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Interval Valued $(\in, \in \vee \mathrm{q})$-Fuzzy Ideal in Rings

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Abstract

The notion of an interval-valued $(\in, \in \vee q)$-fuzzy subring(ideal,prime) in ring is introduced and their characterizations are investigated.

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1 Introduction

Fuzzy set was initiated by Zadeh[10] and so many researchers were conducted on the generalizations of the notion of fuzzy sets. In [11], Zadeh made an extension of the concept of a fuzzy set by an interval-valued fuzzy set. Liu applied the concept of fuzzy sets to the theory of rings and introduced the notions of fuzzy subring and fuzzy ideal of a ring[8]. This concept discussed futher by many researchers[1, 3, 4, 6, 7]. In [9], Ming and Ming introduced introduced the cocept of quasi-coincidence of a fuzzy point with a fuzzy suset. Based on quasi-coincidence, S.K. Bhakat and P.Das[2] introduced a new type fuzzy subring(ideal,prime) of ring called an $(\in, \in \vee q)$-fuzzy subring(ideal,prime).

In this paper, we concenterate on the quasi-coincidence of a fuzzy interval value with an interval valued fuzzy set and introduced the notions of $(\in, \in \vee q)$-fuzzy subring(ideal,prime) which is an extende notion of $(\in, \in \vee q)$-fuzzy subring(ideal,prime). And we give some interesting properties are investigated.

2 Prelminaries

Let R be a ring. By a *subring* of R we mean a nonempty subset S of R such that S is closed under the operations of addition and multiplication in R. A subring I of a ring R is called an *ideal* of R if for all $x \in R, r \in I$, $rx, xr \in I$.

Note that a nonempty subset I of a ring R is an ideal if and only if if it satisfies: (i) for all $a, b \in I$, $a - b \in I$, (ii) for all $a \in I, r \in R$, $ra, ar \in I$.

An ideal I of a ring R is called a *prime* if for all $a, b \in R$, $ab \in I$ implies $a \in I$ or $b \in I$.

We now review some fuzzy logic concepts. Let X be a set. A *fuzzy set* in X is a function $\mu : X \to [0, 1]$.

Definition 2.1. *[8] A fuzzy set μ of a ring R is called a fuzzy subring(ideal) of R if it satisfies:*

(i) $\forall x, y \in R,\ \mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,

(ii) $\forall x, y \in R,\ \mu(xy) \geq \min\{\mu(x), \mu(y)\}(\mu(xy) \geq \mu(x)[\mu(yx) \geq \mu(x))$.

Definition 2.2. *[5] A fuzzy ideal μ of a ring R is called a fuzzy prime ideal of R if it saisfies: $\forall x, y \in R, \mu(xy) = \mu(x)$ or $\mu(xy) = \mu(y)$.*

A fuzzy set μ in a set X of the form

$$\mu(y) := \begin{cases} t \in (0, 1] & \text{if } y = x, \\ 0 & \text{if } y \neq x, \end{cases}$$

is called a *fuzzy point* with support x and value t and is denoted by x_t.

For a fuzzy point x_t and a fuzzy set μ in a set X, To say that $x_t \in \mu$ (resp. $x_t \mathrm{q} \mu$) means that $\mu(x) \geq t$ (resp. $\mu(x) + t > 1$), and in this case, x_t is said to *belong to* (resp. *be quasi-coincident with*) a fuzzy set μ. To say that $x_t \in\vee \mathrm{q}\, \mu$ (resp. $x_t \in\wedge \mathrm{q}\, \mu$) means that $x_t \in \mu$ or $x_t \mathrm{q} \mu$ (resp. $x_t \in \mu$ and $x_t \mathrm{q} \mu$) [9].

Based on belongness and quasi-coincidence, S.K. Bhakat and P.Das introduced the notion of $(\in, \in \vee \mathrm{q})$-fuzzy subrings and ideals of a ring[2].

The notion of interval valued fuzzy set was introduced by Zadeh[10, 11]. To consider the notion of interval valued fuzzy set, we need following notations.

