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Failure Mode Screening Using Fuzzy Set Theory

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Abstract. The fuzzy sets possess natural capability to capture measurement uncertainty as a part of experimental data. Therefore the uncertainties of various kinds involved in system failure analysis may be adequately addressed using the concept of fuzzy sets. Fonseca et al developed and implemented fuzzy reasoning algorithm via an expert system to assess the likelihood of equipment failure mode. The algorithm developed by them is a fuzzy mathematical formulation which linearly relates the presence of different failure causing factors (FCFs) categorized as critical, important and related. It has been observed that the failure data may not be collected accurately and ignorance of uncertainty of any sort may lead to substantial errors. Unlike Fonseca et al, we propose that the FCFs should be measured by a fuzzy technique with desired accuracy. Also, it is quite obvious that the weights of FCFs are vital in failure mode screening. Thus the weights of FCFs should be linguistic and to be assigned appropriate fuzzy numbers in place of crisp numbers. The proposed technique is implemented to the same example taken by Fonseca et al.

Keywords: Failure mode, Fuzzy ranking, Expert system, Fuzzification, Defuzzification

1. Introduction:

Reliable engineering plays vital and appreciable role in the design and development of a technical system. Earlier considerable work has been done in the discipline of reliability engineering for developing a reliable system for military application, space exploration etc. In earlier stage the reliability of system was described in terms of various statistical measures. Zadeh [10] suggested a paradigm shift from the theory of total denial and affirmation to a theory of grading, to give new concept fuzzy sets. Fuzzy sets can express the gradual transition of the system from a working state to a failed state. Cai et al [8] was the first to observe that, because of uncertainties and inaccuracy of data, the estimation of precise values of probability is very difficult in many systems. Thus in [8] Cai et al introduced different form of fuzzy reliability on replacing two basic assumptions of conventional reliability with (i) Fuzzy state assumption and (ii) Possibility assumption. Cheng et al [2] introduced a new method for fuzzy reliability evaluation with the help of interval of confidence. Singer [4] applied fuzzy set theory in fault tree analysis, where relative frequencies of the basic events are represented by different fuzzy numbers. In [5] Fonseca and Knapp observed that conventional analytic techniques developed to address issues such as mathematical and statistical models require the knowledge of precise numerical probabilities and component functional dependencies, information which is rarely available to practitioners in real life. This type of difficulty can be handled by the concept of approximate reasoning and therefore they developed a fuzzy scheme for failure mode screening.

In our work after delve we propose a technique to screen out the failure modes as follows.

First, is the classification and fuzzification of failure causing factors (FCFs). Fonseca et al developed a fuzzy scheme for failure mode screening for plugged or fouled tubes of a machine and observed four FCFs as (i) High temperature of shell substance, (ii) low cross flow velocity, (iii) High liquid viscosity of shell and (iv) Acidity or basicity of shell substance.

These four FCFs are categorized as critical, important and related factors. In the present paper too, we have adopted the similar classification and fuzzification of FCFs of the system of discourse.

Second the measurement of FCFs is crucial for failure analysis of a system. Unlike Fonseca et al, the FCFs are measured by a parameter estimation technique based on fuzzy sets [3], where appropriate fuzzy numbers are assigned to the different numerical values of FCFs taken by an expert using conventional methods.

The remainder of this paper includes the analysis of weight of FCFs and possibility of failure mode. Since in real life, during the typical consultation with experts the expert may answer in form of linguistic terms like ‘nearly’, ‘close to’ etc. Thus the weights of FCFs in different failure modes are also fuzzified that gives us the possibility of a failure mode as a fuzzy number. Using the ordering of fuzzy numbers [6], we compare the possibility of failure mode with another fuzzy number representing the threshold value. Finally we present few concluding comments to demonstrate that if fuzzy technique is used, one may get more realistic results.

2. Classification and fuzzification of failure causing factors

i. Classification of factors provoking system failure The failure of a system may be provoked by various factors termed as failure causing factors (FCFs). In our discussion too the factors provoking system failure are categorized as ***critical, important*** and ***related*** factors. A critical factor is a situation whose impact on the development of a failure mode can be categorized as determinant. A critical factor is different from important and related factor in the sense that its occurrence in its full strength confirms the failure unlike to the other factors. An important factor also plays a vital role in the development of a failure mode, but do not cause the failure of the system, even if it occurs with its full strength. On the other hand a related factor has a very low contribution in the development of a failure mode.

ii. Fuzzification of failure causing factors The FCFs may be fuzzified by associating adequate fuzzy set to these factors. This facilitates us to quantify the contribution of a particular factor by a fuzzy number between 0 and 1

Let f_i be the factor causing the failure of a system. Then this factor may be fuzzified as:

$$f_i(x) = \begin{cases} 0 & for\ \ x \le a\ \ or\ x \ge c \\ \dfrac{x-a}{b-a} & for\ \ a \le x \le b \\ 1 & for\ \ b \le x \le c \\ \dfrac{d-x}{d-c} & for\ c \le x \le d \end{cases} \qquad \text{--- (1)}$$

where x denote the numerical value of factor *i*. This factor *i* may be triangular/trapezoidal fuzzy number or fuzzy number of any other type. The fuzzification of these factors depends on the nature of occurrence during any experimental or functioning mode.

