Basic DEA Models in the Full Fuzzy Position

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Abstract

In this paper basic data envelopment analysis (DEA) models are proposed for evaluation decision making units (DMUs) in the case that inputs, outputs, variables and constraints are fuzzy, (full fuzzy). Because the forms of basic DEA models are in the form of LP, so each of basic DEA models where are full fuzzy, are converted to a MOLP. Therefore, we can solve MOLP by use of lexicography or sum weighting methods. So, we can evaluate DMU by use of gained fuzzy answers.

Keywords: DEA, Full Fuzzy Linear Programming, Multiple Objective Linear Programming

1 Introduction

Data envelopment analysis (DEA) is a methodology that has been widely used to evaluate the relative efficiency of a set of decision-making units (DMUs) involved in a production process. DEA models provide efficiency scores that assess the performance of the different DMUs in terms of either the use of several inputs or the production of certain outputs (or even simultaneously). Most of DEA efficiency scores vary in (0; 1], the unity value being reserved to efficient units. In the particular case of the radial models, the CCR (Charnes, Cooper and Rhodes [2]) and the BCC (Banker, Charnes and Cooper [1]) models yield efficiency scores both in input and in output orientation, although nonoriented DEA efficiency scores can also be defined. Fuzzy mathematical programming provides us with a tool to deal with the natural uncertainty inherent to some production processes. Alternatively, other authors propose chance constrained programming formulations of DEA as stochastic approaches to deal with variations in data. We can find several fuzzy approaches to the assessment of

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efficiency in the DEA literature. Sengupta [5] considers fuzzy both objective and constraints and analyzes the resulting fuzzy DEA model by using Zimmermann’s method [8]. Triantis and Girod [7] use Carlsson and Korhonen method [3] in an application developed in the context of a preprint and packaging line which inserts commercial pamphlets into newspapers. The organization of the present paper is as follows. In section 2, Data Envelopment Analysis introduction and parametric model which cause the basic model of DEA, are discussed. In the section 3.4: fuzzy preliminaries and converting a full fuzzy Lp to a MOLP is presented. Section 5, basic DEA models in the full fuzzy case are converted to corresponding MOLP. Section 6 provides a numerical example, and the conclusion is drawn in section 7.

2 MOLP and DEA

In this section parameters forms of basic DEA models which are presented by Korhonen in [3]. Assume we have n decision making units (DMU) each consuming m inputs and producing s outputs. Let \( X \in \mathbb{R}^{m \times n} \) and \( Y \in \mathbb{R}^{s \times n} \) be the matrices, consisting of nonnegative elements, containing the observed input and output measures for the DMUs. We denote by \( x_j \) (the jth column of \( X \)) the vector of inputs consumed by \( DMU_j \), and by \( x_{ij} \) the quantity of input \( i \) consumed by \( DMU_j \). A similar notation is used for outputs.

Consider the following multiple objective linear program (MOLP):

\[
\begin{align*}
\max & \quad u = U\lambda \\
\text{s.t.} & \quad \lambda \in \Lambda = \{\lambda | \lambda \in \mathbb{R}^n_+, A\lambda \leq b\},
\end{align*}
\]

where \( U = (Y, -X)^t \), \( \Lambda \subset \mathbb{R}^n \) is a feasible set, matrix \( A \in \mathbb{R}^{k \times n} \) and vector \( b \in \mathbb{R}^k \). The set of feasible values of vector \( u \in \mathbb{R}^{m+s} \) is called a feasible region and denoted by \( T = \{u | u = U\lambda, \lambda \in \Lambda\} \) With this model, our purpose is to find a feasible linear combination of the input/output vectors of the existing DMUs, which simultaneously maximizes all outputs and minimizes all inputs. Specifically, in the MCDM-literature (see Steuer, 1986), the concept of efficiency is used to refer to the solutions in the decision variable space \( \mathbb{R}^n \) (set \( \Lambda \) ) and the concept of dominance is often used to refer to the solutions in the criterion space \( \mathbb{R}^{m+s} \) (set \( T \)).

**Definition 2.1.** A point \( u^* = U\lambda \in T \) is efficient (nondominated) if and only if there does not exist another \( u \in T \) such that \( u \geq u^* \), and \( u \neq u^* \).

