Cordial and 3-Equitable Labeling for Some Star Related Graphs

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Abstract

We present here cordial and 3-equitable labeling for the graphs obtained by joining apex vertices of two stars to a new vertex. We extend these results for $k$ copies of stars.

Mathematics Subject Classification: 05C78

Keywords: Cordial labeling, 3-equitable labeling

1. Introduction

We begin with simple, finite, connected, undirected graph $G = (V, E)$. In the present work $K_{1,n}$ denote the star. Vertex corresponds to $K_1$ is called an apex vertex. For all other terminology and notations we follow Harary[7].
We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1 Consider two stars $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ then $G = < K_{1,n}^{(1)} : K_{1,n}^{(2)} >$ is the graph obtained by joining apex vertices of stars to a new vertex $x$.

Note that $G$ has $2n + 3$ vertices and $2n + 2$ edges.

Definition 1.2 Consider $k$ copies of stars namely $K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}, \ldots K_{1,n}^{(k)}$. Then the $G = < K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} : \ldots : K_{1,n}^{(k)} >$ is the graph obtained by joining apex vertices of each $K_{1,n}^{(p-1)}$ and $K_{1,n}^{(p)}$ to a new vertex $x_{p-1}$ where $2 \leq p \leq k$.

Note that $G$ has $k(n + 2) - 1$ vertices and $k(n + 2) - 2$ edges.

Definition 1.3 If the vertices of the graph are assigned values subject to certain conditions is known as graph labeling.

Most interesting graph labeling problems have three important characteristics.

1. a set of numbers from which the labels are chosen.
2. a rule that assigns a value to each edge.
3. a condition that these values must satisfy.

For detail survey on graph labeling one can refer Gallian[6]. Vast amount of literature is available on different types of graph labeling. According to Beineke and Hegde[2] graph labeling serves as a frontier between number theory and structure of graphs.

Labeled graph have variety of applications in coding theory, particularly for missile guidance codes, design of good radar type codes and convolution codes with optimal autocorrelation properties. Labeled graph plays vital role in the study of X-Ray crystallography, communication network and to determine optimal circuit layouts. A detail study of variety of applications of graph labeling is given by Bloom and Golomb[3].

Definition 1.4 Let $G = (V, E)$ be a graph. A mapping $f : V(G) \rightarrow \{0, 1\}$ is called binary vertex labeling of $G$ and $f(v)$ is called the label of the vertex $v$ of $G$ under $f$.

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of $G$ having labels 0 and 1 respectively under $f$ and let $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively under $f^*$.

Definition 1.5 A binary vertex labeling of a graph $G$ is called a cordial
Cordial and 3-equitable labeling

The concept of cordial labeling was introduced by Cahit [4]. Many researchers have studied cordiality of graphs. e.g. Cahit [4] proved that tree is cordial. In the same paper he proved that $K_n$ is cordial if and only if $n \leq 3$. Ho et al. [8] proved that unicyclic graph is cordial unless it is $C_{4k+2}$. Andar et al. [1] discussed cordiality of multiple shells. Vaidya et al. [9], [10], [11] have also discussed the cordiality of various graphs.

**Definition 1.6** Let $G = (V,E)$ be a graph. A mapping $f : V(G) \rightarrow \{0,1,2\}$ is called ternary vertex labeling of $G$ and $f(v)$ is called label of the vertex $v$ of $G$ under $f$.

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \rightarrow \{0,1,2\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1), v_f(2)$ be the number of vertices of $G$ having labels 0, 1, 2 respectively under $f$ and $e_f(0), e_f(1), e_f(2)$ be the number of edges having labels 0, 1, 2 respectively under $f^*$.

**Definition 1.7** A ternary vertex labeling of a graph $G$ is called a 3-equitable labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $0 \leq i, j \leq 2$. A graph $G$ is 3-equitable if it admits 3-equitable labeling.

The concept of 3-equitable labeling was introduced by Cahit [5]. Many researchers have studied 3-equatability of graphs. e.g. Cahit [5] proved that $C_n$ is 3-equitable except $n \equiv 3 mod(6)$. In the same paper he proved that an Eulerian graph with number of edges congruent to $3 mod(6)$ is not 3-equitable. Youssef [12] proved that $W_n$ is 3-equitable for all $n \geq 4$.

