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Generalized Λ-Sets and λ-Sets in Bitopological Spaces

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Abstract

The aim of this paper is to introduce the concept of generalized Λ-sets in bitopological spaces and define the associated closure operator. Also we shall introduce the concept of λ-sets in bitopological spaces and establish a new decomposition of pairwise continuity.

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1- Introduction

H.Maki [4], introduced the concept of generalized $\wedge$-sets in topological spaces and defined the associated closure operator $C^{\wedge}$. G.A. Francisco et al [1] introduced the concept of λ-sets in topological spaces. T.Fukutake [2] introduced the concept of generalized closed sets in bitopological spaces and a new operator $(\tau_i, \tau_j) - Cl^*$.

The aim of this paper is to introduce the concepts of generalized Λ-sets and λ-sets in bitopological spaces and define the associated closure operator $ij - cl^{\wedge}$ on the space as in analogy of [2] by generalizing the results in [1,4] to bitopological spaces.

Throughout this paper (X,τ_1,τ_2) and (Y,σ_1,σ_2) (or briefly, X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of X, by i-$cl(A)$ we denote the closure of A with respect to τ_i (or σ_i) and $X \setminus A = A^c$ will be denote the complement of A. Also $i,j = 1,2$ and $i \neq j$.

2- ij-g.$\wedge$ -sets.

Definition 2.1. Let B be a subset of a bitopological space (X,τ_1,τ_2) we define $B^{\wedge_i} = \cap\{U : B \subset U, U \in \tau_i\}$ and $B^{\vee_i} = \cup\{F : F \subset B, X \setminus F \in \tau_i\}$

Remark 2.2. Let A, B and B_k be subsets of a bitopological spac (X,τ_1,τ_2) for every $k \in I$ (an index set), then we have

(1) $B \subset B^{\wedge_i}$.

(2) If $A \subset B$ then $A^{\wedge_i} \subset B^{\wedge_i}$.

(3) $\left(B^{\wedge_i}\right)^{\wedge_i} = B^{\wedge_i}$.

(4) $\left(\cup_{k\in I} B_k\right)^{\wedge_i} = \cup_{k\in I} B_k^{\wedge_i}$.

(5) If $B \in \tau_i$ then $B = B^{\wedge_i}$.

(6) $(X \setminus B)^{\wedge_i} = X \setminus B^{\vee_i}$.

The converse of (5) above is not true, in general, however, since the intersection of a finite number of τ_i -open sets is τ_i -open, then in finite spaces the converse of (5) is true.

Remark 2.3. We have the following formulas $\left(\cap_{k\in I} B_k\right)^{\wedge_i} \subset \cap_{k\in I} B_k^{\wedge_i}$ and $\left(\cup_{k\in I} B_k\right)^{\vee_i} \supset \cup_{k\in I} B_k^{\vee_i}$; however, in general, $\left(B_1 \cap B_2\right)^{\wedge_i} \neq B_1^{\wedge_i} \cap B_2^{\wedge_i}$ as the following example shows.

Example 2.4. Let $X = \{a,b,c\}$, $\tau_1 = \{X,\phi,\{a\},\{b,c\}\}$ and $\tau_2 = \{X,\phi,\{b\}\}$. Let $B_1 = \{b\}$ and $B_2 = \{ c\}$. Then $\left(B_1 \cap B_2\right)^{\wedge_1} = \phi$ but $B_1^{\wedge_1} \cap B_2^{\wedge_1} = \{b,c\}$.

Definition 2.5. A subset *B* of a bitopological space (X,τ_1,τ_2) is called a $\wedge_i$-set (resp. a $\vee_i$-set) if $B = B^{\wedge_i}$ (resp. $B = B^{\vee_i}$). Then we have the following properties:

(1) ϕ and *X* are $\wedge_i$-sets.

(2) Every union of $\wedge_i$-sets is a $\wedge_i$-set.

(3) Every intersection of $\wedge_i$-sets is a $\wedge_i$-set.

