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# On D-Recurrent Spaces with Semi-Symmetric Recurrent-Metric Connection

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**Abstract**

The properties of Riemannian spaces admitting a semi-symmetric metric connection and semi-symmetric recurrent-metric connection were studied in [1-9]. On the other hand, the notion of the recurrent Riemannian space was introduced and studied in [7]. Furthermore, $D$-recurrent spaces with semi-symmetric connection were investigated in [8].

In this work, we introduce $D$-recurrent spaces with semi-symmetric recurrent-metric connection denoted by $(M_n, g, D)$ and obtain some properties of the curvature tensor $L^m_{ijk}$ of $(M_n, g, D)$. For such a space, it is shown that $D_l L_{ijkh} = \rho_l \, L_{ijkh}$, $(\rho_l \neq 0)$, where $L_{ijkh}$ is the curvature tensor corresponding to the $D$ connection and $\rho_l$ is a non-zero covariant vector field. Also, an example of these spaces is given.

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# 1 Introduction

Let $(M_n, g)$ $(n > 2)$ be an $n$-dimensional differentiable manifold of class $C^\infty$ with metric tensor $g$, the Riemannian connection $\nabla$ and, a smooth linear connection $D$ on $M_n$.

If, for any smooth vector fields $X$ and $Y$ on $M_n$, the torsion tensor $T$ of $D$ satisfies the relation

$$T(X,Y) = \pi(Y)X - \pi(X)Y, \tag{1.1}$$

where $\pi$ is a smooth linear differential form, then the connection $D$ is said to be semi-symmetric [9].

If, in addition, the connection $D$ satisfies the following condition

$$D_X g = 2\mu(X)g, \tag{1.2}$$

where $\mu$ is a smooth linear differential form, then the connection $D$ is said to be semi-symmetric recurrent-metric ([5],[9]).

Let $\left\{ {m \atop ji} \right\}$ be the coefficients of the Riemannian connection $\nabla$ and, $D$ a semi-symmetric recurrent-metric connection on $M_n$ with coefficients

$$\Gamma^m_{ji} = \left\{ {m \atop ji} \right\} + \delta^m_j \lambda_i - \mu_j \delta^m_i - g_{ji}\lambda^m, \tag{1.3}$$

where we put $\pi_i = \lambda_i - \mu_i$ and $\lambda^m = \lambda_t g^{tm}$ [5].

The curvature tensor $L^m_{kji}$ of the connection $D$ and the curvature tensor of the Riemannian connection $\nabla$, respectively, are defined by [5]

$$L^m_{kji} = \partial_k \Gamma^m_{ji} - \partial_j \Gamma^m_{ki} + \Gamma^m_{kt}\Gamma^t_{ji} - \Gamma^m_{jt}\Gamma^t_{ki}\,, \qquad \left(\partial_k = \frac{\partial}{\partial x^k}\right), \tag{1.4}$$

$$R^m_{kji} = \partial_k \left\{ {m \atop ji} \right\} - \partial_j \left\{ {m \atop ki} \right\} + \left\{ {m \atop kt} \right\}\left\{ {t \atop ji} \right\} - \left\{ {m \atop jt} \right\}\left\{ {t \atop ki} \right\}. \tag{1.5}$$

Let $L_{kjih} = L^m_{kji}\, g_{mh}$ and $R_{kjih} = R^m_{kji}\, g_{mh}$.

Substituting (1.3) in (1.4), we obtain the following equation for the curvature tensor $L^m_{kji}$ of $M_n$ with the semi-symmetric recurrent-metric connection, in short $(M_n, g, D)$ .

$$L_{kjih} = R_{kjih} + g_{jh}\lambda_{ki} - g_{kh}\lambda_{ji} + \lambda_{jh}g_{ki} - \lambda_{kh}g_{ji} + g_{ih}(\mu_{jk} - \mu_{kj}), \tag{1.6}$$

where we put

$$\lambda_{ki} = \nabla_k \lambda_i - \lambda_k \lambda_i + \frac{1}{2}\, g_{ki}\, \lambda_t \lambda^t, \tag{1.7}$$

$$\mu_{jk} = \nabla_j \mu_k - \mu_j \mu_k + \frac{1}{2}\, g_{jk}\, \mu_t \mu^t. \tag{1.8}$$