In this paper these factors have been fuzzzified on assigning them trapezoidal fuzzy numbers of distinct shapes. The shapes of these fuzzy numbers depend on the nature of the involved factor i.e. what sort of behaviour a factor imparts with the change in its numerical value. Fuzzification of the precipitating factors has been done similar to that of Fonseca et al in [5], where for the plugged or fouled tubes of Machine A the following FCFs has been defined. .

(a) High Temperature of shell substance {150-800^{0}F}. Corresponding trapezoidal fuzzy number to this factor in accordance to their magnitude limit is given as:

$$f_a\,(x) = \begin{cases} 0 & if \quad x \le 150^0 F, \\ .0015x - .23 & if \quad 150^0 F < x < 800^0 F \\ 1 & if \quad x \ge 800^0 F \end{cases} \qquad \text{--- (2)}$$

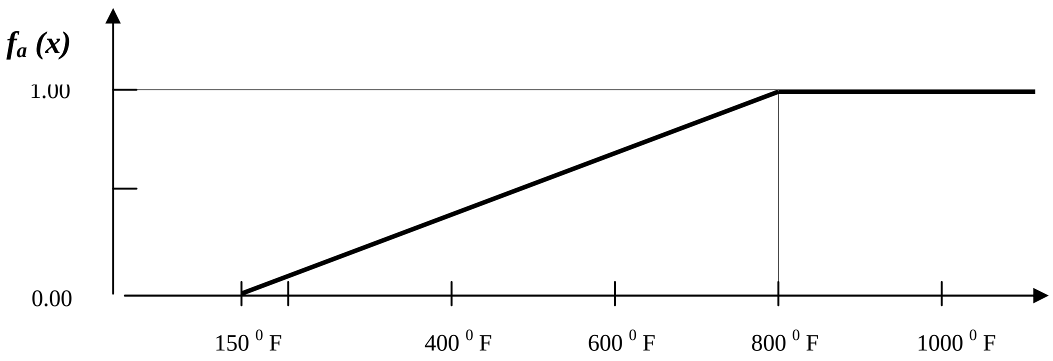

Figure 1 Fuzzification of factor 'a'

(b)Low cross flow velocity on shell {15-3 f/s}: The corresponding trapezoidal fuzzy number in accordance to its magnitude is described as:

$$f_b(x) = \begin{cases} 0 & if \;\; x \ge 15\, ft/s, \\ -.083x + 1.25 & if \;\; 15\, ft/s > x > 3\; ft/s, \\ 1 & if \;\; x \le 3\; ft/s, \end{cases} \quad \text{--- (3)}$$

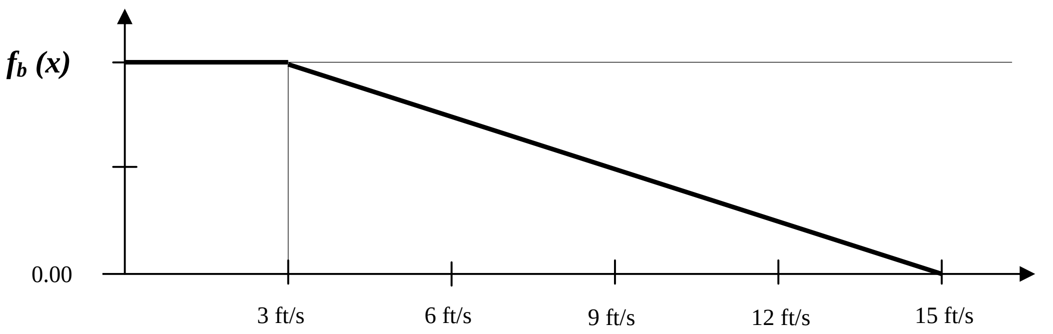

Figure 2 Fuzzification of factor 'b'

(c) High liquid viscosity on shell {100-700SSU}: This factor is fuzzified by assigning the following fuzzy number:

$$f_c\;(x) = \begin{cases} 0 & if \;\; x \le 100\, SSU, \\ .0016x - .16 & if \;\; 100\, SSU < x < 700\, SSU \\ 1 & if \;\; x \ge 700\, SSU \end{cases} \quad \text{--- (4)}$$

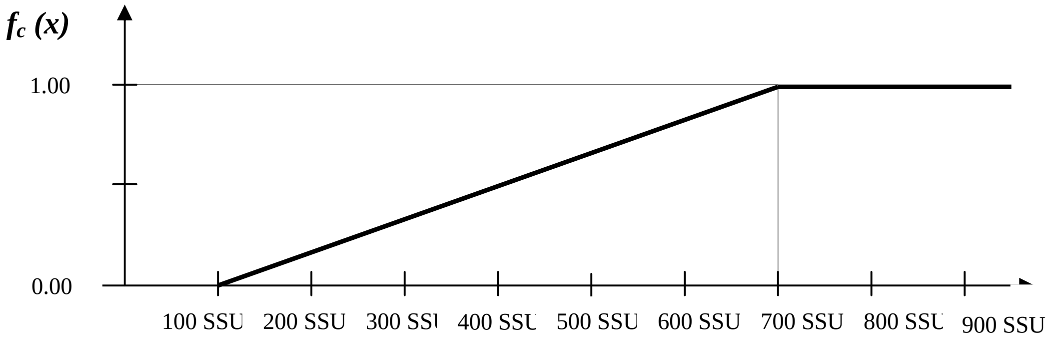

Figure 3 Fuzzification of factor ‘c’