(4) A subset *B* is a $\wedge_i$-set if and only if $X\backslash B$ is a $\vee_i$-set.

Thus, we conclude that the family of all $\wedge_i$-sets of *X* form a topology on *X* finer than τ_i.

To show that not every $\wedge_i$-set is τ_i-open and so the converse of (5) in Remark 2.2 is not true, in general, let *X* = *R* be the set of all real numbers, τ_1 = the usual topology and τ_2 = the indiscrete topology on *R*. Then $\{0\}$ is a $\wedge_1$-set but not τ_1-open.

It is easy to see that in a bitopological space (X,τ_1,τ_2) if every singleton set is a $\wedge_i$-set, then (X,τ_1,τ_2) is pairwise T_1 in the sense of Reilly [5].

Proposition 2.6. If a bitopological space (X,τ_1,τ_2) is pairwise T_1 in the sense of Reilly [5], then every subset of *X* is a $\wedge_i$-set.

Proof : Let *B* be a subset of *X* and $x \in X\backslash B$. Since *X* is pairwise T_1, then $\{x\}$ is τ_i-closed and so $X\backslash\{x\}$ is a τ_i-open set containing *B*. This implies that $x \notin B^{\wedge_i}$. Hence we have $B^{\wedge_i} \subset B$ and therefore $B^{\wedge_i} = B$.

Definition 2.7. A subset *B* of a bitopological space (X,τ_1,τ_2) is called an *ij-g.*$\wedge$-set if $B^{\wedge_i} \subset F$ whenever $B \subset F$ and *F* is τ_j-closed . A subset *B* is called an *ij-g.*$\vee$-set of *X* if $X\backslash B$ is an *ij-g.*$\wedge$-set . Let $ij - D^{\wedge}$ (resp. $ij - D^{\vee}$) denote the set of all *ij-g.*$\wedge$-sets (resp. *ij-g.*$\vee$-sets) of *X* .

Remark 2.8. . In a bitopological space (X,τ_1,τ_2), we have

(1) every $\wedge_i$-set is an *ij-g.*$\wedge$-set.

(2) every $\vee_i$-set is an *ij-g.*$\vee$-set.

(3) if $B_k \in ij-D^{\wedge}$, then $\cup_{k\in I} B_k \in ij-D^{\wedge}$ for any index set I.

(4) if $B_k \in ij-D^{\vee}$, then $\cap_{k\in I} B_k \in ij-D^{\vee}$ for any index set I.

The intersection of two *ij-g*.∧-sets need not be an *ij-g*.∧-set as can be shown by the following example

Example 2.9. Let $X=\{a,b,c,d\}$, $\tau_1=\{X,\phi,\{a\},\{b\},\{a,b\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$ and $\tau_2=\{X,\phi,\{b\},\{c,d\},\{b,c,d\}\}$. Then $\{a,b,c\}$ and $\{a,b,d\}$ are 21-g.∧-sets but their intersection $\{a,b\}$ not 21-g.∧-set.

The converse of Remark 2.8(1) is not true, in general, since in Example 2.9, $\{b,c\}$ and $\{a,b,d\}$ are both 12-g.∧-sets but not $\wedge_1$-sets.

Proposition 2.10. Let *x* be a point of a bitopological space (X,τ_1,τ_2), then

(1) $\{x\}$ is a τ_j-open set or $X\backslash\{x\}$ is an *ij-g*.∧-set.

(2) $\{x\}$ is a τ_j-open set or $\{x\}$ is an *ij-g*.∨-set.

Proof: (1) Suppose that $\{x\}$ is not a τ_j-open set. Then the only a τ_j-closed set *F* containing $X\backslash\{x\}$ is *X*. Thus $(X\backslash\{x\})^{\wedge_i} \subset F = X$ and $X\backslash\{x\}$ is an *ij-g*.∧-set.

(2) Follows directly by (1) and the definition.