**Proposition.** For the curvature tensor $L_{kjih}$ of the semi-symmetric recurrent-metric connection, the following properties hold:

$$i) \quad L_{kjih} = -L_{jkih}, \tag{1.9}$$

$$ii) \quad L_{kkih} = 0. \tag{1.10}$$

**Proof.** i) Interchanging the indices $k$ and $j$ in (1.6), we get

$$L_{jkih} = R_{jkih} + g_{kh}\lambda_{ji} - g_{jh}\lambda_{ki} + \lambda_{kh}g_{ji} - \lambda_{jh}g_{ki} + g_{ih}(\mu_{kj} - \mu_{jk}) = -L_{kjih}.$$

ii) By taking $k = j$ in (1.6), we obtain

$$L_{kkih} = R_{kkih} + g_{kh}\lambda_{ki} - g_{kh}\lambda_{ki} + \lambda_{kh}g_{ki} - \lambda_{kh}g_{ki} + g_{ih}(\mu_{kk} - \mu_{kk}) = 0.$$

**Definition.** The space $(M_n, g, D)$ is called $D$-recurrent if there exists a non-zero covariant vector field $\rho_l$ such that

$$D_l\, L^m_{ijk} = (\rho_l - 2\mu_l)\, L^m_{ijk}, \tag{1.11}$$

where $\mu_l$ is the component of a covariant vector field satisfying the condition of recurrent metric connection (1.2).

**Lemma.** If the space $(M_n, g, D)$ is $D$-recurrent, then

$$D_l\, L_{ijkh} = \rho_l\, L_{ijkh}. \tag{1.12}$$

**Proof.** Multiplying (1.11) by $g_{mh}$ we obtain

$$\begin{aligned}(D_l\, L^m_{ijk})\, g_{mh} &= (\rho_l - 2\mu_l)\, L^m_{ijk}\, g_{mh} \\ &= \rho_l\, L^m_{ijk}\, g_{mh} - L^m_{ijk}\, (2\mu_l\, g_{mh}).\end{aligned}$$

Using (1.2) we get

$$(D_l\, L^m_{ijk})\, g_{mh} + L^m_{ijk}\, (D_l\, g_{mh}) = \rho_l\, L^m_{ijk}\, g_{mh},$$

from which $D_l\, L_{ijkh} = \rho_l\, L_{ijkh}$.

In the next section, we will present an example of a D-recurrent space with semi-symmetric recurrent-metric connection that we have introduced.

# 2 An Example of D-Recurrent Spaces

Let each Latin index run over $1, 2, ..., n$ and each Greek index $2, 3, ..., n-1$. We define the metric $g$ in $(M_n, g, D)$, $(n \geq 4)$ by the formula [2,3]

$$ds^2 = \varphi\,(dx^1)^2 + k_{\alpha\beta}dx^\alpha dx^\beta + 2dx^1dx^n, \tag{2.1}$$

where $[k_{\alpha\beta}]$ is a symmetric non-singular matrix with constant entries, $\varphi$ is a scalar function and independent of $x^n$.

In the metric (2.1), the only non-vanishing components of the Christoffel symbols, the Riemannian curvature tensor $R_{hijk}$ and Ricci tensor $R_{ij}$, respectively, are the following, [6]

$$\left\{ \begin{array}{c} \beta \\ 11 \end{array} \right\} = -\frac{1}{2}k^{\beta\alpha}\varphi_{.\alpha}\ , \quad \left\{ \begin{array}{c} n \\ 11 \end{array} \right\} = \frac{1}{2}\varphi_{.1}\ , \quad \left\{ \begin{array}{c} n \\ 1\alpha \end{array} \right\} = \frac{1}{2}\varphi_{.\alpha},$$

$$\text{and} \qquad R_{1\alpha\beta1} = \frac{1}{2}\varphi_{.\alpha\beta}\ , \qquad R_{11} = \frac{1}{2}k^{\alpha\beta}\,\varphi_{.\alpha\beta}, \tag{2.2}$$

where (.) denotes the partial differentiation with respect to coordinates and $[k^{\alpha\beta}]$ is the inverse matrix of $[k_{\alpha\beta}]$.