(d) Acidity or basicity of shell substance {pH: 0-4 or 10-14}: To fuzzify this factors the following fuzzy set is used.

$$f_d(x) = \begin{cases} -.25x+1 & if\ \ 0 < x < 4, \\ 0 & if\ \ 4 \le x \le 10, \\ .25x-2.5 & if\ \ 10 < x < 14 \\ 1 & if\ \ x = 0\, or\, x = 14, \end{cases} \qquad \text{--- (5)}$$

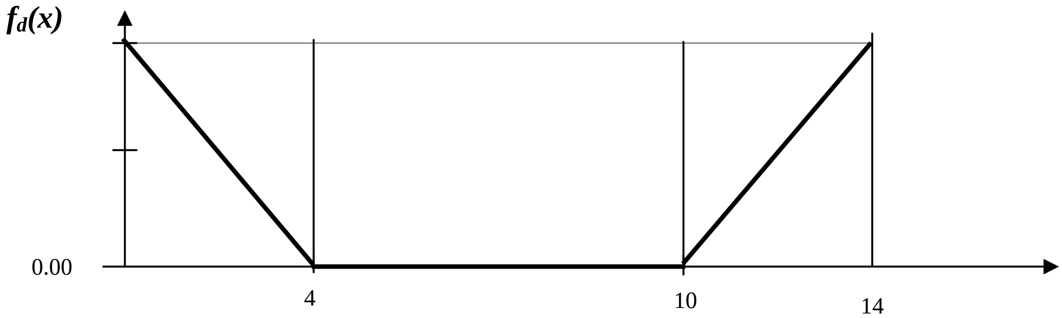

Figure 4 Fuzzification of factor‘d’

3. Measure of failure causing factors

It is a well-known fact that the system analysis is completely based on failure causing factors and therefore the measurement of these factors is of great importance. Although the best effective tools are usually used in the measurement of these factors, and the experts get very alert to note their value. But at the time of measurement of FCFs, a major part of uncertainties might be avoided for the sake of simplicity and some errors might not be considered. For example suppose during the consultation with expert the temperature is ‘about 300^0F’, then it’s not realistic to consider this value as 300^0F exactly, since it may have any value in the neighborhood of 300^0F.

Also if multiple failure data values need to be obtained under similar operating conditions or if operating conditions, in all cases, cannot be exactly defined and contain vagueness concerning the description. In other way it may involve human judgment, evaluation and decision at certain stages, which may be vague.

Under the above potential source of error, it is appropriate to deal with the measurement of failure causing factors by fuzzy techniques. Here we employ the technique developed by Pandey and Tyagi [3] using the concepts of trapezoidal and triangular fuzzy numbers [1, 11] to measure FCFs.

Using this method the FCFs are measured in the following steps.
(a) FCF is estimated according to some existing procedure. The process must be repeated so that more than one number is available for estimating the failure rate.
(b) Numbers resulting from step (a) are fuzzified by assigning appropriate fuzzy numbers obtained to each of them.
(c) Get a single fuzzy set by taking the fuzzy union of the fuzzy numbers obtained through step (b).
(d) Defuzzify the fuzzy set of step (c), to get a single crisp (non fuzzy) number as the final estimate for failure causing factor. Defuzzification [7, 12] is the process that creates a single assessment from fuzzy conclusion set.
(e)

Here we have also categorized the concerned failure causing factors as
(i) Critical factors, (ii) Important factors and (iii) Related factors

Using this technique, first we measure critical factor. If during a consultation of the prototype, the temperature (*T*) of the shell fluid was found to be at 235^0 F, 300^0 F and 340^0 F. Instead of taking their mean, step (b) suggests to define three fuzzy numbers $\tilde{T}_1$, $\tilde{T}_2$ and $\tilde{T}_3$, about 235^0 F, 300^0 F and 340^0 F respectively.

$$\tilde{T}_1(x) = \begin{cases} \dfrac{x-200}{35} & 200 < x \le 235 \\ \dfrac{275-x}{40} & 235 < x \le 275 \end{cases}, \quad \tilde{T}_2(x) = \begin{cases} \dfrac{x-255}{45} & 255 < x \le 300 \\ \dfrac{360-x}{60} & 300 < x \le 360 \end{cases},$$

$$\tilde{T}_3(x) = \begin{cases} \dfrac{x-290}{50} & 290 < x \le 340 \\ \dfrac{385-x}{45} & 340 < x \le 385 \end{cases}$$