Remark 2.11. In a bitopological space (X,τ_1,τ_2)

(1) for a subset $B\subset X$, generally $B^{\wedge_1}$ not equal to $B^{\wedge_2}$ since, in Example 2.9, $\{a,c\}^{\wedge_1}=\{a,c,d\}$ but $\{a,c\}^{\wedge_2}=X$.

(2) generally, $12-D^{\wedge}$ is not equal to $21-D^{\wedge}$ since, in Example 2.9, $12-D^{\wedge}$ = the set of all subsets of *X* while $21-D^{\wedge}=\{\{b\},\{c\},\{d\},\{c,d\},\{b,c\},\{b,d\},\{b,c,d\},$ $\{a,b,c\},\{a,b,d\},X,\phi\}$.

Proposition 2.12.In a bitopological space(X,τ_1,τ_2) if $\tau_i\subset\tau_j$,then $B^{\wedge_i}\subset B^{\wedge_j}$ and $ij-D^{\wedge}\subset ji-D^{\wedge}$, for any subset *B* of *X*.

Proposition 2.13. In a bitopological space (X,τ_1,τ_2) every subset *A* is an *ij-g*.∧-set if and only if every τ_j-closed set is a $\wedge_i$-set .

Proof: Let $A\subset X$ and let *F* be a τ_j-closed set such that $A\subset F$. Then $A^{\wedge_i}\subset F^{\wedge_j}=F$ (since *F* is a $\wedge_i$-set). This shows that *A* is an *ij-g*.∧-set.

Conversely, let *F* be a τ_j-closed set, then *F* is an *ij-g.*$\wedge$-set and so $F^{\wedge_i} \subset F$. Since $F \subset F^{\wedge_i}$, then $F = F^{\wedge_i}$ and so *F* is a $\wedge_i$-set .

Proposition 2.14. In a bitopological space (X,τ_1,τ_2) if *B* is an *ij-g.*$\wedge$-set, then $B^{\wedge_i} \setminus B$ contains no nonempty τ_j-open set .

Proof: Let *U* be a τ_j-open subset of $B^{\wedge_i} \setminus B$, then $B \subset U^c$. Now since U^c is τ_j-closed and *B* is an *ij-g.*$\wedge$-set, then $B^{\wedge_i} \subset U^c$, i,e., $U \subset (B^{\wedge_i})^c$. Thus $U \subset B^{\wedge_i} \cap (B^{\wedge_i})^c = \phi$.

Proposition 2.15. In a bitopological space (X,τ_1,τ_2) if *A* is an *ij-g.*$\wedge$-set and $A \subset B \subset A^{\wedge_i}$, then *B* is an *ij-g.*$\wedge$-set.

3. ij-$\wedge$-closure operator.

By using the family of all *ij-g.*$\wedge$-sets of a bitopological space (X,τ_1,τ_2), we can define the closure operator $ij-cl^{\wedge}$ and the associated topology $\tau^{\wedge}(i,j)$ on X.

Definition 3.1. For a subset *B* of a bitopological space (X,τ_1,τ_2) we define $ij-cl^{\wedge}(B) = \cap\{U : B \subset U, U \in ij-D^{\wedge}\}$ and $ij-\text{int}^{\wedge}(B) = \cup\{F : F \subset B, F \in ij-D^{\vee}\}$.

Proposition 3.2. Let *A*, *B* and B_k be subsets of a bitopological space (X,τ_1,τ_2) for $k \in I$, then

(1) $B \subset ij-cl^{\wedge}(B)$, $ij-cl^{\wedge}(X \setminus B) = X \setminus ij-\text{int}^{\wedge}(B)$ and $ij-cl^{\wedge}(\phi) = \phi$.

(2) $\cup_{k\in I}\, ij-cl^{\wedge}(B_k) = ij-cl^{\wedge}(\cup_{k\in I} B_k)$.

(3) $ij-cl^{\wedge}(ij-cl^{\wedge}(B)) = ij-cl^{\wedge}(B)$.