Let's define the vector components $\lambda_h$ and $\mu_h$, which are contained in the formula of coefficients of the connection $D$ by

$$\lambda_h = \left\{ \begin{array}{cc} \psi(x^1), & for\ h=1 \\ 0, & otherwise \end{array} \right. , \qquad \mu_h = \left\{ \begin{array}{cc} \theta(x^1), & for\ h=1 \\ 0, & otherwise \end{array} \right. , \tag{2.3}$$

where $\psi(x^1)$ and $\theta(x^1)$ are continuous functions of $x^1$ on the interval $I = [a, b]$.

By using the formula (1.3), (2.2) and (2.3), we obtain the following non-zero components of $\Gamma^h_{ij}$

$$\begin{aligned}
\Gamma^1_{11} &= \left\{ \begin{array}{c} 1 \\ 11 \end{array} \right\} + \delta^1_1\lambda_1 - \mu_1\delta^1_1 - g_{11}\lambda^1 = \lambda_1 - \mu_1\,, \\
\Gamma^\beta_{11} &= \left\{ \begin{array}{c} \beta \\ 11 \end{array} \right\} + \delta^\beta_1\lambda_1 - \mu_1\delta^\beta_1 - g_{11}\lambda^\beta = \left\{ \begin{array}{c} \beta \\ 11 \end{array} \right\} = -\frac{1}{2}k^{\beta\alpha}\varphi_{.\alpha}\,, \\
\Gamma^\alpha_{1\alpha} &= \left\{ \begin{array}{c} \alpha \\ 1\alpha \end{array} \right\} + \delta^\alpha_1\lambda_\alpha - \mu_1\delta^\alpha_\alpha - g_{1\alpha}\lambda^\alpha = -\mu_1, \\
\Gamma^\alpha_{\alpha1} &= \left\{ \begin{array}{c} \alpha \\ \alpha1 \end{array} \right\} + \delta^\alpha_\alpha\lambda_1 - \mu_\alpha\delta^\alpha_1 - g_{\alpha1}\lambda^\alpha = \lambda_1, \\
\Gamma^\beta_{1\alpha} &= \left\{ \begin{array}{c} \beta \\ 1\alpha \end{array} \right\} + \delta^\beta_1\lambda_\alpha - \mu_1\delta^\beta_\alpha - g_{1\alpha}\lambda^\beta = \left\{ \begin{array}{ll} -\mu_1, & \text{for } \alpha=\beta \\ 0, & \alpha\neq\beta \end{array} \right. ,
\end{aligned}$$

$$\begin{aligned}
\Gamma^{\beta}_{\alpha 1} &= \left\{ \begin{array}{c} \beta \\ \alpha 1 \end{array} \right\} + \delta^{\beta}_{\alpha}\lambda_1 - \mu_{\alpha}\delta^{\beta}_1 - g_{\alpha 1}\lambda^{\beta} = \begin{cases} \lambda_1, & \text{for } \alpha = \beta \\ 0, & \alpha \neq \beta \end{cases}, \\
\Gamma^{n}_{11} &= \left\{ \begin{array}{c} n \\ 11 \end{array} \right\} + \delta^{n}_{1}\lambda_1 - \mu_1\delta^{n}_1 - g_{11}\lambda^{n} = \left\{ \begin{array}{c} n \\ 11 \end{array} \right\} = \frac{1}{2}\varphi_{.1} - \lambda_1\varphi, \\
\Gamma^{n}_{1\alpha} &= \left\{ \begin{array}{c} n \\ 1\alpha \end{array} \right\} + \delta^{n}_{1}\lambda_\alpha - \mu_1\delta^{n}_\alpha - g_{1\alpha}\lambda^{n} = \left\{ \begin{array}{c} n \\ 1\alpha \end{array} \right\} = \frac{1}{2}\varphi_{.\alpha}, \\
\Gamma^{n}_{\alpha 1} &= \left\{ \begin{array}{c} n \\ \alpha 1 \end{array} \right\} + \delta^{n}_{\alpha}\lambda_1 - \mu_\alpha\delta^{n}_1 - g_{\alpha 1}\lambda^{n} = \left\{ \begin{array}{c} n \\ \alpha 1 \end{array} \right\} = \frac{1}{2}\varphi_{.\alpha}, \\
\Gamma^{n}_{1n} &= \left\{ \begin{array}{c} n \\ 1n \end{array} \right\} + \delta^{n}_{1}\lambda_n - \mu_1\delta^{n}_n - g_{1n}\lambda^{n} = -\mu_1 - \lambda_1, \\
\Gamma^{n}_{\alpha\alpha} &= \left\{ \begin{array}{c} n \\ \alpha\alpha \end{array} \right\} + \delta^{n}_{\alpha}\lambda_\alpha - \mu_\alpha\delta^{n}_\alpha - g_{\alpha\alpha}\lambda^{n} = -\lambda_1.
\end{aligned} \tag{2.4}$$