Let us consider now (feasible) decision making units \( u \in T \) from the perspective of data envelopment analysis. Set \( T \) is called a Production Possibility Set in the DEA-literature. In data envelopment analysis, we are interested in
recognizing efficient DMUs, which are defined as a subset of points of set $T$ satisfying the efficiency condition defined below:  

**Definition 2.2.** A solution $(Y^*, X^*) = (y^*, x^*)$, $\lambda^* \in \Lambda$, is efficient iff there does not exist another $(y, x) \in T$ such that $y \geq y^*, x \leq x^*$, and $(y, x) \neq (y^*, x^*)$.

Basic models which are presented by khorhonen in [5] are as following:

$$
\sum_{j=1}^{n} \lambda_j y_{rj} - \sigma w^y_r - s^+_r = g^y_r, \quad \forall r \\
\lambda_j \in \Lambda, s^-_i \geq 0, s^+_r \geq 0, \quad \forall i, \forall r, \forall j
$$

where $\varepsilon > 0$ is a so-called non-Archimedean element defined to be smaller than any positive real number.

$$
\mu_t \geq 0, u_r \geq 0, v_i \geq 0, \quad \forall i, \forall r, \forall t
$$

Vector $g^x$ consists of aspiration levels for inputs and $g^y$ of aspiration levels for outputs ($g = (g^x, g^y)$). Vectors $w^x > 0$ and $w^y > 0$ ($w = (w^x, w^y)$) are the weighting vectors for inputs and outputs, respectively. Let $\lambda$'s denote the optimal value of the models $Z^*$ and $W^*$. Therefore we can obtain all the basic DEA models form (2.2) by substituting following table parameters.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>$w^x$</th>
<th>$g^x$</th>
<th>$w^y$</th>
<th>$g^y$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output-Oriented CCR-model</td>
<td>0</td>
<td>$X_p$</td>
<td>0</td>
<td>$Y_p$</td>
<td>$R^+_n$</td>
</tr>
<tr>
<td>Input-Oriented CCR-model</td>
<td>$X_p$</td>
<td>0</td>
<td>0</td>
<td>$Y_p$</td>
<td>$R^+_n$</td>
</tr>
<tr>
<td>Output-Oriented BCC-model</td>
<td>0</td>
<td>$X_p$</td>
<td>$Y_p$</td>
<td></td>
<td>$\Lambda$</td>
</tr>
<tr>
<td>Input-Oriented BCC-model</td>
<td>$X_p$</td>
<td>0</td>
<td>0</td>
<td>$Y_p$</td>
<td>$\Lambda$</td>
</tr>
<tr>
<td>Combined CCR-model</td>
<td>$X_p$</td>
<td>$X_p$</td>
<td>$Y_p$</td>
<td>$Y_p$</td>
<td>$R^+_n$</td>
</tr>
<tr>
<td>Combined BCC-model</td>
<td>$X_p$</td>
<td>$X_p$</td>
<td>$Y_p$</td>
<td>$Y_p$</td>
<td>$\Lambda$</td>
</tr>
<tr>
<td>General Combined model</td>
<td></td>
<td>$X_p$</td>
<td></td>
<td>$Y_p$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Modifications of Model (2.2) for Different (Primal) DEA Models

The input oriented models are usually in DEA solved as a minimization problem by writing $w^x = -X_p$ and modifying the objective function accordingly.
A DMU is efficient iff \( Z^* = W^* = 1 \) and all \( S^- , S^+ \) equal zero; otherwise it is inefficient. All efficient DMUs lie on the frontier, which is defined as subset of points of set \( T \) satisfying the efficiency condition above.

3 Preliminary

Definition 3.1. We represent an arbitrary fuzzy number by an ordered pair of functions \( \tilde{u} = (u(r), \overline{u}(r)) \), \( 0 \leq r \leq 1 \), which satisfy the following requirements:

- \( \underline{u}(r) \) is a bounded left continuous nondecreasing function over \([0,1]\).
- \( \overline{u}(r) \) is a bounded left continuous nonincreasing function over \([0,1]\).
- \( \underline{u}(r) \) and \( \overline{u}(r) \) are right continuous in 0
- \( \underline{u}(r) \leq \overline{u}(r) \), \( 0 \leq r \leq 1 \).
- \( \underline{u}(r) = \overline{u}(r) = 0 \) for \( r < 0 \) or \( r > 1 \)

A crisp number \( \alpha \) is simply represented by \( \underline{u}(r) = \overline{u}(r) = \alpha \), \( 0 \leq r \leq 1 \).

