Research Announcements.

Unsolved Problems

Kahar El-Hussein

College of Sciences. Aljouf University, KSA
Department of Mathematics
Kahar-Hussein@Lycos.com, khali_kh@yahoo.com

Abstract

0.1 It is well known (theorem of I.E. Segal) that if $G$ is a locally compact unimodular group and $\tilde{G}$ is the set of irreducible unitary representation $G$, then there exists a unique positive measure $\mu$ on $\tilde{G}$ such that

$$\int_G f(x)dx = \int_{\tilde{G}} \text{tr}(\lambda(f)(\text{tr}(\lambda(f))))^*d\mu(\lambda),$$

where $dX$ is the Haar measure on $G$ and the operator-valued function

$$\lambda(f) = \int_G f(x)\lambda(f)dx$$

is called the Fourier transform on $G$. This being so:

0.2 Suppose that the Fourier transform on the $n$-dimensional analytic Lie group $G$ is defined and denoted by $F$.

Now the question: Can we solve the major problems in analysis on $G$, which are solved by the Fourier transform on the group $\mathbb{R}^n$?

1 Introduction

1.1 We know that the major problem in analysis and algebra are the following

1. $F(L^2(G))$ (Plancherel formula).
2. $F(M^+(G))$ (Bochner theorem).
3. $F(D(G))$ and $F(E'(G))$ (Paley-Weiner theorem).
4. Solvability of Invariant Partial Differential Equations (Existence, Regularity, Sobolev spaces,...).
5. Classification of all maximal Ideals in $L^1(G)$ (Problem of spectral theory).

6. Classification of all closed Ideals in $L^1(G)$ (Problem of spectral Synthesis).

1.2. By finding $\hat{G}$, except problem 1. which has been solved completely for
nilpotent Lie groups (Kilikov), semi-simple Lie groups (Harish Chandara) and
solvable Lie groups (Bernart), very few results were obtained for problem
2. (Existence theorem for invariant partial operator) by attempts of some
mathematicians as [1],[4],[7,8],[14]. In fact the part that has been solved is
indeed nothing but partial differential equations with constant coefficients.

2 Research Announcements

2.1. Far away from the hard theory of irreducible unitary representations of
analytic Lie group $G$, the main ideals of my research announcement describing
new results on the $(2n+1)$–Heisenberg group $H^n$ are the following

1. Consider the classical Fourier transform on the $(2n+1)$–vector group
$\mathbb{R}^{2n+1}$. as the Fourier transform on $G$.

2. Transfer problems on the $(2n+1)$-dimensional Heisenberg group $H^n$ to
the $(2n+1)$–dimensional vector group $\mathbb{R}^{2n+1}$, by means of an embedding
of both groups into the $(3n+1)$–dimensional, so I called it mixed group
[9].

2.2. Analysis over the vector groups is easier, this should lead to obtain our
results and solving the major problems 1. 2. 3. 4. 5. and 6. on the $(2n + 1)$–Heisenberg group $H^n$.

In particular we have obtained a relation between the Cauchy Remain operator

$$Q = -i \frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}$$  \hspace{1cm} (1)

and the Lewey operator

$$P = -i \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} - 2(x_1 + ix_2) \frac{\partial}{\partial x_3}$$  \hspace{1cm} (2)

which gives us the local solvability of $P$. 
3 Unsolved problems

I believe with trust that we can solve the major problems on any simply connected nilpotent Lie group $G$.

References


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