Substituting (2.3) in the formula of $\lambda_{jk}$ and $\mu_{jk}$ and using (2.2) we see that the only non-zero components are

$$\lambda_{11} = \nabla_1\lambda_1 - \lambda_1\lambda_1 = \frac{\partial\lambda_1}{\partial x^1} - \lambda_a\left\{ \begin{array}{c} a \\ 11 \end{array} \right\} - \lambda_1^2 = \psi'(x^1) - \psi^2(x^1), \tag{2.5}$$

$$\mu_{11} = \nabla_1\mu_1 - \mu_1\mu_1 = \frac{\partial\mu_1}{\partial x^1} - \mu_a\left\{ \begin{array}{c} a \\ 11 \end{array} \right\} - \mu_1^2 = \theta'(x^1) - \theta^2(x^1). \tag{2.6}$$

Recall that the covariant derivatives of the tensors $\lambda_{ij}$ and $\mu_{ij}$ with respect to the connection $D$ are given

$$D_k\lambda_{ij} = \frac{\partial\lambda_{ij}}{\partial x^k} - \lambda_{aj}\,\Gamma^a_{ik} - \lambda_{ia}\,\Gamma^a_{jk}\,, \qquad D_k\mu_{ij} = \frac{\partial\mu_{ij}}{\partial x^k} - \mu_{aj}\,\Gamma^a_{ik} - \mu_{ia}\,\Gamma^a_{jk}\,.$$

One can easily show that the only non-zero components $D_l\,\lambda_{11}$ and $D_l\,\mu_{11}$ are

$$\begin{aligned}
D_1\lambda_{11} &= (\psi'' - 2\psi\,\psi') - 2(\psi' - \psi^2)(\psi - \theta), \\
D_1\mu_{11} &= (\theta'' - 2\theta\,\theta') - 2(\theta' - \theta^2)(\psi - \theta).
\end{aligned} \tag{2.7}$$

For the metric (2.1),

if we consider $k_{\alpha\beta}$ as $\delta_{\alpha\beta}$ and $\varphi = k_{\alpha\beta}x^\alpha x^\beta (x^1)^4\, e^{\int_a^{x^1}(4\psi(t)-2\theta(t))dt}$, $x^1 \in I = [a,b]$,

then we obtain

$$\varphi = \sum_{\alpha=2}^{n-1} x^\alpha x^\alpha (x^1)^4\, e^{\int_a^{x^1}(4\psi(t)-2\theta(t))dt}, \quad x^1 \in I. \tag{2.8}$$

Hence,

$$\varphi_{.\alpha\alpha} = 2\,(x^1)^4\, e^{\int_a^{x^1}(4\psi(t)-2\theta(t))dt}, \quad \text{and } \varphi_{.\alpha\beta}=0 \text{ for } \alpha\neq\beta. \tag{2.9}$$

It follows from (2.2) and (2.8) that the only non-zero components of $R_{hijk}$ are

$$R_{1\alpha\alpha 1} = (x^1)^4\, e^{\int_a^{x^1}(4\psi(t)-2\theta(t))dt}. \tag{2.10}$$

On the other hand, by using the covariant derivative of the curvature tensor with respect to the connection $D$, we find

$$D_l L_{hijk} = \frac{\partial L_{hijk}}{\partial x^l} - L_{aijk}\,\Gamma^a_{hl} - L_{hajk}\,\Gamma^a_{il} - L_{haik}\,\Gamma^a_{jl} - L_{hija}\,\Gamma^a_{kl}.$$

Also, by means of (2.4) and (2.9), it can be easily shown that the only non-zero components of $D_l\,R_{hijk}$ are

$$D_1\,R_{1\alpha\alpha 1} = 4(x^1)^3\, e^{\int_a^{x^1}(4\psi(t)-2\theta(t))dt}. \tag{2.11}$$