Definition 3.2. The fuzzy number \( \tilde{u} = (M_\tilde{u}, L_\tilde{u}, U_\tilde{u}) \) is a non symmetric triangular fuzzy number \( NSTFN \), which \( M_\tilde{u}, L_\tilde{u}, U_\tilde{u} \in \mathbb{R} \), \( M_\tilde{u} \) is center of \( \tilde{u} \) and \( L_\tilde{u} \) and \( U_\tilde{u} \) are the left and right margins of \( \tilde{u} \).

Definition 3.3. let \( \tilde{u} = (M_\tilde{u}, L_\tilde{u}, U_\tilde{u}), \tilde{t} = (M_\tilde{t}, L_\tilde{t}, U_\tilde{t}) \) in \( NSTFN \) and \( k \in \mathbb{R} \), by using extension principal we can define:

1. \( \tilde{u} = \tilde{t} \) if and only if \( M_\tilde{u} = M_\tilde{t} \) and \( L_\tilde{u} = L_\tilde{t} \) and \( U_\tilde{u} = U_\tilde{t} \).
2. \( \tilde{u} + \tilde{t} = (M_\tilde{u} + M_\tilde{t}, L_\tilde{u} + L_\tilde{t}, U_\tilde{u} + U_\tilde{t}) \).
3. \[
    k\tilde{u} = \begin{cases} 
    (kM_\tilde{u}, kL_\tilde{u}, kU_\tilde{u}), & k \geq 0 \\
    (kM_\tilde{u}, -kU_\tilde{u}, -kL_\tilde{u}), & k < 0 
    \end{cases} \quad (3.2)
\]

Definition 3.4. let \( \tilde{u} = (M_\tilde{u}, L_\tilde{u}, U_\tilde{u}) \in NSTFN \), parametric form of \( \tilde{u} \) is \( u(r) = M_\tilde{u} - L_\tilde{u} + L_\tilde{u}r \) and \( \overline{u}(r) = M_\tilde{u} + U_\tilde{u} - U_\tilde{u}r, 0 \leq r \leq 1 \) which \( M_\tilde{u}, L_\tilde{u}, U_\tilde{u} \in \mathbb{R} \).

Definition 3.5. in the case particular:
if \( r = 1 \) then \( M_\tilde{u} = \text{core}(\tilde{u}) = \overline{u}(1) = \underline{u}(1) \)
if \( r = 0 \) then \( L_\tilde{u} = M_\tilde{u} - \underline{u}(0) \geq 0 \) and \( U_\tilde{u} = \overline{u}(0) - M_\tilde{u} \geq 0 \)

Definition 3.6. For two fuzzy numbers in parametric forms \( \tilde{t} = (t(r), \overline{t}(r)) \)

\( \tilde{u} = (u(r), \overline{u}(r)) \) we have:
Basic DEA models

\[ \tilde{u} = h = (\underline{h}(r), \overline{h}(r)) \]
where
\[ \underline{h}(r) = \min \{ \underline{t}(r) \underline{m}(r), \underline{t}(r) \overline{m}(r), \overline{t}(r) \underline{m}(r), \overline{t}(r) \overline{m}(r) \} \]
\[ \overline{h}(r) = \max \{ \underline{t}(r) \underline{m}(r), \underline{t}(r) \overline{m}(r), \overline{t}(r) \underline{m}(r), \overline{t}(r) \overline{m}(r) \} \]

for example for two positive non symmetric triangular fuzzy numbers
\[ \tilde{t} = (\tilde{t}_1 + \tilde{t}_2(1-r), \tilde{t}_1 + \tilde{t}_2(1-r)) \]

4 Full fuzzy linear programming

In this section we discuss briefly method which Hosseinzadeh Lotfi et.al.[4] is presented for solving following FFLP:

\[
\begin{align*}
\max &\quad \tilde{C}\tilde{X} \\
\text{s.t.} &\quad \tilde{A}\tilde{X} = \tilde{b} \\
\tilde{X} &\geq 0, \quad \tilde{X} \in N.S.T^n
\end{align*}
\]

which \(0 = (0,0,0)\) is the singleton zero, \(\tilde{X} = (X, L, U), \tilde{b} = (b, L_b, U_b), \tilde{C} = (C, L_C, U_C), \tilde{A} = (A, L_A, U_A)\). \(A \in E^{m \times n}, C, b \) and \(\tilde{X} \in E^n\) and \(A, C, \tilde{X}, b\) are arbitrary fuzzy matrix and fuzzy number vectors. \(E\) is the euclidean space of fuzzy numbers.