(4) if $A \subset B$, then $ij-cl^{\wedge}(A) \subset ij-cl^{\wedge}(B)$.

Proof : The proofs of (1) and (4) are obvious.

(2) Suppose that there exists a point $x \notin ij-cl^{\wedge}(\cup_{k\in I} B_k)$. Then there exists a subset $U \in ij-D^{\wedge}$ such that $\cup_{k\in I} B_k \subset U$ and $x \notin U$. Thus for each $k \in I$, we have $x \notin \cup_{k\in I}\, ij-cl^{\wedge}(B_k)$. Conversely, let $x \notin \cup_{k\in I}\, ij-cl^{\wedge}(B_k)$. Then there

exists subsets $U_k \in ij-D^{\wedge}$, for all $k\in I$, such that $x \notin U_k$ and $B_k \subset U_k$. Let $U = \cup_{k\in I} U_k$, then $x \notin U$, $U \in ij-D^{\wedge}$ and $\cup_{k\in I} B_k \subset U$. This shows that $x \notin ij-cl^{\wedge}(\cup_{k\in I} B_k)$.

(3) It is clear by the definition that $ij-cl^{\wedge}(B) \subset ij-cl^{\wedge}(ij-cl^{\wedge}(B))$. Now let $x \notin ij-cl^{\wedge}(B)$, then there exists $U \in ij-D^{\wedge}$ such that $x \notin U$ and $B \subset U$.Since $U \in ij-D^{\wedge}$ then $ij-cl^{\wedge}(B) \subset U$,thus $x \notin ij-cl^{\wedge}(ij-cl^{\wedge}(B))$ .this $ij-cl^{\wedge}(ij-cl^{\wedge}(B)) \subset ij-cl^{\wedge}(B)$.

By Proposition 3.2, we have the following theorem

Theorem 3.3. The operator $ij-cl^{\wedge}$ defined above is a Kuratowski operator on *X*.

From Theorem 3.3 $ij-cl^{\wedge}$ defines a new topology on X .

Definition 3.4. Let $\tau^{\wedge}(i,j)$ be the topology on *X* generated by $ij-cl^{\wedge}$ in the usual manner, i.e., $\tau^{\wedge}(i,j) = \{E \subset X : ij-cl^{\wedge}(X \setminus E) = X \setminus E\}$. We define a family $ij-F^{\wedge}$ by $ij-F^{\wedge} = \{B : ij-cl^{\wedge}(B) = B\}$. Then $ij-F^{\wedge} = \{B : X \setminus B \in \tau^{\wedge}(i,j)\}$. And let $\tau^{\vee}(i,j) = \{B \subset X : ij-\operatorname{int}^{\vee}(B) = B\}$.

Proposition 3.5. Let $F_i^{\wedge}$ (resp. $\tau_i^{\vee}$) be the family of all $\wedge_i$-sets (resp. $\vee_i$-sets) of (X,τ_1,τ_2) ,Then

(1) $\tau^{\wedge}(i,j) = \tau^{\vee}(i,j)$.

(2) $\tau_i \subset F_i^{\wedge} \subset ij-D^{\wedge} \subset ij-F^{\wedge}$.

(3) $F_i \subset \tau_i^{\vee} \subset ij-D^{\vee} \subset \tau^{\wedge}(i,j)$, where F_i is the family of all τ_i-closed subsets of X .

Proof: (1) It is clear by Remark 2.2 (6) and Definition 3.4. To prove (2), let $B \in \tau_i$, then ,by Remark 2.2(5) B is $\wedge_i$-set ,i.e, $B \in F_i^{\wedge}$. By Remark 2.8(1) $B \in ij-D^{\wedge}$ for each $B \in F_i^{\wedge}$. For each $B \in ij-D^{\wedge}$ we have $B = ij-cl^{\wedge}(B)$. This implies that $B \in ij-F^{\wedge}$. The proof of (3) follows directly by Remark2.2 (5,6), Remark 2.8(2) and Definition 3.4.