Substituting (2.5), (2.6), (2.9), and (2.10) in (1.11), we obtain the non-zero components of the curvature tensor $L_{hijk}$ and their covariant derivatives with respect to the connection $D$ as follows

$$\begin{aligned} L_{1\alpha 1\alpha} &= R_{1\alpha 1\alpha} + g_{\alpha\alpha}\lambda_{11}, \\ L_{\alpha 11\alpha} &= -L_{1\alpha 1\alpha} = R_{\alpha 11\alpha} - g_{\alpha\alpha}\lambda_{11}, \\ L_{1\alpha\alpha 1} &= R_{1\alpha\alpha 1} - g_{\alpha\alpha}\lambda_{11}, \\ L_{\alpha 1\alpha 1} &= -L_{1\alpha\alpha 1} = R_{\alpha 1\alpha 1} + g_{\alpha\alpha}\lambda_{11}, \\ L_{1n11} &= -\lambda_{11}, \\ L_{n111} &= -L_{1n11} = \lambda_{11}, \end{aligned} \tag{2.12}$$

$$\begin{aligned} D_kL_{1\alpha 1\alpha} &= D_kR_{1\alpha 1\alpha} + D_k(g_{\alpha\alpha}\lambda_{11}) = \begin{cases} D_1R_{1\alpha 1\alpha} + D_1(g_{\alpha\alpha}\lambda_{11}), & k=1 \\ 0, & k\neq 1 \end{cases}, \\ D_kL_{\alpha 11\alpha} &= D_kR_{\alpha 11\alpha} - D_k(g_{\alpha\alpha}\lambda_{11}) = \begin{cases} D_1R_{\alpha 11\alpha} - D_1(g_{\alpha\alpha}\lambda_{11}), & k=1 \\ 0, & k\neq 1 \end{cases}, \\ D_kL_{1\alpha\alpha 1} &= D_kR_{1\alpha\alpha 1} - D_k(g_{\alpha\alpha}\lambda_{11}) = \begin{cases} D_1R_{\alpha\alpha 1} - D_1(g_{\alpha\alpha}\lambda_{11}), & k=1 \\ 0, & k\neq 1 \end{cases}, \\ D_kL_{\alpha 1\alpha 1} &= D_kR_{\alpha 1\alpha 1} + D_k(g_{\alpha\alpha}\lambda_{11}) = \begin{cases} D_1R_{\alpha 1\alpha 1} + D_1(g_{\alpha\alpha}\lambda_{11}), & k=1 \\ 0, & k\neq 1 \end{cases}, \\ D_kL_{1n11} &= -D_k\lambda_{11} = \begin{cases} -D_1\lambda_{11}, & k=1 \\ 0, & k\neq 1 \end{cases}, \\ D_kL_{n111} &= D_k\lambda_{11} = \begin{cases} D_1\lambda_{11}, & k=1 \\ 0, & k\neq 1. \end{cases} \end{aligned} \tag{2.13}$$

Since $D_l\, R_{1\alpha\alpha 1} = 0$ and $D_l\, \lambda_{11} = 0$ for $l \neq 1$ , we observe that $D_l\, L_{hijk} = 0$ for $l \neq 1$.

Moreover, using the metric recurrency condition we obtain

$$D_1\, g_{11} = 2\mu_1\, g_{11}, \tag{2.14}$$

$$D_1\, g_{1n} = 2\mu_1\, g_{1n}, \tag{2.15}$$

from which we get

$$\varphi_{.1} - 2\varphi\psi = 0, \tag{2.16}$$

$$\theta = -\psi. \tag{2.17}$$

If we suppose that $\rho = (-4/x^1, 0, 0, ..., 0)$, $(x^1 \neq 0)$ then $D$-recurrency condition (1.12) reduces to $D_1\, L_{hijk} = \rho_1\, L_{hijk}$ which gives the following equations

$$D_1\,(g_{\alpha\alpha}\lambda_{11}) = \rho_1\, g_{\alpha\alpha}\lambda_{11}, \tag{2.18}$$

$$D_1\, R_{1\alpha\alpha 1} = \rho_1\, R_{1\alpha\alpha 1}. \tag{2.19}$$

Substituting (2.5), (2.6) and (2.17) in (2.18) we obtain the following differential equation

$$\psi'' - 4\psi\psi' + 2\psi^3 = 0, \tag{2.20}$$

where $(')$ denotes the derivative with respect to $x^1$.