In the present investigations we prove that graphs $< K^{(1)}_{1,n} : K^{(2)}_{1,n} >$ and $< K^{(1)}_{1,n} : K^{(2)}_{1,n} : K^{(3)}_{1,n} : \ldots : K^{(k)}_{1,n} >$ are cordial as well as 3-equitable.

2. **Main Results**

**Theorem-2.1:** Graph $< K^{(1)}_{1,n} : K^{(2)}_{1,n} >$ is cordial.

**Proof:** Let $v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, \ldots v_n^{(1)}$ be the pendant vertices $K^{(1)}_{1,n}$ and $v_1^{(2)}, v_2^{(2)}, v_3^{(2)}, \ldots v_n^{(2)}$ be the pendant vertices $K^{(2)}_{1,n}$. Let $c_1$ and $c_2$ be the apex vertices of $K^{(1)}_{1,n}$ and $K^{(2)}_{1,n}$ respectively and they are adjacent to a new common vertex $x$. Let $G = < K^{(1)}_{1,n} : K^{(2)}_{1,n} >$. We define binary vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

For any $n \in N$ and $i = 1,2,\ldots n$ where $N$ is set of natural numbers.

In this case we define labeling as follows

**Case 1:** If $n$ even

For $j = 1,2$

\[
f(v_i^{(j)}) = 0; \text{ if } 1 \leq i \leq \frac{n}{2}
\]

\[
= 1; \quad \frac{n+2}{2} \leq i \leq n
\]

\[
f(c_1) = 0;
\]

\[
f(c_2) = 1;
\]
\[ f(x) = 0; \]

**Case 2:** If \( n \) odd

For \( j = 1, 2 \)

\[ f(v_i^{(j)}) = 0; \text{ if } 1 \leq i \leq \frac{n-1}{2} \]

\[ = 1; \frac{n+1}{2} \leq i \leq n \]

\[ f(c_1) = f(c_2) = f(x) = 0; \]

The labeling pattern defined above covers all possible arrangement of vertices. The graph \( G \) satisfies the conditions \( |v_f(0) - v_f(1)| \leq 1 \) and \( |e_f(0) - e_f(1)| \leq 1 \) as shown in Table 1. i.e. \( G \) admits cordial labeling.

<table>
<thead>
<tr>
<th>( n )</th>
<th>Vertex Condition</th>
<th>Edge Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n \in \mathbb{N} )</td>
<td>( v_f(0) = v_f(1) + 1 )</td>
<td>( e_f(0) = e_f(1) )</td>
</tr>
</tbody>
</table>

**Table 1**

For better understanding of the above defined labeling pattern, consider following illustration.

**Illustration 2.2** Consider \( G = < K_{1,7}^{(1)} : K_{1,7}^{(2)} >. \) Here \( n = 7 \). The cordial labeling is as shown in Figure 1.

![Figure 1](image)

Above result can be extended for \( k \)-copies of \( K_{1,n} \) as follows.

**Theorem 2.3** Graph \( < K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} : \ldots : K_{1,n}^{(k)} > \) is cordial.

**Proof:** Let \( K_{1,n}^{(j)} \) be \( k \) copies of star \( K_{1,n} \), \( v_i^{(j)} \) be the pendant vertices of \( K_{1,n}^{(j)} \) and \( c_j \) be the apex vertex of \( K_{1,n}^{(j)} \) (here \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, k \)). Let \( x_1, x_2, \ldots, x_{k-1} \) be the vertices such that \( c_{p-1} \) and \( c_p \) are adjacent to \( x_{p-1} \) where \( 2 \leq p \leq k \). Consider \( G = < K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} : \ldots : K_{1,n}^{(k)} >. \) To define binary vertex labeling \( f : V(G) \rightarrow \{0, 1\} \) we consider following cases.