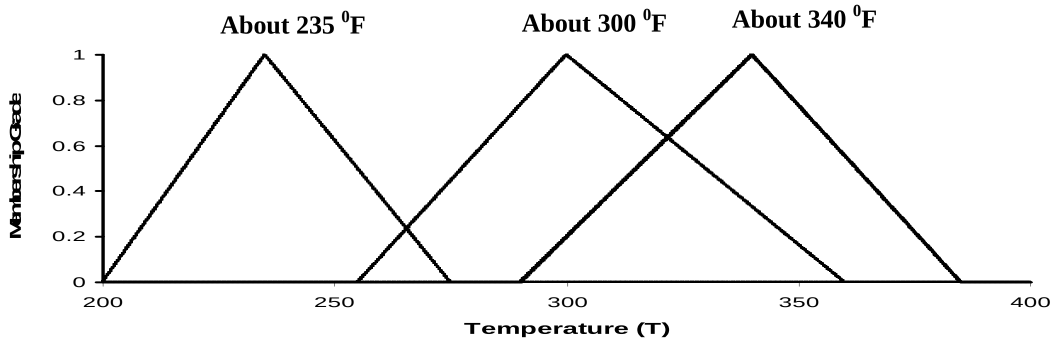

Figure 5

Then as per step(c) we take the fuzzy union of $\tilde{T}_1$, $\tilde{T}_2$ and $\tilde{T}_3$ given by a single fuzzy number $\tilde{T}$.

$$\tilde{T}(x) = \begin{cases} \dfrac{x-200}{35} & 200 < x \le 235 \\ \dfrac{275-x}{40} & 235 < x \le 265.59 \\ \dfrac{x-255}{45} & 265.59 < x \le 300 \\ \dfrac{360-x}{60} & 300 < x \le 321.82 \\ \dfrac{x-290}{50} & 321.82 < x \le 340 \\ \dfrac{385-x}{45} & 340 < x \le 385. \end{cases} \quad \text{--- (6)}$$

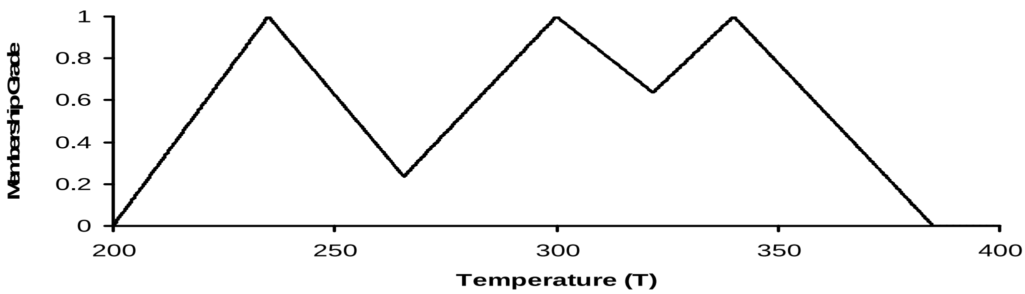

Figure 6

The last step (d) calls for the defuzzification of $\tilde{T}$. Several techniques for defuzzification are available [7, 12]. Centroid method of defuzzication is moderately common technique that considers the area covered by the fuzzy number and returns the center of gravity of the covered area as the required non-fuzzy number.

$$T(non-\ fuzzy\, number) = \frac{\int_C x\tilde{T}(x)\ dx}{\int_C \tilde{T}(x)\, dx} \qquad \text{--- (7)}$$

Using (7) the defuzzification of $\tilde{T}(x)$ yields T= 278.6593

In a similar manner we can measure the remaining FCFs. If the speed of the shell fluid (v) was found to be at 8 ft/s, 12ft/s and 15 ft/s on consultation with prototype expert system. We define three fuzzy numbers $\tilde{v}_1$, $\tilde{v}_2$ and $\tilde{v}_3$, almost 8ft/s, 12ft/s, and 15ft/s.respectively.

$$\tilde{v}_1(x) = \begin{cases} \frac{x-3}{4} & 3 < x < 7 \\ 1 & 7 < x < 9 \\ \frac{12-x}{3} & 9 < x < 12 \end{cases}, \quad \tilde{v}_2(x) = \begin{cases} \frac{x-6}{5} & 6 < x < 11 \\ 1 & 11 < x < 13 \\ \frac{16-x}{3} & 13 < x < 16 \end{cases},$$

$$\tilde{v}_3(x) = \begin{cases} \frac{x-11}{3} & 11 < x < 14 \\ 1 & 14 < x < 16 \\ \frac{20-x}{4} & 16 < x < 20 \end{cases}$$