Let us consider the transformations

$$\psi' = \phi \tag{2.21}$$

$$\psi'' = \dot{\phi}\,\phi \tag{2.22}$$

then the differential equation (2.20) takes the form

$$\dot{\phi}\,\phi - 4\psi\,\phi + 2\psi^3 = 0, \tag{2.23}$$

where $\dot{\phi} = d\phi/d\psi$.

When we compare (2.23) with Abel's equation of the 2nd kind it is seen that [10]

$$\begin{aligned} \phi\,\dot{\phi} &= P(\psi)\,\phi + Q(\psi), \\ P &= 4\psi, \qquad Q = -2\psi^3. \end{aligned} \tag{2.24}$$

After transformations

$$\phi = \psi^2\,\kappa(\psi), \qquad \zeta = \ln\psi \tag{2.25}$$

and by using the derivative of $\kappa$ with respect to $\zeta$

$$\frac{d\kappa}{d\zeta} = \frac{\kappa_\psi}{\zeta_\psi} = \psi\left[\frac{\dot{\phi}}{\psi^2} - \frac{2\phi}{\psi^3}\right] \tag{2.26}$$

we have the separable differential equation

$$\frac{d\kappa}{d\zeta} = 4 - \frac{2}{\kappa} - 2\kappa = -\frac{2}{\kappa}(\kappa - 1)^2. \tag{2.27}$$

We can solve the above differential equation for $\zeta$ as

$$\zeta = \frac{1}{2}\left[\frac{1}{\kappa - 1} - \ln(\kappa - 1)\right] + C_1, \qquad \kappa \neq 1 \tag{2.28}$$

and by replacing $\kappa = \phi/\psi^2$ and $\zeta = \ln\psi$ we find the following first order ODE

$$\ln\psi = \frac{1}{2}\left[\frac{1}{\psi'/\psi^2 - 1} - \ln(\frac{\psi'}{\psi^2} - 1)\right] + C_1 \tag{2.29}$$

with an arbitrary integration constant $C_1$.

After arranging the above equation according to $\psi$ and $\psi'$, the solution takes the form

$$\frac{\psi^2}{\psi' - \psi^2} = \ln[C_1(\psi' - \psi^2)] \tag{2.30}$$

and this equation needs to be integrated once more to find the general solution. However, it is not possible to solve it explicitly for $\psi'$. So, we take a particular solution for the choice, $\kappa = 1$ in (2.27) and this leads to the solution

$$\psi(x^1) = -\frac{1}{x^1 + c} = -\theta, \qquad x^1 \neq -c, \tag{2.31}$$

where $c$ is a constant.

Finally, we have determined the functions $\psi$ and $\theta$, therefore, the connection coefficients are determined such that the metric recurrency condition (1.2) and $D$-recurrency condition (1.12) are satisfied for the space $(M_n, g, D)$, having the metric (2.1).

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## REFERENCES

[1] O.C. Andonie, D. Smaranda, Certaines connexions sem-symetriques, *Tensor, N. S.,* **31**, (1977), 8-12.

[2] U.C. De, B.K. Mazumder , On pseudo Ricci Symmetric Spaces, *Tensor, N.S.,* **60**, (1998), 135-138.

[3] U.C. De, S. Bandyopadhyay, On weakly symmetric Riemannian spaces, *Publ. Math. Debrecen,* **54**, (1999), 377-381.

[4] Y.X. Liang, On semi-symmetric and metric-recurrent connection, *Journal of Xiamen University (natural science),* **27**, (1988), 248-253.

[5] Y.X. Liang, On semi-symmetric and recurrent-metric connection, *Tensor, N. S.,* **55**, (1994), 107-112.

[6] W. Roter, On conformally related conformally recurrent metrics 1, Some general results, *Colloquium Mathematicum*, **47**, (1982), 39-46,

[7] A.G. Walker, On Ruse's spaces of recurrent curvature , *Proc. Lond. Math. Soc.* , **52**, (1950), 36-64,

[8] S.A. Uysal and R. Doğan, Ö., On recurrent spaces with semi-symmetric Singh connection, *Indian J. Pure Appl. Math.* **34(7)** (2003) 1061-1065.

[9] K. Yano, On semi-symmetric metric connection, *Rev. Roumanie Math. Prues Appl.,* **15**, (1970), 1579-1586

[10] D. Zwillinger, Handbook of Differential Equations, 3rd ed. Boston, MA: Academic Press, p.120, 1997.

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