By assume triangular form model (4.5), FFLP is converted to following:

\[
\begin{align*}
\max &\quad (\tilde{C}, L_C, U_C)(\tilde{X}, L, U) \\
\text{s.t.} &\quad (\tilde{A}, L_A, U_A)(\tilde{X}, L, U) = (b, L_b, U_b) \\
&\quad (\tilde{X}, L, U) \geq 0, \\
&\quad (\tilde{X}, L, U) \in N.S.T^n
\end{align*}
\]

We must solve a maximization problem for the core of the model (4.6), and two problems for the right and left hand side the core of the (4.6). A minimization problem for the left and a maximization problem for the right hand side margin.

So above FFLP is converted to following MOLP:
For solving MOLP model (4.7), we can use MOLP solving methods which are discussed in [5]. Therefore by using lexicography method we can optimize $F_0(\tilde{X})$ objective function on the feasible optimal region and then on the obtained region we can maximize $F_1(\tilde{X}) - F_2(\tilde{X})$ function or we can maximize $MF_0(\tilde{X}) + F_1(\tilde{X}) - F_2(\tilde{X})$ function on feasible region. (M is large number)

5 fuzzy DEA model (FDEA)

Assume that there are $n$, DMUs to be evaluated. Each DMU consumes varying amounts of $m$ different inputs to produce $s$ different outputs. without loss of generality, we assume that all the input and output data cannot be exactly obtained due to the existence of uncertainty. They are approximately known and can be by positive triangular fuzzy numbers. Especially, $DMU_j$ consumes amounts $\tilde{X}_j = \{\tilde{x}_{ij}\}$ of inputs ($i=1,2,...,m$) and produces amounts $\tilde{Y}_j = \{\tilde{y}_{rj}\}$ of outputs ($r=1,2,...,s$). All inputs and outputs are assumed to be nonnegative, namely

$$M_{\tilde{x}_{ij}} - L_{\tilde{x}_{ij}} \geq 0 \text{, } i = 1, 2, ..., m \text{, } j = 1, 2, ..., n$$

$$M_{\tilde{y}_{rj}} - L_{\tilde{y}_{rj}} \geq 0 \text{, } r = 1, 2, ..., s \text{, } j = 1, 2, ..., n$$

but at least one input and output are positive. So we assume model (2.2) in the full fuzzy case.

$$\begin{align*}
\text{Max} \quad & (M_{\tilde{\sigma}}, L_{\tilde{\sigma}}, U_{\tilde{\sigma}}) + e \left[ \sum_{i=1}^{m} (M_{\tilde{\sigma}_i^-}, L_{\tilde{\sigma}_i^-}, U_{\tilde{\sigma}_i^-}) + \sum_{r=1}^{s} (M_{\tilde{\sigma}_r^+}, L_{\tilde{\sigma}_r^+}, U_{\tilde{\sigma}_r^+}) \right] \\
\text{s.t.} \quad & \sum_{j=1}^{n} (M_{\tilde{\lambda}_j}, L_{\tilde{\lambda}_j}, U_{\tilde{\lambda}_j}) (M_{\tilde{x}_{ij}}, L_{\tilde{x}_{ij}}, U_{\tilde{x}_{ij}}) + (M_{\tilde{\sigma}}, L_{\tilde{\sigma}}, U_{\tilde{\sigma}}) (M_{\tilde{w}_i^x}, L_{\tilde{w}_i^x}, U_{\tilde{w}_i^x}) \\
& \quad + (M_{\tilde{\sigma}_i^-}, L_{\tilde{\sigma}_i^-}, U_{\tilde{\sigma}_i^-}) =* (M_{\tilde{g}_{i^x}}, L_{\tilde{g}_{i^x}}, U_{\tilde{g}_{i^x}}) \quad \forall i \\
& \sum_{j=1}^{n} (M_{\tilde{\lambda}_j}, L_{\tilde{\lambda}_j}, U_{\tilde{\lambda}_j}) (M_{\tilde{y}_{rj}}, L_{\tilde{y}_{rj}}, U_{\tilde{y}_{rj}}) - (M_{\tilde{\sigma}}, L_{\tilde{\sigma}}, U_{\tilde{\sigma}}) (M_{\tilde{w}_r^y}, L_{\tilde{w}_r^y}, U_{\tilde{w}_r^y}) \\
& \quad - (M_{\tilde{\sigma}_r^+}, L_{\tilde{\sigma}_r^+}, U_{\tilde{\sigma}_r^+}) =* (M_{\tilde{g}_{r^y}}, L_{\tilde{g}_{r^y}}, U_{\tilde{g}_{r^y}}) \quad \forall r \\
& (M_{\tilde{\sigma}_i^-}, L_{\tilde{\sigma}_i^-}, U_{\tilde{\sigma}_i^-}) \geq* 0, \quad (M_{\tilde{\sigma}_r^+}, L_{\tilde{\sigma}_r^+}, U_{\tilde{\sigma}_r^+}) \geq* 0, \quad (M_{\tilde{\lambda}_j}, L_{\tilde{\lambda}_j}, U_{\tilde{\lambda}_j}) \geq* 0 \quad \forall i, \forall r, \forall j \quad (5.5)
\end{align*}$$