By an interval number $\widehat{a}$, we mean an interval $[\underline{a}, \overline{a}]$, where $0 \leq \underline{a} \leq \overline{a} \leq 1$. The interval $[a, a]$ can be simply identified with the number $a \in [0, 1]$. Let $D[0, 1]$ denotes the set of all interval numbers. Consider the interval numbers $\widehat{a}_i = [\underline{a}_i, \overline{a}_i]$, $\widehat{b}_i = [\underline{b}_i, \overline{b}_i] \in D[0, 1], i \in I$, we define

$\mathrm{rmin}(\widehat{a}_i, \widehat{b}_i) = [\min\{\underline{a}_i, \underline{b}_i\}, \min\{\overline{a}_i, \overline{b}_i\}]$,

$\mathrm{rmax}(\widehat{a}_i, \widehat{b}_i) = [\max\{\underline{a}_i, \underline{b}_i\}, \max\{\overline{a}_i, \overline{b}_i\}]$,

$\mathrm{rinf}\widehat{a}_i = \left[\bigwedge_{i\in I} \underline{a}_i, \bigwedge_{i \in I} \overline{a}_i\right], \}\mathrm{rsup}\widehat{a}_i = \left[\bigvee_{i \in I} \underline{a}_i, \bigvee_{i\in I} \overline{a}_i\right]$.

We also define the symbols"$\leq$", "$=$", "$<$" in case of two interval numbers in $D[0, 1]$.

(1) $\widehat{a}_1 \leq \widehat{a}_2$ if and only if $\underline{a}_1 \leq \underline{a}_2$ and $\overline{b}_1 \leq \overline{b}_2$.

(2) $\widehat{a}_1 = \widehat{a}_2$ if and only if $\underline{a}_1 = \underline{a}_2$ and $\overline{b}_1 = \overline{b}_2$.

(3) $\widehat{a}_1 < \widehat{a}_2$ if and only if $\underline{a}_1 < \underline{a}_2$ and $\overline{b}_1 < \overline{b}_2$.

Under these notation, the concept of an interval-valued fuzzy set defined on a non-empty set X as objects having the form

$$\mathrm{A} = \{(x, [\underline{\mu_{\mathrm{A}}}(x), \overline{\mu_{\mathrm{A}}}(x)])\},\ \forall\, x \in X,\ \text{(briefly, denoted by } A = [\underline{\mu_{\mathrm{A}}}, \overline{\mu_{\mathrm{A}}}]),$$

where $\underline{\mu_{\mathrm{A}}}$ and $\overline{\mu_{\mathrm{A}}}$ are two fuzzy sets in X such that $\underline{\mu_{\mathrm{A}}}(x) \leq \overline{\mu_{\mathrm{A}}}(x)$ for all $x \in X$. Let $\widehat{\mu_{\mathrm{A}}}(x) = \left[\underline{\mu_{\mathrm{A}}}(x), \overline{\mu_{\mathrm{A}}}(x)\right]$, $\forall\, x \in X$. Then $\widehat{\mu_{\mathrm{A}}}(x) \in D[0,1]$, $\forall\, x \in X$, and therefore the interval-valued fuzzy set A is given by

$$\mathrm{A} = \{(x, \widehat{\mu_{\mathrm{A}}}(x))\},\ \forall\, x \in X,\ \text{ where } \widehat{\mu_{\mathrm{A}}} : X \to D[0,1].$$

For a given interval valued fuzzy sets A and B in a set X, we define

- $\mathrm{A} \subseteq \mathrm{B} \Leftrightarrow (\forall x \in X)\,(\underline{\mu_{\mathrm{A}}}(x) \leq \underline{\mu_{\mathrm{B}}}(x),\ \overline{\mu_{\mathrm{A}}}(x) \leq \overline{\mu_{\mathrm{B}}}(x))$.
- $\mathrm{A} = \mathrm{B} \Leftrightarrow \mathrm{A} \subseteq \mathrm{B}$ and $\mathrm{B} \subseteq \mathrm{A}$.
- $\mathrm{A} \cap \mathrm{B} = \{(x, \left[\min\{\underline{\mu_{\mathrm{A}}}(x), \underline{\mu_{\mathrm{B}}}(x)\}, \min\{\overline{\mu_{\mathrm{A}}}(x), \overline{\mu_{\mathrm{B}}}(x)\}\right]) \mid x \in X\}$
- $\mathrm{A} \cup\ \mathrm{B} = \{(x, \left[\max\{\underline{\mu_{\mathrm{A}}}(x), \underline{\mu_{\mathrm{B}}}(x)\}, \max\{\overline{\mu_{\mathrm{A}}}(x), \overline{\mu_{\mathrm{B}}}(x)\}\right]) \mid x \in X\}$.