**Case 1:** \( n \in \mathbb{N} \) even and \( k \) where \( k \in \mathbb{N} - \{1, 2\} \).

In this case we define labeling function \( f \) as

For \( j = 1, 2, \ldots, k \)

\[ f(v_i^{(j)}) = 0; \text{ if } 1 \leq i \leq \frac{n}{2}. \]
= 1; if \( n + 2 \leq i \leq n \).

\[
\begin{align*}
  f(c_j) &= 1; \text{if } j \text{ even.} \\
  &= 0; \text{if } j \text{ odd.}
\end{align*}
\]

\[
\begin{align*}
  f(x_j) &= 1; \text{if } j \text{ even, } j \neq k. \\
  &= 0; \text{if } j \text{ odd, } j \neq k.
\end{align*}
\]

**Case 2:** \( n \in N - \{1, 2\} \) odd and \( k \) where \( k \in N - \{1, 2\} \).

In this case we define labeling function \( f \) as

For \( j = 1, 2, \ldots k \)

\[
\begin{align*}
  f(v^{(j)}_i) &= 0; \text{if } 1 \leq i \leq \frac{n-1}{2}. \\
  &= 1; \text{if } \frac{n+1}{2} \leq i \leq n.
\end{align*}
\]

\[
\begin{align*}
  f(c_j) &= 1; \text{if } j \text{ even.} \\
  &= 0; \text{if } j \text{ odd.}
\end{align*}
\]

\[
\begin{align*}
  f(x_j) &= 0, \text{ } j \neq k.
\end{align*}
\]

The labeling pattern defined above covers all the possibilities. In each case, the graph \( G \) under consideration satisfies the conditions \( |v_f(0) - v_f(1)| \leq 1 \) and \( |e_f(0) - e_f(1)| \leq 1 \) as shown in Table 2. i.e. \( G \) admits cordial labeling.

Let \( n = 2a + b \) and \( k = 2c + d \) where \( a \in N \cup \{0\}, c \in N \)

<table>
<thead>
<tr>
<th>( b )</th>
<th>( d )</th>
<th>Vertex Condition</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
<td>( v_f(0)=v_f(1)+1 )</td>
<td>( e_f(0)=e_f(1) )</td>
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<td>1</td>
<td>0</td>
<td>( v_f(0)+1=v_f(1) )</td>
<td>( e_f(0)=e_f(1) )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( v_f(0)=v_f(1) )</td>
<td>( e_f(0)+1=e_f(1) )</td>
</tr>
</tbody>
</table>

**Table 2**

For better understanding of the above defined labeling pattern, consider following illustration.

**Illustration 2.4** Consider \( G = < K_{1,6}^{(1)} : K_{1,6}^{(2)} : K_{1,6}^{(3)} > \). Here \( n = 6 \) and \( k = 3 \).

The cordial labeling is as shown in Figure 2. It is the case 1 of Theorem 2.3.

![Figure 2](image_url)

**Theorem 2.5** Graph \( < K_{1,n}^{(1)} : K_{1,n}^{(2)} > \) is 3-equitable.

**Proof:** Let \( v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, \ldots v_n^{(1)} \) be the pendant vertices \( K_{1,n}^{(1)} \) and \( v_1^{(2)}, v_2^{(2)}, v_3^{(2)}, \ldots v_n^{(2)} \) be the pendant vertices \( K_{1,n}^{(2)} \). Let \( c_1 \) and \( c_2 \) be the apex vertices of \( K_{1,n}^{(1)} \)
and \( K_{1,n}^{(2)} \) respectively and they are adjacent to a new common vertex \( x \). Let \( G = < K_{1,n}^{(1)} : K_{1,n}^{(2)} > \). To define ternary vertex labeling \( f : V(G) \rightarrow \{0, 1, 2\} \) we consider the following cases.

**Case 1:** \( n \equiv 0(\text{mod}3) \)

In this case we define labeling \( f \) as:

For \( j = 1, 2 \)

\[ f(v_i^{(j)}) = \begin{cases} 0; & i \equiv 0(\text{mod}3) \\ 1; & i \equiv 1(\text{mod}3) \\ 2; & i \equiv 2(\text{mod}3), 1 \leq i \leq n-1 \end{cases} \]

\( f(v_n^{(1)}) = 1; \)

\( f(v_n^{(2)}) = f(c_1) = f(x) = 0; \)

\( f(c_2) = 2; \)

**Case 2:** \( n \equiv 1(\text{mod}3) \)

In this case we define labeling \( f \) as:

For \( j = 1, 2 \)

\[ f(v_i^{(j)}) = \begin{cases} 0; & i \equiv 0(\text{mod}3) \\ 1; & i \equiv 1(\text{mod}3) \\ 2; & i \equiv 2(\text{mod}3) \end{cases} \]

\( f(c_1) = f(x) = 0; \)

\( f(c_2) = 2; \)

**Case 3:** \( n \equiv 2(\text{mod}3) \)

In this case we define labeling \( f \) as:

For \( j = 1, 2 \)

\[ f(v_i^{(j)}) = \begin{cases} 0; & i \equiv 0(\text{mod}3) \\ 1; & i \equiv 1(\text{mod}3) \\ 2; & i \equiv 2(\text{mod}3) \end{cases} \]

\( f(c_1) = f(c_2) = f(x) = 0; \)

The labeling pattern defined above covers all possible arrangement of vertices. In each case, the graph \( G \) under consideration satisfies the conditions \( |v_f(i) - v_f(j)| \leq 1 \) and \( |e_f(i) - e_f(j)| \leq 1 \) for all \( 0 \leq i, j \leq 2 \) as shown in Table 3. i.e. \( G \) admits 3-equitable labeling.

Let \( n = 3a + b \) and \( a \in N \cup \{0\} \)

<table>
<thead>
<tr>
<th>( b )</th>
<th>Vertex Condition</th>
<th>Edge Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( v_f(0) = v_f(1) = v_f(2) = 1 )</td>
<td>( e_f(0) + e_f(1) + e_f(2) = 1 )</td>
</tr>
<tr>
<td>1</td>
<td>( v_f(0) = v_f(1) = 1 = v_f(2) = 1 )</td>
<td>( e_f(0) = e_f(1) = e_f(2) )</td>
</tr>
<tr>
<td>2</td>
<td>( v_f(0) = v_f(1) = v_f(2) )</td>
<td>( e_f(0) = e_f(1) = e_f(2) = 1 )</td>
</tr>
</tbody>
</table>

Table 3

For better understanding of the above defined labeling pattern, consider following illustration.

**Illustration 2.6** Consider a graph \( G = < K_{1,8}^{(1)} : K_{1,8}^{(2)} > \) Here \( n = 8 \) i.e \( n \equiv 2(\text{mod}3) \). The corresponding 3-equitable labeling is shown in Figure 3. It
is the case related to case -3

Figure 3

Above result can be extended for \( k \)-copies of \( K_{1,n} \) as follows.

**Theorem 2.7** Graph \( < K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} : \ldots : K_{1,n}^{(k)} > \) is 3-equitable.

**Proof:** Let \( K_{1,n}^{(j)}, j = 1, 2, \ldots k \) be \( k \) copies of star \( K_{1,n} \). Let \( v_i^{(j)} \) be the pendant vertices of \( K_{1,n}^{(j)} \) where \( i = 1, 2, \ldots n \) and \( j = 1, 2, \ldots k \). Let \( c_j \) be the apex vertex of \( K_{1,n}^{(j)} \) where \( j = 1, 2, \ldots k \). Let \( G =< K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} : \ldots : K_{1,n}^{(k)} > \) and \( x_1, x_2, \ldots, x_{k-1} \) are the vertices as stated in Theorem 2.3. To define ternary vertex labeling \( f : V(G) \rightarrow \{0, 1, 2\} \) we consider following cases.

**Case 1:** For \( n \equiv 0(\text{mod}3) \)

In this case we define labeling function \( f \) as follows

**Subcase 1:** For \( k \equiv 0(\text{mod}3) \)

\[
\begin{align*}
f(v_i^{(j)}) &= 0; \text{if } i \equiv 1(\text{mod}3) \\
&= 1; \text{if } i \equiv 2(\text{mod}3) \\
&= 2; \text{if } i \equiv 0(\text{mod}3), i \leq n - 1 \\
f(v_n^{(j)}) &= 1; \text{if } j \equiv 1, 2(\text{mod}3) \\
&= 2; \text{if } j \equiv 0(\text{mod}3) \\
f(c_j) &= 0; \text{if } j \equiv 1, 2(\text{mod}3) \\
&= 2; \text{if } j \equiv 0(\text{mod}3) \\
f(x_j) &= 2; \text{if } j \leq n - 1
\end{align*}
\]

**Subcase 2:** For \( k \equiv 1(\text{mod}3) \)

\[
\begin{align*}
f(v_i^{(1)}) &= 0; \text{if } i \equiv 1(\text{mod}3) \\
&= 1; \text{if } i \equiv 2(\text{mod}3) \\
&= 2; \text{if } i \equiv 0(\text{mod}3) \\
f(c_1) &= 2 \\
f(x_1) &= 0
\end{align*}
\]

For remaining vertices take \( j = k - 1 \) and use the pattern of subcase 1.