FFLP model (5.8) contains parameters $(M_{\tilde{w}_i^x}, L_{\tilde{w}_i^x}, U_{\tilde{w}_i^x})$, $(M_{\tilde{w}_r^y}, L_{\tilde{w}_r^y}, U_{\tilde{w}_r^y})$, $(M_{\tilde{g}_{i^x}}, L_{\tilde{g}_{i^x}}, U_{\tilde{g}_{i^x}})$, $(M_{\tilde{g}_{r^y}}, L_{\tilde{g}_{r^y}}, U_{\tilde{g}_{r^y}})$. Hence by converting FFLP model (5.8) to MOLP model (5.9) which is a corresponding models (2.2) for DEA basic models, it is obtained as following:
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\[ M_{\bar{y}_i} = M_{\bar{g}_i}^* \quad \forall i \]

\[
\sum_{j=1}^{n} M_{\bar{y}_r,j} M_{\bar{X}_j} - M_{\bar{w}_r,y} M_{\bar{\sigma}} - \\
M_{s_r} + = M_{\bar{g}_r,y} \quad \forall r
\]

\[
\sum_{j=1}^{n} M_{\bar{x}_i,j} L_{\bar{X}_j}^* + M_{\bar{w}_i} L_{\bar{\sigma}} + \\
L_{s_i} - + \sum_{j=1}^{n} L_{\bar{x}_i,j} (M_{\bar{X}_j} - L_{\bar{X}_j}) + L_{\bar{w}_i} (M_{\bar{\sigma}} - L_{\bar{\sigma}}) = L_{\bar{g}_i} - \quad \forall i
\]

\[
\sum_{j=1}^{n} M_{\bar{y}_r,j} L_{\bar{X}_j} - M_{\bar{w}_r,y} L_{\bar{\sigma}} - \\
L_{s_r} + + \sum_{j=1}^{n} L_{\bar{y}_r,j} (M_{\bar{X}_j} - L_{\bar{X}_j}) + U_{\bar{w}_r} (M_{\bar{\sigma}} + U_{\bar{\sigma}}) = L_{\bar{g}_r} - \quad \forall r
\]

\[
\sum_{j=1}^{n} M_{\bar{x}_i,j} U_{\bar{X}_j} + M_{\bar{w}_i} U_{\bar{\sigma}} + \\
U_{s_i} - + \sum_{j=1}^{n} U_{\bar{x}_i,j} (M_{\bar{X}_j} + U_{\bar{X}_j}) + U_{\bar{w}_i} (M_{\bar{\sigma}} + U_{\bar{\sigma}}) = L_{\bar{g}_i} - \quad \forall i
\]

\[
\sum_{j=1}^{n} M_{\bar{y}_r,j} U_{\bar{X}_j} - M_{\bar{w}_r,y} U_{\bar{\sigma}} - \\
U_{s_r} + + \sum_{j=1}^{n} U_{\bar{y}_r,j} (M_{\bar{X}_j} + U_{\bar{X}_j}) + L_{\bar{w}_r} (M_{\bar{\sigma}} + U_{\bar{\sigma}}) = L_{\bar{g}_r} - \quad \forall r
\]

\[
M_{\bar{X}_j} - L_{\bar{X}_j} \geq 0, M_{\bar{s}_i} - - L_{\bar{s}_i} \geq 0, M_{\bar{s}_r} + - L_{\bar{s}_r} \geq 0 \\
L_{\bar{X}_j} \geq 0, \bar{U}_{\bar{X}_j} \geq 0, L_{\bar{s}_i} \geq 0, L_{\bar{s}_r} \geq 0, U_{\bar{s}_i} \geq 0, U_{\bar{s}_r} \geq 0, \quad \forall i, \forall r, \forall j
\]