3 Interval valued $(\in, \in \vee \mathrm{q})$-fuzzy subrings and fuzzy ideals

In what follows, R is a ring unless otherwise specified. An interval valued fuzzy set A in R of the form

$$\widehat{\mu_{\mathrm{A}}}(y) = \begin{cases} \widehat{a} \neq [0,0] & \text{if } y = x, \\ & \text{if } y \neq x \end{cases}$$

is called an fuzzy interval value with support x and interval value $\widehat{a}$ and is denoted by $\mathcal{U}(x; \widehat{a})$

Throught this paer, we assume that $\widehat{\mu_{\mathrm{A}}}(x) = \left[\underline{\mu_{\mathrm{A}}}(x), \overline{\mu_{\mathrm{A}}}(x)\right]$ must satisfy the following two properties:

(1) Any two interval numbers of $D[0,1]$ are comparable;

(2) $\mathrm{rmin}\{\underline{\mu_{\mathrm{A}}}(x), \overline{\mu_{\mathrm{A}}}(x)\} < [0.5, 0.5]$ or $\mathrm{rmin}\{\underline{\mu_{\mathrm{A}}}(x), \overline{\mu_{\mathrm{A}}}(y)\} \geq [0.5, 0.5]$ for all $x \in R$.

We say that a fuzzy interval value $\mathcal{U}(x; \widehat{a})$ belong to (resp. is a quasi-coincident with) an interval valued fuzzy set A, written by $\mathcal{U}(x; \widehat{a}) \in \mathrm{A}$(resp.$\mathcal{U}(x; \widehat{a})\mathrm{q}\mathrm{A}$) ,if $\widehat{\mu_{\mathrm{A}}}(x) \geq \widehat{a}$(resp. $\widehat{\mu_{\mathrm{A}}}(x) + \widehat{a} > [1,1]$.) If $\widehat{\mu_{\mathrm{A}}}(x) < \widehat{a}$(resp. $\widehat{\mu_{\mathrm{A}}}(x) + \widehat{a} \leq [1,1]$.), then we write $\mathcal{U}(x; \widehat{a}) \notin \mathrm{A}$(resp.$\mathcal{U}(x; \widehat{a})\overline{\mathrm{q}}\mathrm{A}$). If $\mathcal{U}(x; \widehat{a}) \in \mathrm{A}$ or $\mathcal{U}(x; \widehat{a})\mathrm{q}\mathrm{A}$, then we write $\mathcal{U}(x; \widehat{a}) \in\vee\, \mathrm{q}\mathrm{A}$. The symble $\overline{\in \vee \mathrm{q}}$ means $\in \vee\, \mathrm{q}$ does not hold.

Definition 3.1. *An interval valued fuzzy set* A *in* R *is called a* $(\in, \in\vee\mathrm{q})$-*fuzzy subring of* R *if for all* $x, y \in R$ *and* $a, b \in (0, 1]$,

(S1) $\mathcal{U}(x;\widehat{a})$ *and* $\mathcal{U}(y;\widehat{b}) \in \mathrm{A}$ *imply* $\mathcal{U}(x+y;\mathrm{rmin}\{\widehat{a},\widehat{b}\}) \in\vee\,\mathrm{qA}$,

(S2) $\mathcal{U}(x;\widehat{a}) \in \mathrm{A}$ *imply* $\mathcal{U}(-x;\widehat{a}) \in\vee\,\mathrm{qA}$,

(S3) $\mathcal{U}(x;\widehat{a})$ *and* $\mathcal{U}(y;\widehat{b}) \in \mathrm{A}$ *imply* $\mathcal{U}(xy;\mathrm{rmin}\{\widehat{a},\widehat{b}\} \in\vee\,\mathrm{qA}$.