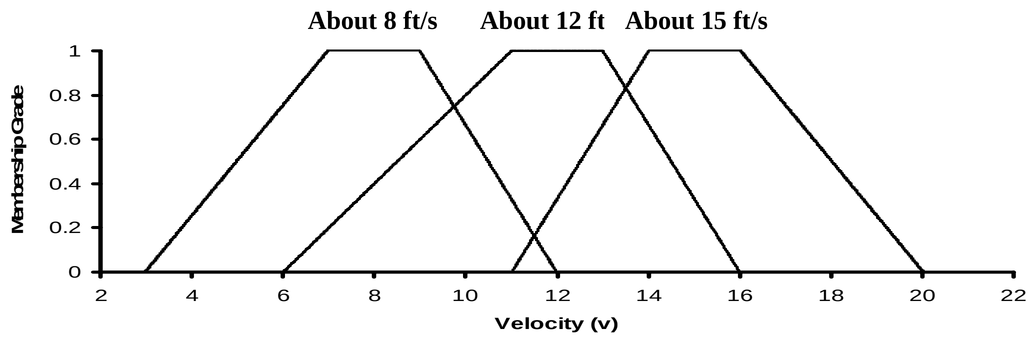

Figure 7

In accordance with the step (c) the union of these fuzzy numbers is given by $\tilde{v}$ where

$$\tilde{v}(x)=\begin{cases}\dfrac{x-3}{4} & 3<x\le 7\\ 1 & 7<x\le 9\\ \dfrac{12-x}{3} & 9<x\le 9.75\\ \dfrac{x-6}{5} & 9.75<x\le 11\\ 1 & 11<x\le 13\\ \dfrac{16-x}{3} & 13<x\le 13.5\\ \dfrac{x-11}{3} & 13.5<x\le 14\\ 1 & 14<x\le 16\\ \dfrac{20-x}{4} & 16<x\le 20\end{cases} \qquad \text{--- (8)}$$

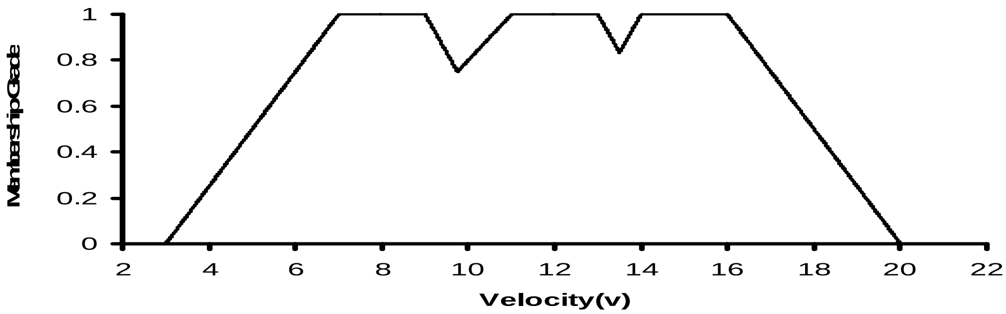

Figure 8

The defuzzification of (8) gives *v*= 11.5191.

Likewise another important factor viscosity of the shell fluid was found to be 600SSU, 650SSU and 680SSU on consultation with the prototype experts system. Three fuzzy numbers are defined as $\tilde{V}_1$, $\tilde{V}_2$ and $\tilde{V}_3$, near to 600SSU, 650SSU and 680SSU respectively.

$$\tilde{V}_1(x)=\begin{cases}\dfrac{x-580}{10} & 580<x\le 590\\ 1 & 590<x\le 610\\ \dfrac{620-x}{10} & 610<x\le 620\end{cases},\qquad \tilde{V}_2(x)=\begin{cases}\dfrac{x-610}{25} & 610<x\le 635\\ 1 & 635<x\le 660\\ \dfrac{670-x}{10} & 660<x\le 670\end{cases}$$

$$\tilde{V}_3(x) = \begin{cases} \dfrac{x-650}{25} & 650 < x \le 675 \\ 1 & 675 < x \le 700 \\ \dfrac{720-x}{20} & 700 < x \le 720 \end{cases}$$

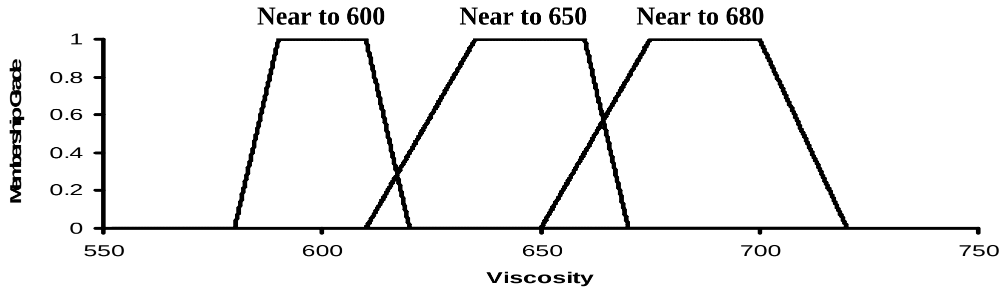

Figure 9

The fuzzy set $\tilde{V}$ gives the union of fuzzy numbers $\tilde{V}_1$, $\tilde{V}_2$ and $\tilde{V}_3$

$$\tilde{V}(x) = \begin{cases} \dfrac{x-580}{10} & 580 < x \le 590 \\ 1 & 590 < x \le 610 \\ \dfrac{620-x}{10} & 610 < x \le 617.14286 \\ \dfrac{x-610}{25} & 617.14286 < x \le 635 \\ 1 & 635 < x \le 660 \\ \dfrac{670-x}{10} & 660 < x \le 664.2857 \\ \dfrac{x-650}{25} & 664.2857 < x \le 675 \\ 1 & 675 < x \le 700 \\ \dfrac{720-x}{20} & 700 < x \le 720 \end{cases} \quad \text{--- (9)}$$

