following corollary to the above theorem.

4.3. Corollary. *Let $A = (\mu_A, \nu_A)$ be an IFSM of M and N be a submodule of M. Assume that R is a field. Then the IFS $Q = (\mu_Q, \nu_Q)$, defined as in the above theorem is an IFSM of the quotient module M/N.*

References

[1] K.T. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20:87–96, 1986.

[2] K.T. Atanassov. New operations defined over the intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 61:137–142, 1994.

[3] L. Atanassov. On intuitionistic fuzzy versions of L. Zadeh's extention principle. *Notes on Intuitionistic Fuzzy Sets*, 13(3):33–36, 2006.

[4] D. Coker. An introduction to intuitionistic fuzzy topological spaces. *Fuzzy Sets and Systems*, 88:81–89, 1997.

[5] B. Davvaz, W.A. Dudek, and Y.B. Jun. Intuitionistic fuzzy Hv-submodules. *Inform. Sci.*, 176:285–300, 2006.

[6] T.W. Hungerford. *Algebra*. Springer Verlag, 1974.

[7] Paul Isaac and Pearly P. John. On intuitionistic fuzzy submodules of a module. *Int. Jl. of Mathematical Sciences and Applications*, 1(3):1447–1454, September 2011.

[8] Pearly P. John and Paul Isaac. IFSM's of an R-module - A study. *To appear in "Int. Math. Forum"*.

[9] J.N. Mordeson and D.S. Malik. *Fuzzy Commutative Algebra*. World Scientific, 1998. ISBN 981-02-3628-X.

[10] C.V. Negoita and D.A. Ralescu. *Applications of Fuzzy Sets and System Analysis*. Birkhauser, Basel, 1975.

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