Theorem 3.2. A *is an interval valued* $(\in, \in\vee q)$-*fuzzy subring of* R *if and only if for all* $x, y \in R$ *the following three conditions are satisfied:*

(S4) $\widehat{\mu_{\mathrm{A}}}(x+y) \geq \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y), [0.5, 0.5]\}$,

(S5) $\widehat{\mu_{\mathrm{A}}}(-x) \geq \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), [0.5, 0.5]\}$,

(S6) $\widehat{\mu_{\mathrm{A}}}(xy) \geq \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y), [0.5, 0.5]\}$.

Proof. (S1) $\Rightarrow$ (S4) Assume that (S4) is not valid, then there exists $x, y \in R$ such that $\widehat{\mu_{\mathrm{A}}}(x+y) < \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y), [0.5, 0.5]\}$. We consider the following two caeses:

$$\text{(i)}\ \ \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y)\} < [0.5, 0.5] \ \text{ and } \ \text{(ii)}\ \ \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y)\} \geq [0.5, 0.5].$$

For the case (i) we have $\widehat{\mu_{\mathrm{A}}}(x+y) < \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y)\})$. Choose $\widehat{a}$ such that $\widehat{\mu_{\mathrm{A}}}(x+y) < \widehat{a} < \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y)\})$. Then $\mathcal{U}(x;\widehat{a}), \mathcal{U}(y;\widehat{a}) \in \mathrm{A}$. Since $\widehat{\mu_{\mathrm{A}}}(x+y) < \widehat{a}$ and $\widehat{\mu_{\mathrm{A}}}(x+y) + \widehat{a} < [1, 1]$, we have $\mathcal{U}(x+y;\widehat{a}) \notin \mathrm{A}$ and $\mathcal{U}(x+y;\widehat{a})\overline{\mathrm{q}}\mathrm{A})$. From this implies that $\mathcal{U}(x+y;\widehat{a})\overline{\in\vee\,\mathrm{q}}\mathrm{A})$, which contradictis (S1). For the (ii) case we have $\widehat{\mu_{\mathrm{A}}}(x+y) < [0.5, 0.5] \leq \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y)\}$. Then $\mathcal{U}(x; [0.5, 0.5]), \mathcal{U}(x; [0.5, 0.5]) \in \mathrm{A}$. But $\mathcal{U}(x+y; [0.5, 0.5])\overline{\in\vee\,\mathrm{q}}\mathrm{A})$ which contradictis (S1). Therefore (S4) holds.
(S4) $\Rightarrow$ (S1) Let $x, y \in R$ and $a, b \in (0, 1]$ be such that $\mathcal{U}(x;\widehat{a}) \in \mathrm{A}$ and $\mathcal{U}(y;\widehat{b}) \in \mathrm{A}$. Then $\widehat{\mu_{\mathrm{A}}}(x) \geq \widehat{a}$ and $\widehat{\mu_{\mathrm{A}}}(y) \geq \widehat{b}$. Now we have $\widehat{\mu_{\mathrm{A}}}(x+y) \geq \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y), [0.5, 0.5]\} \geq \mathrm{rmin}\{\widehat{a}, \widehat{b}, [0.5, 0.5]\}$. If $\mathrm{rmin}\{\widehat{a}, \widehat{b}\} < [0.5, 0.5]$ we have $\widehat{\mu_{\mathrm{A}}}(x+y) \geq \mathrm{rmin}\{\widehat{a}, \widehat{b}\}$. If $\mathrm{rmin}\{\widehat{a}, \widehat{b}\} \geq [0.5, 0.5]$, which implies that $\widehat{\mu_{\mathrm{A}}}(x+y) \geq [0.5, 0.5]$, according to $\widehat{\mu_{\mathrm{A}}}(x+y) + \mathrm{rmin}(\widehat{a}, \widehat{b}) \geq [1, 1]$. Hence $\mathcal{U}(x+y;\mathrm{rmin}\{\widehat{a}, \widehat{b}\}) \in\vee\,\mathrm{qA}$. Therefore (S3) holds.
(S2) $\Rightarrow$ (S5) Assume that (S5) is not valid, then there exists $x \in R$ such that $\widehat{\mu_{\mathrm{A}}}(-x) < \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), [0.5, 0.5]\}$. We consider the following two caeses:

$$\text{(i)}\ \ \widehat{\mu_{\mathrm{A}}}(x) < [0.5, 0.5] \ \text{ and } \ \text{(ii)}\ \ \widehat{\mu_{\mathrm{A}}}(x) \geq [0.5, 0.5].$$