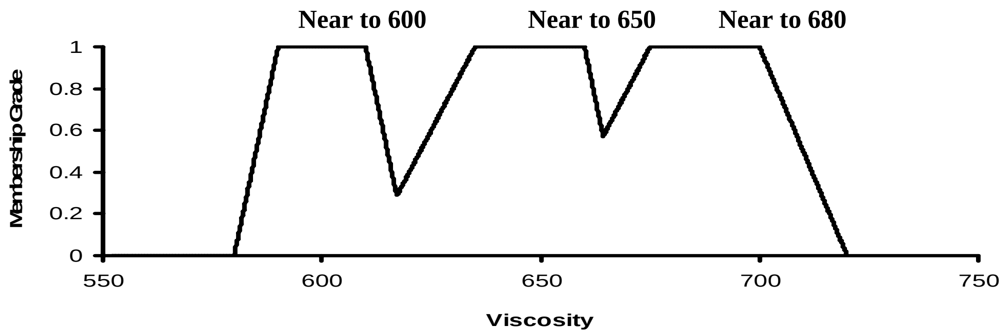

Figure 10

After defuzzification of (9) we get V= 649.1908 .

At last it is observed that the values of pH of the shell fluid are 1, 3 and 5. The pH of the shell is fuzzified by assigning the three fuzzy numbers about1, 3 and 5.

$$p\tilde{H}_1(x) = \begin{cases} 2x & 0 < x \le .5 \\ 1 & .5 < x \le 1.6 \\ \dfrac{2-x}{.4} & 1.6 < x \le 2 \end{cases}, \qquad p\tilde{H}_2(x) = \begin{cases} x-1.5 & 1.5 < x \le 2.5 \\ 1 & 2.5 < x \le 3.8 \\ \dfrac{4.5-x}{.7} & 3.8 < x \le 4.5 \end{cases}$$

$$p\tilde{H}_3(x) = \begin{cases} x-4 & 4 < x \le 5 \\ 1 & 5 < x \le 6 \\ \dfrac{6.9-x}{.9} & 6 < x \le 6.9 \end{cases}$$

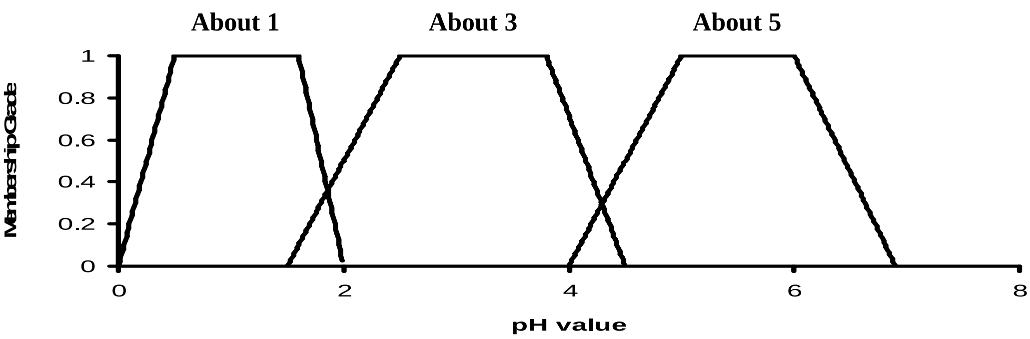

Figure 11

In accordance with the method discussed earlier the union of these fuzzy numbers is given by $p\tilde{H}$, where

$$p\tilde{H}(x)=\begin{cases}2x & 0<x\le .5\\ 1\quad . & .5<x\le 1.6\\ \dfrac{2-x}{.4} & 1.6<x\le 1.857\\ x-1.5 & 1.857<x\le 2.5\\ 1\quad . & 2.5<x\le 3.8\\ \dfrac{4.5-x}{.7} & 3.8<x\le 4.294\\ x-4 & 4.294<x\le 5\\ 1\quad . & 5<x\le 6\\ \dfrac{6.9-x}{.9} & 6<x\le 6.9\end{cases} \qquad \text{--- (10)}$$

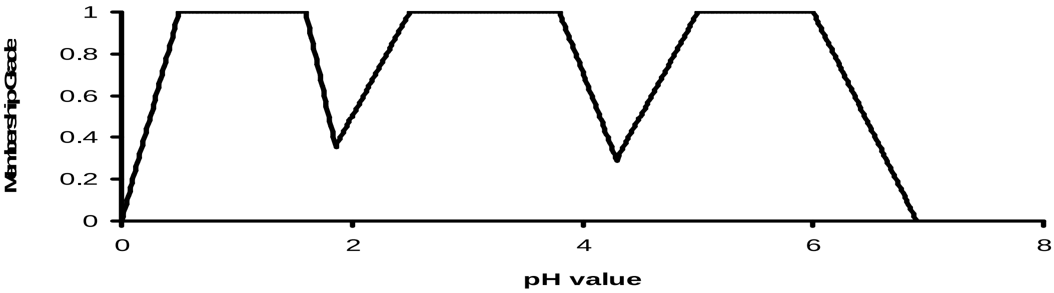

Figure 12

On defuzzification of (10) we get pH = 3.3478

4. Fuzzification of weights of FCFs and threshold value

(i) Fuzzification of weights of FCFs: The role of weight of FCFs can not be ruled out in failure mode screening. Thus to get more realistic results we have fuzzified the weights of different failure causing factors.