With substituting parameters \((M_{\bar{w}_i}^*, L_{\bar{w}_i}^*, U_{\bar{w}_i}^*) = (M_{\bar{x}_ip}, L_{\bar{x}_ip}, U_{\bar{x}_ip})\), \((M_{\bar{w}_r,y}, L_{\bar{w}_r,y}, U_{\bar{w}_r,y}) = (0, 0, 0)\), \((M_{\bar{g}_i}^*, L_{\bar{g}_i}^*, U_{\bar{g}_i}^*) = (0, 0, 0)\), \((M_{\bar{g}_r,y}, L_{\bar{g}_r,y}, U_{\bar{g}_r,y}) = (M_{\bar{y}_{rp}}, L_{\bar{y}_{rp}}, U_{\bar{y}_{rp}})\). In model (5.9), corresponding MOLP of model CCR in input oriented in the full fuzzy case for evaluating DMU \(p\) is following:

\[ M_{\bar{y}_i} = 0 \quad \forall i \]

\[
\sum_{j=1}^{n} M_{\bar{y}_r,j} M_{\bar{X}_j} - \\
M_{s_r} + = M_{\bar{g}_r,p} \quad \forall r
\]

\[
\sum_{j=1}^{n} M_{\bar{x}_i,j} L_{\bar{X}_j} + M_{\bar{x}_ip} L_{\bar{\sigma}} + \\
\]
\[
\begin{align*}
L\tilde{s}_i^- + \sum_{j=1}^n L\tilde{x}_{ij} (M\tilde{\lambda}_j - L\tilde{\lambda}_j) + L\tilde{x}_{ip} (M\tilde{\sigma} - L\tilde{\sigma}) &= L\tilde{x}_{ip}, \quad \forall i \\
L\tilde{s}_r^+ + \sum_{j=1}^n L\tilde{y}_{rj} (M\tilde{\lambda}_j - L\tilde{\lambda}_j) + U\tilde{w}_r (M\tilde{\sigma} - L\tilde{\sigma}) &= M\tilde{y}_{rp}, \quad \forall r \\
\sum_{j=1}^n M\tilde{x}_{ij} U\tilde{\lambda}_j + M\tilde{x}_{ip} U\tilde{\sigma} + U\tilde{s}_i^- + \sum_{j=1}^n U\tilde{x}_{ij} (M\tilde{\lambda}_j + U\tilde{\lambda}_j) + U\tilde{x}_{ip} (M\tilde{\sigma} + U\tilde{\sigma}) &= 0, \quad \forall i \\
\sum_{j=1}^n M\tilde{y}_{rj} U\tilde{\lambda}_j + U\tilde{s}_r^+ + \sum_{j=1}^n U\tilde{y}_{rj} (M\tilde{\lambda}_j + U\tilde{\lambda}_j) &= U\tilde{y}_{rp}, \quad \forall r \\
M\tilde{\lambda}_j - L\tilde{\lambda}_j &\geq 0, M\tilde{s}_i^- - L\tilde{s}_i^- \geq 0, M\tilde{s}_r^+ - L\tilde{s}_r^+ \geq 0 \\
L\tilde{\lambda}_j &\geq 0, U\tilde{\lambda}_j \geq 0, L\tilde{s}_i^- \geq 0, U\tilde{s}_i^- \geq 0, L\tilde{s}_r^+ \geq 0, U\tilde{s}_r^+ \geq 0, \quad \forall i, \forall r, \forall j
\end{align*}
\]