For the case (i) we have $\widehat{\mu_{\mathrm{A}}}(-x) = \widehat{r} < \widehat{\mu_{\mathrm{A}}}(x) = \widehat{t}$. Choose $\widehat{a}$ such that $\widehat{r} < \widehat{a} < \widehat{t}$ and $\widehat{r} + \widehat{t} < [1, 1]$. Then $\mathcal{U}(x; [0.5, 0.5]) \in A$, but $\mathcal{U}(-x; [0.5, 0.5])\overline{\in\vee\,\mathrm{q}}\mathrm{A})$ which contradictis (S1). For the (ii) case we have $\widehat{\mu_{\mathrm{A}}}(-x) < \mathrm{rmin}\{\widehat{a}(x), [0.5, 0.5]\})$.

Then $\mathcal{U}(x;[0.5,0.5])\in \mathrm{A}$,but $\mathcal{U}(-x;[0.5,0.5])\overline{\in\vee\mathrm{q}}\mathrm{A})$ which contradictis (S1). Therefore (S4) holds.
(S5) $\Rightarrow$ (S2) Let $x\in R$ and $a\in(0,1]$ be such that $\mathcal{U}(x;\widehat{a})\in \mathrm{A}$. Then $\widehat{\mu_{\mathrm{A}}}(x)\geq \widehat{a}$. Now we have $\widehat{\mu_{\mathrm{A}}}(-x)\geq \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x),[0.5,0.5]\}\geq \mathrm{rmin}\{\widehat{a},[0.5,0.5]\}$. If $\widehat{a}(-x)\geq\widehat{a}$ we have $\widehat{a}\leq[0.5,0.5]$ If $\widehat{a}(-x)\geq[0.5,0.5]$, which implies that $\widehat{\mu_{\mathrm{A}}}(-x)\geq[0.5,0.5]$. Hence $\mathcal{U}(-x;\widehat{a})\in\vee\,\mathrm{qA}$. Therefore (S3) holds.
(S3) $\Rightarrow$ (S6) This proof is similar to (S2) $\Longleftrightarrow$ (S5) □

Definition 3.3. *An interval valued fuzzy set* A *in* R *is called an* $(\in,\in\vee\mathrm{q})$*-fuzzy ideal of* R *if for all* $x,y\in R$ *and* $t,\in(0,1]$,

(1) A *is an interval valued* $(\in,\in\vee\mathrm{q})$*-fuzzy subring of* R*,*

(2) $\mathcal{U}(x;\widehat{t})\in\mathrm{A}$ *and* $y\in R$ *imply* $\mathcal{U}(xy;\widehat{t}),\mathcal{U}(yx;\widehat{t})\in\vee\,\mathrm{qA}$

Theorem 3.4. A *is an interval valued* $(\in,\in\vee\mathrm{q})$*-fuzzy ideal of* R *if and only if for all* $x,y\in R$ *the following three conditions are satisfied:*

(1) $\widehat{\mu_{\mathrm{A}}}(x-y)\geq \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x),\widehat{\mu_{\mathrm{A}}}(y),[0.5,0.5]\}$,

(2) $\widehat{\mu_{\mathrm{A}}}(xy),\widehat{\mu_{\mathrm{A}}}(yx)\geq \mathrm{rmin}\{\widehat{\mu_{\mathrm{A}}}(x),[0.5,0.5]\}$.

Proof. Straightforward. □

Definition 3.5. *An interval valued fuzzy set* A *in* R *is called an* $(\in,\in\vee\mathrm{q})$*-fuzzy prime ideal of* R *if it saisfies:* $\forall x,y\in R, t\in(0,1]$,

$$\mathcal{U}(xy;\widehat{t})\in\mathrm{A}\ \mathit{imply}\mathcal{U}(x;\widehat{t})\in\vee\,\mathrm{qA}\ \mathit{or}\ \mathcal{U}(y;\widehat{t})\in\vee\,\mathrm{qA}.$$

Lemma 3.6. *Let* A *be a subset of* R*. A characteristic function* χ_A *is an interval valued* $(\in,\in\vee\mathrm{q})$*-fuzzy ideal of* R *if and only if* A *is an ideal of* R*.*