In our work we define three fuzzy numbers $\tilde{w}_{cr}$, $\tilde{w}_{im}$ and $\tilde{w}_{re}$ close to 3, 2 and 1 respectively to fuzzify the weights of failure causing factors in [5]

$$\tilde{w}_{cr}(x)=\begin{cases}\dfrac{x-1.5}{1.25} & 1.5<x\le 2.75\\ 1 & 2.75<x\le 3.5\\ 4.5-x & 3.5<x\le 4.5\end{cases}, \qquad \tilde{w}_{im}(x)=\begin{cases}\dfrac{x-.75}{.75} & .75<x\le 1.5\\ 1 & 1.5<x\le 2.5\\ \dfrac{3-x}{.5} & 2.5<x\le 3\end{cases},$$

$$\tilde{w}_{re}(x)=\begin{cases}\frac{x}{.5} & 0<x\le .5\\ 1 & .5<x\le 1.25\\ \frac{2-x}{.75} & 1.25<x\le 2\end{cases}$$

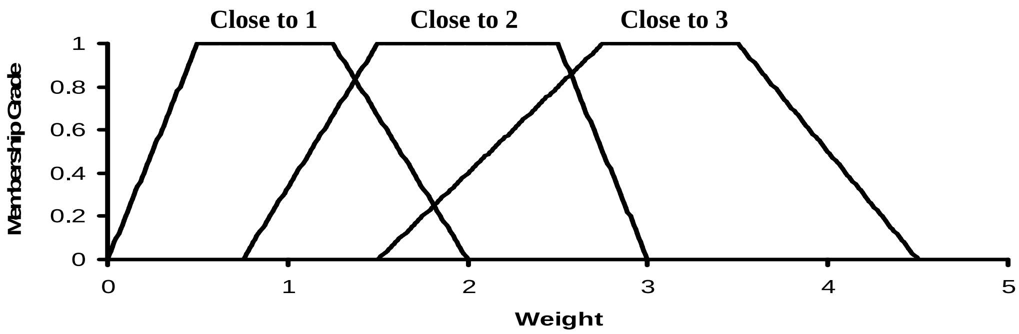

Figure 13

(ii) Threshold Value

The threshold value for a failure mode should represent minimum amount of evidence that the analyst is willing to accept regarding the prompt manifestation of the failure mode. This evidence is completely based on magnitude of the impact of distinct failure causing factors provoking the failure mode [5]. In this paper we employ the fuzzy number $\tilde{T}_h(x)$ "about 5" to fuzzify the threshold value for all failure modes contained in the knowledge base of the system.

$$\tilde{T}_h(x)=\begin{cases}\frac{x-2}{2.5} & 2<x\le 4.5\\ 1 & 4.5<x\le 5.75\\ \frac{7.5-x}{1.75} & 5.75<x\le 7.5\end{cases} \quad \text{--- (11)}$$

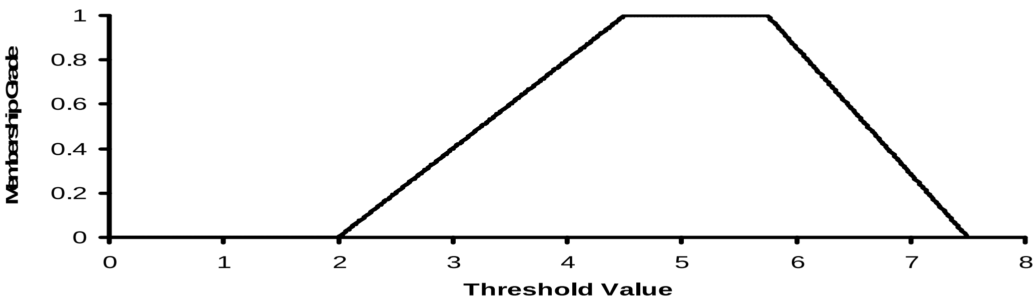

Figure 14

5. Possibility of occurrence of a failure mode

The possibility of failure mode involving different failure causing factors with different weights is computed by the following expression.

$$P_O(x) = \left[\tilde{w}_{cr} * \sum_{\forall i=critical} f_{cr}(x) + \tilde{w}_{im} * \sum_{\forall j=important} f_{im}(x) + \tilde{w}_{re} * \sum_{\forall k=related} f_{re}(x) \right] \quad \text{--- (12)}$$

= (1.5, 2.75, 3.5, 4.5)*0.18799 + (075, 1.5, 2.5, 3)*1.17262+ (0, 0.5, 1.25, 2)*0.16305