Therefore for solving model (5.8) we can maximize \(Z_0\) objective function and then on optimal feasible space maximize minus of the first and second objective functions.
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Hence by converting FFLP model to MOLP model (5.11) which is a corresponding models (2.3) for DEA basic models, it is obtained as following:

\[
\text{s.t. } \sum_{r=1}^{s} M_{\bar{u}_r} M_{\bar{y}_{rj}} - \sum_{i=1}^{m} M_{\bar{v}_i} M_{\bar{x}_{ij}} + M_{\bar{l}_j} = 0
\]

\[
\sum_{r=1}^{s} M_{\bar{u}_r} M_{\bar{w}_{y,r}} + \sum_{i=1}^{m} M_{\bar{v}_i} M_{\bar{w}_{x,i}} = 1
\]

\[
\sum_{r=1}^{m} L_{\bar{u}_r} M_{\bar{y}_{rj}} - \sum_{i=1}^{m} L_{\bar{v}_i} M_{\bar{x}_{ij}} + \sum_{r=1}^{s} M_{\bar{u}_r} L_{\bar{y}_{rj}}
\]

\[- \sum_{i=1}^{s} M_{\bar{v}_i} L_{\bar{x}_{ij}} - \sum_{r=1}^{s} L_{\bar{u}_r} L_{\bar{y}_{rj}} + \sum_{i=1}^{s} L_{\bar{v}_i} L_{\bar{x}_{ij}} + L_{\bar{i}_j} = 0 \quad \forall j \]

\[
\sum_{r=1}^{s} L_{\bar{u}_r} M_{\bar{w}_{y,r}} - \sum_{i=1}^{s} L_{\bar{v}_i} M_{\bar{w}_{x,i}} + \sum_{r=1}^{s} M_{\bar{u}_r} L_{\bar{w}_{x,r}} - \sum_{i=1}^{s} M_{\bar{v}_i} L_{\bar{w}_{x,i}}
\]

\[- \sum_{r=1}^{s} L_{\bar{u}_r} L_{\bar{w}_{y,r}} - \sum_{i=1}^{s} L_{\bar{v}_i} L_{\bar{w}_{x,i}} = 0 \quad \forall i \]

\[
\sum_{r=1}^{s} U_{\bar{u}_r} M_{\bar{y}_{rj}} - \sum_{i=1}^{s} U_{\bar{v}_i} M_{\bar{x}_{ij}} + \sum_{r=1}^{m} M_{\bar{y}_{rj}} U_{\bar{y}_{rj}} - \sum_{i=1}^{m} M_{\bar{v}_i} U_{\bar{x}_{ij}}
\]

\[+ \sum_{r=1}^{s} U_{\bar{u}_r} U_{\bar{y}_{rj}} - \sum_{i=1}^{s} U_{\bar{v}_i} U_{\bar{x}_{ij}} + U_{\bar{i}_j} = 0 \quad \forall j \]

\[
\sum_{r=1}^{s} U_{\bar{u}_r} M_{\bar{w}_{y,r}} + \sum_{i=1}^{s} U_{\bar{v}_i} M_{\bar{w}_{x,i}} + \sum_{r=1}^{s} M_{\bar{u}_r} U_{\bar{w}_{y,r}} + \sum_{i=1}^{s} M_{\bar{v}_i} U_{\bar{w}_{x,i}}
\]

\[+ \sum_{r=1}^{s} U_{\bar{u}_r} U_{\bar{w}_{y,r}} + \sum_{i=1}^{s} U_{\bar{v}_i} U_{\bar{w}_{x,i}} = 0 \]

\[
M_{\bar{x}_{r}} - L_{\bar{u}_r} \geq 0, M_{\bar{y}_{i}} - L_{\bar{v}_i} \geq 0, M_{\bar{x}_{j}}, L_{\bar{y}_{j}} \geq 0,
\]

\[
L_{\bar{u}_r} \geq 0, U_{\bar{u}_r} \geq 0, L_{\bar{v}_i} \geq 0, U_{\bar{v}_i} \geq 0, L_{\bar{x}_j} \geq 0, U_{\bar{x}_j} \geq 0,
\quad \forall i, \forall r, \forall j \]