Proof. Suppose that χ_A is an interval valued $(\in,\in\vee\mathrm{q})$-fuzzy ideal of R. Let $x,y\in A$. Then $\chi_A(x)=[1,1]=\chi_A(y)$, and so

$$\chi_A(x-y)\geq \mathrm{rmin}\{\chi_A(x),\chi_A(y),[0.5,0.5]\}=[0.5,0.5].$$

It follows that $\chi_A(x-y)=[1,1]$ so that $x-y\in A$. On the other hand, if $a\in A$ and $r\in R$, then $\chi_A(ar)\geq \mathrm{rmax}\{\chi_A(a),\chi_A(r)\}=\mathrm{rmax}\{[1,1],\chi_A(r)\}=[1,1]$ and $\chi_A(ra)\geq \mathrm{rmax}\{\chi_A(r),\chi_A(a)\}=\mathrm{rmax}\{\chi_A(r),[1,1],\}=[1,1]$ so that $ar\in A$ and $ra\in A$. Therefore A is an ideal of R.

Conversely, assume that A be an ideal of ring R. It is clear that $\mathcal{U}(\chi_A;[1,1])=A$. We first show that $\chi_A(x-y)\geq \mathrm{rmin}\{\chi_A(x),\chi_A(y),[0.5,0.5]\}$ for all $x,y\in R$. Let $x,y\in R$. If $x,y\in A$, then $x-y\in A$ and so

$$\chi_A(x-y)=[1,1]\geq \mathrm{rmin}\{\chi_A(x),\chi_A(y),[0.5,0.5]\}=[0.5,0.5].$$

If $x, y \notin A$, then $\chi_A(x) = [0, 0] = \chi_A(y)$ and thus

$$\chi_A(x - y) \geq \text{rmin}\{\chi_A(x), \chi_A A(y), [0.5, 0.5]\} = [0.0].$$

If $x \in A$ and $y \notin A$, then $\chi_A(x) = [1, 1]$ and $\chi_A(y) = [0, 0]$. It follows that

$$\chi_A(x - y) \geq \text{rmin}\{\chi_A(x), \chi_A(y), [0.5, 0.5]\} = [0.0].$$

Similarly for the case $x \notin A$ and $y \in A$, we get

$$\chi_A(x - y) \geq \text{rmin}\{\chi_A(x), \chi_A(y), [0.5, 0.5]\}$$

. Similary, it can be shown that $\chi_A(xy), \chi_A(yx) \geq \text{rmin}\{\chi_A(x), [0.5, 0.5]\}$ for all $x, y \in R$. Therefore A is an interval valued $(\in, \in\vee \text{q})$-fuzzy ideal of R, and the proof is complete. □

Theorem 3.7. *Let A be a subset of R. A function χ_A is an interval valued $(\in, \in\vee \text{q})$-fuzzy prime ideal if and only if A is a prime ideal of R.*

Proof. Let χ_A is an interval valued $(\in, \in\vee \text{q})$-fuzzy prime ideal of R. By Lemma 3.6, we know that χ_A is an interval valued $(\in, \in\vee \text{q})$-fuzzy ideal of R. Now we assume that $xy \in A$ for all $x, y \in R$. Then $\chi_A(xy) = [1, 1]$, which implies $\mathcal{U}(xy; \widehat{t}) \in \chi_A$ for all $t \in (0, 1]$. Because of primality of χ_A, we have $\mathcal{U}(x; \widehat{t}) \in \vee \text{q}\chi_A$ or $\mathcal{U}(y; \widehat{t}) \in \vee \text{q}\chi_A$. Note that $\chi_A(x) > [0, 0]$ or $\chi_A(y) > [0, 0]$. This implies that $\chi_A(x) = [1, 1]$ or $\chi_A(y) = [1, 1]$ and hence $x \in A$ or $y \in A$. Therefore A is prime ideal of R. Conversely, A is a prime ideal of R. By Lemma 3.6, we know that χ_A is an interval valued $(\in, \in\vee \text{q})$-fuzzy ideal of R. Let $\mathcal{U}(xy; \widehat{t}) \in \chi_A$ for all $x, y \in R$ and $t \in (0, 1]$. Then $\chi_A(xy) = [1, 1]$, which implies $xy \in A$. Because of primality of A, we have $x \in A$ or $y \in A$. This implies that $\mathcal{U}(x; \widehat{t}) \in \chi_A$ or $\mathcal{U}(y; \widehat{t}) \in\vee \text{q}\chi_A$. Therefore χ_A is an interval valued $(\in, \in\vee \text{q})$-fuzzy prime ideal of R. □