= (1.1615, 2.3565, 3.7934, 4.6900)

$$\text{Thus, } P_O(x) = \begin{cases} \dfrac{x - 1.1615}{1.195} & 1.1615 < x \le 2.3565 \\ 1 & 2.3565 < x \le 3.7934 \\ \dfrac{4.69 - x}{.8966} & 3.7934 < x \le 4.69 \end{cases} \quad \text{--- (13)}$$

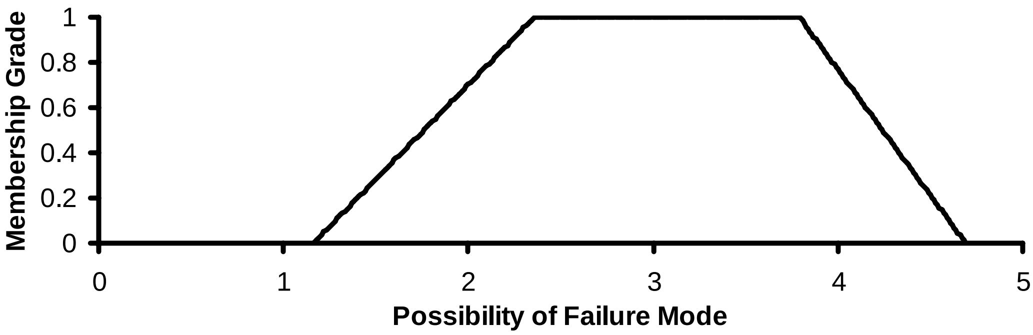

Figure 15

The fuzzy number obtained for Possibility of the failure mode is compared with the threshold value using ordering of fuzzy numbers. Since the computed possibility of failure mode is less than the threshold value, this particular failure mode would not be included in final analysis of the system.

6. Conclusion

For well-defined objects the randomness represents a kind of predictive uncertainty [8], however evaluation of various failure causing factors of a system involve uncertainty (imprecision, vagueness). Thus the measurement of these factors using the techniques based on crisp set theory does not give us realistic results. The contribution of present article reports method of failure mode screening employing a technique, based on fuzzy set theory to measure the failure causing factors. Conclusively, it can be favourably asserted that this approach for failure mode screening provides comprehensive results that can be used to screen out failure modes involving human judgment, unreported times, vague operating conditions to the area of equipment maintenance management through the development of a fuzzy scheme embedded into a fuzzy expert system. Moreover, the weights of the different failure causing factors are fuzzified resulting in the possibility of that particular failure mode. The possibility of failure mode is compared with threshold value using ordering of fuzzy numbers. It is a well known fact that fuzzy set provides us a powerful representation of measurement uncertainties and meaningful representation of vague concepts expressed in natural language. Finally a candid view has been derived from the study that the generalization algorithm developed by us may provide more realistic results.

References

1. A.C.F. Guimarees, N.F.F. Ebecken, Fuzzy FTA: a fuzzy fault tree system for uncertainty analysis, Ann. Nucl. Energy 26 (1999) 523–532.

2. C.H. Cheng, D.L. Mon, (1993), "Fuzzy system reliability analysis by interval of confidence", *Fuzzy Sets and Systems*, Vol. 56 No.1, pp.29-35.

3. D. Pandey and Sanjay Kumar Tyagi “Profust Reliability of a gracefully degradable system”, Fuzzy Sets and Systems, Vol. 158 (2007) 794– 803.
4. D. Singer, “A fuzzy set approach to fault tree analysis”, Fuzzy Sets and Systems 34 (1990) 145-155.

5. Daniel J Fonseca and Gerald M. Knapp, “A fuzzy scheme for failure mode screening”, *Fuzzy Sets and Systems* 121 (2001) 453-457.

6. G.J. Klir, B.Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice-Hall of India, New Delhi, 2002.

7. H.J. Zimmermann, Fuzzy Set Theory and its Application, second ed., Kluwer, Amsterdam, 1991.

8. K.Y. Cai, C.Y. Wen, Street-lighting lamps replacement: a fuzzy viewpoint, in: Fuzzy Sets and Systems, Vol. 37, North-Holland, Amsterdam, 1990, pp. 161–172.

9. K.Y. Cai, Introduction to Fuzzy Reliability, Kluwer Academic Publishers, USA, 1996.

10. L.A. Zadeh, Fuzzy sets, Inform. Control 8 (3) (1965) 338–353.

11. R.S. Chanda, P.K. Bhattacharjee, A reliability approach to transmission expansion using fuzzy fault tree model, Electric Power Systems Res.45 (1998) 101–108.

12. T.J. Ross, Fuzzy Logic with Engineering Applications, International ed., McGraw-Hill Inc., NewYork, 1997.

13. Y. K. Gu, H. Z. Huang, “Fuzzy Mapping Between Physical Domain and Function Domain in Design Process”, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, Vol. 12, (1) 2004, pp 7-20.

14. Z Huang., X Tong., Zuo J Ming, “Posbist fault tree analysis of coherent systems”, *Reliability Engineering and System Safety*, Vol. 84, (2) 2004, pp 141-148.

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