With substituting parameters \((M_{\bar{w}_{x,i}}, L_{\bar{w}_{x,r}}, U_{\bar{w}_{x,i}}) = (M_{\bar{x}_{ip}}, L_{\bar{x}_{ip}}, U_{\bar{x}_{ip}})\), \((M_{\bar{w}_{y,r}}, L_{\bar{w}_{y,r}}, U_{\bar{w}_{y,r}}) = (0, 0, 0)\), \((M_{\bar{y}_{rj}}, L_{\bar{y}_{rj}}, U_{\bar{y}_{rj}}) = (0, 0, 0)\), \((M_{\bar{y}_{rj}}, L_{\bar{y}_{rj}}, U_{\bar{y}_{rj}}) = (M_{\bar{o}_{ip}}, L_{\bar{o}_{ip}}, U_{\bar{o}_{ip}})\). In model (5.9), corresponding MOLP of model multiple CCR min input oriented in the full fuzzy case for evaluating \(DMU_p\) is following:
\[
\begin{align*}
\text{s.t.} \quad & \sum_{r=1}^{s} M_{\tilde{u}_r} M_{\tilde{y}_{rj}} - \sum_{i=1}^{m} M_{\tilde{v}_i} M_{\tilde{x}_{ij}} + M_{\tilde{s}_j} = 0 \quad \forall j \\
& \sum_{i=1}^{m} M_{\tilde{v}_i} M_{\tilde{x}_{ip}} = 1 \\
& \sum_{r=1}^{s} L_{\tilde{u}_r} M_{\tilde{y}_{rj}} - \sum_{i=1}^{m} L_{\tilde{v}_i} M_{\tilde{x}_{ij}} + \sum_{r=1}^{s} M_{\tilde{u}_r} L_{\tilde{y}_{rj}} - \sum_{i=1}^{m} M_{\tilde{v}_i} L_{\tilde{x}_{ij}} \\
& \quad - \sum_{r=1}^{s} L_{\tilde{u}_r} L_{\tilde{y}_{rj}} + \sum_{i=1}^{m} L_{\tilde{v}_i} L_{\tilde{x}_{ij}} + L_{\tilde{s}_j} = 0 \quad \forall j \\
& \sum_{i=1}^{m} L_{\tilde{v}_i} M_{\tilde{x}_{ip}} + \sum_{i=1}^{m} M_{\tilde{v}_i} L_{\tilde{x}_{ip}} - \sum_{i=1}^{m} L_{\tilde{v}_i} L_{\tilde{x}_{ip}} = 0 \\
& \sum_{r=1}^{s} U_{\tilde{u}_r} M_{\tilde{y}_{rj}} - \sum_{i=1}^{m} U_{\tilde{v}_i} M_{\tilde{x}_{ij}} + \sum_{r=1}^{s} M_{\tilde{u}_r} U_{\tilde{y}_{rj}} - \sum_{i=1}^{m} M_{\tilde{v}_i} U_{\tilde{x}_{ij}} \\
& \quad + \sum_{r=1}^{s} U_{\tilde{u}_r} U_{\tilde{y}_{rj}} - \sum_{i=1}^{m} U_{\tilde{v}_i} U_{\tilde{x}_{ij}} + U_{\tilde{s}_j} = 0 \quad \forall j \\
& \sum_{i=1}^{m} U_{\tilde{v}_i} M_{\tilde{x}_{ip}} + \sum_{i=1}^{m} M_{\tilde{v}_i} U_{\tilde{x}_{ip}} + \sum_{i=1}^{m} U_{\tilde{v}_i} U_{\tilde{x}_{ip}} = 0 \\
& M_{\tilde{u}_r} - L_{\tilde{u}_r} \geq 0, M_{\tilde{v}_i} - L_{\tilde{v}_i} \geq 0, M_{\tilde{s}_j}, L_{\tilde{s}_j} \geq 0, \\
& L_{\tilde{u}_r} \geq 0, U_{\tilde{u}_r} \geq 0, L_{\tilde{v}_i} \geq 0, U_{\tilde{v}_i} \geq 0, L_{\tilde{s}_j} \geq 0, U_{\tilde{s}_j} \geq 0, \\
& \forall i, \forall r, \forall j
\end{align*}
\]

Therefore for solving model (5.12) we can maximize \( W_0 \) objective function and then on optimal feasible space maximize minus of the first and second objective functions.

In above models, efficiency according to DMU is a number triangular that it obtain by solving a model. Therefore we can classify them by comparing number triangular corresponding to DMUs.
6 Conclusion

By use of full fuzzy linear programming, each of basic DEA models is converted to a MOLP. So general forms of basic DEA models in the full fuzzy case is suggested. Hence using basic DEA modelsin the full fuzzy case for evaluating DMUs in the place only using fuzzy inputs and outputs in more useful and more real. Therefore searching efficient frontier and other methods of solving FFLP for basic DEA models can be an interesting and useful issue for future research.

References


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