Let R be a ring. Then, for an interval valued fuzzy set A of R, the *level subset* of A in R is defined to be the following subset of R,

$$\mathcal{U}(\mathrm{A}; \widehat{t}) = \{x \in R \mid \widehat{\mu_{\mathrm{A}}}(x) \geq \widehat{t}\} \,\text{for}\, t \in (0, 1].$$

Theorem 3.8. *Let* A *be an interval valued fuzzy subset in R. Then A is an interval valued $(\in, \in\vee \text{q})$-fuzzy ideal of R if and only if $\mathcal{U}(\mathrm{A}; \widehat{t})$ is an ideal of R for every $t \in (0, 0.5]$.*

Proof. Assume that A is an interval valued $(\in, \in\vee \text{q})$-fuzzy ideal of R and let $t \in (0, 0.5]$ be such that $x, y \in \mathcal{U}(\mathrm{A}; \widehat{t})$ Then

$$\widehat{\mu_{\mathrm{A}}}(x - y) \geq \text{rmin}\{\widehat{\mu_{\mathrm{A}}}(x), \widehat{\mu_{\mathrm{A}}}(y), [0.5, 0.5]\} \geq \text{rmin}\{\widehat{t}, [0.5, 0.5]\} = \widehat{t}$$

and so $x - y \in \mathcal{U}(A; \widehat{t})$. Let $x \in \mathcal{U}(A; \widehat{t}])$ and $r \in R$. Then we have

$$\widehat{\mu_A}(xr), \widehat{\mu_A}(rx) \geq \text{rmin}\{\widehat{\mu_A}(x), [0.5, 0.5]\} \geq \text{rmin}\{\widehat{t}, [0.5, 0.5]\} = \widehat{t}.$$

Hence $xr, rx \in \mathcal{U}(A; \widehat{t})$. Thus $\mathcal{U}(A; \widehat{t})$ is an ideal of R for every $t \in (0, 0.5]$. Conversely, let A be an interval valued fuzzy subset in ring R such that $\mathcal{U}(A; \widehat{t})$ is an ideal of R for every $t \in (0, 0.5]$. Assume that A is not an interval valued $(\in, \in\vee q)$-fuzzy ideal of R ,then there exists $x, y \in R$ such that $\widehat{\mu_A}(x - y) < \text{rmin}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y), [0.5, 0.5]\}$. Then we can choose $\widehat{t}$ such that $\widehat{\mu_A}(x-y) < \widehat{t} < \text{rmin}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y), [0.5, 0.5]\}$. Thus $x, y \in \mathcal{U}(A; \widehat{t})$. Since $\mathcal{U}(A; \widehat{t})$ is an ideal of R, we have $x - y \in \mathcal{U}(A; \widehat{t})$. Thus $\widehat{\mu_A}(x - y) \geq \widehat{t}$, a contradiction. Hence $\widehat{\mu_A}(x - y) \geq \text{rmin}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y), [0.5, 0.5]\}$ for all $x, y \in R$. Similary, we get $\widehat{\mu_A}(xy), \widehat{\mu_A}(yx) \geq \text{rmin}\{\widehat{\mu_A}(x), [0.5, 0.5]\}$ for all $x, y \in R$. Therefore A is an interval valued $(\in, \in\vee q)$-fuzzy ideal of R. □

Theorem 3.9. *Let* A *be an interval valued fuzzy subset of* R*. Then* A *is an interval valued* $(\in, \in\vee q)$*-fuzzy prime ideal of* R*. if and only if* $\widehat{\mu_A}$ *satisfies the following assertions:*

$$(\forall x, y \in R)\,(\text{rmax}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y)\} \geq \text{rmin}\{\widehat{\mu_A}(xy), [0.5, 0.5]\}).$$