**Subcase 3:** For \( k \equiv 2(\text{mod}3) \)

For \( j = 1, 2 \)

\[
f(v_i^{(j)}) = 0; \text{if } i \equiv 1(\text{mod}3)
\]
= 1; if \( i \equiv 2 \pmod{3} \)
= 2; if \( i \equiv 0 \pmod{3} \), \( 1 \leq i \leq n - 1 \)
\[ f(v_1^{(1)}) = 1 \]
\[ f(v_1^{(2)}) = f(c_2) = f(x_j) = 2 \]
\[ f(c_1) = 0 \]

For remaining vertices take \( j = k - 2 \) and use the pattern of subcase 1.

**Case 2:** For \( n \equiv 1 \pmod{3} \)

In this case we define labeling function \( f \) as follows

**Subcase 1:** For \( k \equiv 0 \pmod{3} \)

**Subcase 1.1:** For \( n = 1 \)
\[ f(v_1^{(j)}) = 2; \text{ if } j \equiv 0 \pmod{3} \]
\[ = 1; \text{ if } j \equiv 1, 2 \pmod{3} \]
\[ f(c_j) = 2; \text{ if } j \equiv 1 \pmod{3} \]
\[ = 1; \text{ if } j \equiv 2 \pmod{3} \]
\[ = 0; \text{ if } j \equiv 0 \pmod{3} \]
\[ f(x_j) = 0; j \neq k \]

**Subcase 1.2:** For \( n > 1 \)
\[ f(v_i^{(j)}) = 0; \text{ if } i \equiv 0 \pmod{3} \]
\[ = 1; \text{ if } i \equiv 1 \pmod{3} \]
\[ = 2; \text{ if } i \equiv 2 \pmod{3}, i \leq n - 2 \]
\[ f(v_{n-1}^{(j)}) = 0; \text{ if } j \equiv 1, 2 \pmod{3} \]
\[ = 2; \text{ if } j \equiv 0 \pmod{3} \]
\[ f(c_j) = 1 \]
\[ f(x_j) = 0; j \equiv 0 \pmod{3} \]

**Subcase 2:** For \( k \equiv 1 \pmod{3} \)
\[ f(v_i^{(1)}) = 0; \text{ if } i \equiv 0 \pmod{3} \]
\[ = 1; \text{ if } i \equiv 1 \pmod{3} \]
\[ = 2; \text{ if } i \equiv 2 \pmod{3} \]
\[ f(c_1) = 0 \]
\[ f(x_1) = 2 \]

For remaining vertices take \( j = k - 1 \) and use the pattern of subcase 1.1 or subcase 1.2 if \( n = 1 \) or \( n > 1 \) respectively.

**Subcase 3:** For \( k \equiv 2 \pmod{3} \).

For \( j = 1, 2 \)
\[ f(v_i^{(j)}) = 0; \text{ if } i \equiv 0 \pmod{3} \]
\[ = 1; \text{ if } i \equiv 1 \pmod{3} \]
\[ = 2; \text{ if } i \equiv 2 \pmod{3} \]
\[ f(c_1) = f(x_2) = 2 \]
\[ f(c_2) = f(x_1) = 0 \]
\[ f(x_1) = 2; \text{ if } n = 1 \]
\[ f(x_1) = 0; \text{ if } n > 1 \]

For remaining vertices take \( j = k - 2 \) and use the pattern of subcase 1.1 or subcase 1.2 if \( n = 1 \) or \( n > 1 \) respectively.