Proof. Let A be an interval valued $(\in, \in\vee q)$-fuzzy prime ideal of R. If there exist $x, y \in R$ such that $\text{rmax}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y)\} < \text{rmin}\{\widehat{\mu_A}(xy), [0.5, 0.5]\}$. Then we can choose $\widehat{t}$ such that $\text{rmax}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y)\} < \widehat{t} < \text{rmin}\{\widehat{\mu_A}(xy), [0.5, 0.5]\}$. Then $\mathcal{U}(xy; \widehat{t}) \in A$. Since $\widehat{\mu_A}(x), \widehat{\mu_A}(y) < \widehat{t}$ and $\widehat{\mu_A}(x) + \widehat{t}, \widehat{\mu_A}(y) + \widehat{t} < [1, 1]$, we have $\mathcal{U}(x; \widehat{t})\overline{\in\vee q}A$ and $\mathcal{U}(y; \widehat{t})\overline{\in\vee q}A)$, a contradiction. Conversely, let $\text{rmax}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y)\} \geq \text{rmin}\{\widehat{\mu_A}(xy), [0.5, 0.5]\}$ for all $x, y \in R$. Then $\text{rmax}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y)\} \geq \text{rmin}\{\widehat{t}, [0.5, 0.5]\} = [0.5, 0.5]$ or $\widehat{t}$ according as $\widehat{t}.[0.5, 0.5]$ or $\widehat{t} \leq [0.5, 0.5]$. This imply $\mathcal{U}(x; \widehat{t}) \in\vee qA$ or $\mathcal{U}(y; \widehat{t}) \in\vee qA$. Therefore A is an interval valued $(\in, \in\vee q)$-fuzzy prime ideal of R. □

Theorem 3.10. *Let* A *be an interval valued fuzzy subset of* R*. An interval valued* $(\in, \in\vee q)$*-fuzzy ideal is interval valued* $(\in, \in\vee q)$*-fuzzy prime ideal if and only if* $\mathcal{U}(A; \widehat{t})$ *is a prime ideal of* R *for all* $t \in (0, 0.5]$

Proof. Assume that A is an interval valued $(\in, \in\vee q)$-fuzzy prime ideal of R. By Theorem3.9, we know that $\mathcal{U}(A; \widehat{t})$ is an ideal of R for every $t \in (0, 0.5]$. Let $xy \in \mathcal{U}(A; \widehat{t})$. Since A is an interval valued $(\in, \in\vee q)$-fuzzy prime ideal of R,

$$\text{rmax}\{\widehat{\mu_A}(x), \widehat{\mu_A}(y)\} \geq \text{rmin}\{\widehat{\mu_A}(xy), [0.5, 0.5]\} \geq \text{rmin}\{\widehat{t}, [0.5, 0.5]\} = \widehat{t}$$

and so $\mathcal{U}(x; \widehat{t}) \in A$ or $\mathcal{U}(y; \widehat{t}) \in A$. Theorefore $\mathcal{U}(A; \widehat{t})$ is an prime ideal of R for every $t \in (0, 0.5]$. Conversely, let $\mathcal{U}(A; \widehat{t})$ is a prime ideal of R for every

$t \in (0, 0.5]$. By Theorem3.9, we know that A is an interval valued $(\in, \in \vee \mathrm{q})$-fuzzy ideal of R. Let $\widehat{t} \leq [0.5, 0.5]$ Assume that $\mathcal{U}(xy; \widehat{t}) \in A$. Since $\mathcal{U}(\mathrm{A}; \widehat{t})$ is a prime ideal of R, $x \in \mathcal{U}(\mathrm{A}; \widehat{t})$ or $y \in \mathcal{U}(\mathrm{A}; \widehat{t})$. and so $\mathcal{U}(x; \widehat{t}) \in A$ or $\mathcal{U}(y; \widehat{t}) \in A$. If $t > [0.5, 0.5]$, since $\mathcal{U}(\mathrm{A}; [0.5, 0.5])$ is prime, we have $x \in \mathcal{U}(\mathrm{A}; [0.5, 0.5])$ or $y \in \mathcal{U}(\mathrm{A}; [0.5, 0.5])$. Hence $\mathcal{U}(x; \widehat{t}) \in \vee \mathrm{qA}$ or $\mathcal{U}(y; \widehat{t}) \in \vee \mathrm{qA}$. Therefore A is an interval valued $(\in, \in \vee \mathrm{q})$-fuzzy ideal of R. □

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