**Case 3:** For \( n \equiv 2(\text{mod}3) \).

In this case we define labeling function \( f \) as follows

**Subcase 1:** For \( k \equiv 0(\text{mod}3) \)
\[ f(v_i^{(j)}) = 0; \text{ if } i \equiv 0(\text{mod}3) \]
\[ = 1; \text{ if } i \equiv 1(\text{mod}3) \]
\[ = 2; \text{ if } i \equiv 2(\text{mod}3), \text{ } i \leq n - 1 \]
\[ f(v_n^{(j)}) = 1; \text{ if } j \equiv 1(\text{mod}3) \]
\[ = 2; \text{ if } j \equiv 0, 2(\text{mod}3) \]
\[ f(c_j) = 2; \text{ if } j \equiv 1(\text{mod}3) \]
\[ = 0; \text{ if } j \equiv 0, 2(\text{mod}3) \]
\[ f(x_j) = 0; \text{ if } j \equiv 1, 2(\text{mod}3) \]
\[ = 2; \text{ if } j \equiv 0(\text{mod}3) \]

**Subcase 2:** For \( k \equiv 1(\text{mod}3) \)
\[ f(v_i^{(1)}) = 0; \text{ if } i \equiv 0(\text{mod}3) \]
\[ = 1; \text{ if } i \equiv 1(\text{mod}3) \]
\[ = 2; \text{ if } i \equiv 2(\text{mod}3), \text{ } i \leq n \]
\[ f(c_1) = 0 \]
\[ f(x_1) = 2 \]

For remaining vertices take \( j = k - 1 \) and use the pattern of subcase 1.

**Subcase 3:** For \( k \equiv 2(\text{mod}3) \)

For \( j = 1, 2 \)
\[ f(v_i^{(j)}) = 0; \text{ if } i \equiv 0(\text{mod}3) \]
\[ = 1; \text{ if } i \equiv 1(\text{mod}3) \]
\[ = 2; \text{ if } i \equiv 2(\text{mod}3), \text{ } i \leq n \]
\[ f(c_1) = 2 \]
\[ f(c_2) = f(x_j) = 0. \]

For remaining vertices take \( j = k - 2 \) and use the pattern of subcase 1.

The labeling pattern defined above covers all possible arrangement of vertices. In each case, the graph \( G \) under consideration satisfies the conditions \(|v_f(i) - v_f(j)| \leq 1 \) and \(|e_f(i) - e_f(j)| \leq 1 \) for all \( 0 \leq i, j \leq 2 \) as shown in Table 4. i.e. \( G \) admits 3-equitable labeling.

Let \( n = 3a + b \) and \( k = 3c + d \) where \( a \in N \cup \{0\}, c \in N \).
Table 4

<table>
<thead>
<tr>
<th>b</th>
<th>d</th>
<th>Vertex Condition</th>
<th>Edge Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>(v_f(0)+v_f(1)=v_f(2)+1)</td>
<td>(e_f(0)+1=e_f(1)=e_f(2)+1)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>(v_f(0)=v_f(1)=v_f(2)+1)</td>
<td>(e_f(0)=e_f(1)=e_f(2)+1)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>(v_f(0)=v_f(1)=v_f(2)+1)</td>
<td>(e_f(0)=e_f(1)=e_f(2)+1)</td>
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</tr>
<tr>
<td>2</td>
<td>2</td>
<td>(v_f(0)=v_f(1)=v_f(2)+1)</td>
<td>(e_f(0)=e_f(1)=e_f(2)+1)</td>
</tr>
</tbody>
</table>

For better understanding of the above defined labeling pattern, consider following illustration.

**Illustration 2.8** Consider a graph \(G = < K_{1,5}^{(1)} : K_{1,5}^{(2)} : K_{1,5}^{(3)} : K_{1,5}^{(4)} >\). Here \(n = 5\) and \(k = 4\). The corresponding 3-equitable labeling is as shown in *Figure 4*.

![Figure 4](image)

3. **Concluding Remarks**

Labeled graph is the topic of current interest for many researchers as it has diversified applications. We discuss here cordial labeling and 3-equitable labeling of some star related graphs. This approach is novel and contribute two new graphs to the theory of cordial graphs as well as 3-equitable graphs. The derived labeling pattern is demonstrated by means of elegant illustrations which provides better understanding of the derived results. The results reported here are new and will add new dimension in the theory of cordial and 3-equitable graphs.

**References**


Received: November, 2008