Remark 3.6. Containment relations in Proposition 3.5 may not be proper, since in Example 2.9, $\{a,b\} \in 21-F^{\wedge}$ but $\{a,b\} \notin 21-D^{\wedge}$ and $\{a,b,c\} \in 21-D^{\wedge}$ but $\{a,b,c\}$ not a $\wedge_2$-set.

Remark 3.7. Let (X,τ_1,τ_2) as in Example 2.9 . Then $21-cl^{\wedge}(\{a,d\})$ = $\{a,b,d\}$ and $(\tau_2,\tau_1)-cl^{*}(\{a,d\}) = \{a,d\}$, where $(\tau_i,\tau_j)-cl^{*}$ is the closure operator due to T. Fukutake [2]. In this space $21-D^{\wedge}$ = $\{X,\{b\},\{c\},\{d\},\{c,d\},\{b,c\},\{b,d\},\{b,c,d\},\{a,b,c\},\{a,b,d\}\}$ and $\tau^{\wedge}(2,1)$ = $\{X,\phi,\{a\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$. On the other hand $D(\tau_2,\tau_1)$ = the power set of X and $\tau^{*}(\tau_2,\tau_1)$ = the indiscrete topology on X , where $D(\tau_i,\tau_j)$ is the family of all (τ_i,τ_j)-g-closed sets (A is called (τ_i,τ_j)-g-closed if and only if j-cl(A) $\subset G$ whenever $A \subset G$ and G is τ_i-open [2]) and $\tau^{*}(\tau_i,\tau_j)$ is the topology on X generated by $(\tau_i,\tau_j)-cl^{*}$ operator [2]. These show that $(\tau_i,\tau_j)-cl^{*}$ operator does not coincide with $ij-cl^{\wedge}$ operator and $\tau^{*}(\tau_i,\tau_j)$ and $\tau^{\wedge}(i,j)$ are different topologies on X , in general.

Proposition 3.8. In a bitopological space (X,τ_1,τ_2) if U is ij-g-$\wedge$-set, then $ij-cl^{\wedge}(U)$ =U .

Corollary 3.9. In a bitopological space (X,τ_1,τ_2), if E is an ij-g. $\wedge$-set, then E is $\tau^{\wedge}(i,j)$-closed.

Proposition 3.10. For a bitopological space (X,τ_1,τ_2) , we have

(1) if $F_i = \tau^{\wedge}(i,j)$, then $\tau_i = ij-D^{\wedge}$.

(2) if every ij-g.$\wedge$-set is τ_i-open, then $\tau_i^{\vee} = \tau^{\wedge}(i,j)$.

(3) if every ij-g.$\wedge$-closed set is τ_i-closed, then $\tau_i = \tau^{\wedge}(i,j)$.

Proposition 3.11. In a bitopological space (X,τ_1,τ_2) , if $\tau_j = \tau^{\wedge}(i,j)$, then every singleton $\{x\}$ of X is $\tau^{\wedge}(i,j)$-open .

Proof: Suppose that $\{x\}$ is not τ_j-open , then by Proposition 2.10(1) , $X\backslash\{x\}$ is ij-g.$\wedge$-set. Thus $\{x\} \in \tau^{\wedge}(i,j)$. Now if $\{x\} \in \tau_j = \tau^{\wedge}(i,j)$, then $\{x\}$ is $\tau^{\wedge}(i,j)$-open.

4. ij-λ-sets .

Definition 4.1. A subset A of a bitopological space (X,τ_1,τ_2) is called ij-λ-closed if $A=L\cap F$, where L is $\wedge_i$-set and F is a τ_j-closed set . The complement of an ij-λ-closed set will be called ij-λ-open.

Remark 4.2. The concepts of ij-g.∧-sets and ij-λ-closed sets are independent, since let $X=\{a,b,c,d\}$, $\tau_1 = \{X,\phi,\{a\},\{b\},\{a,b\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$ and $\tau_2 = \{X, \phi,\{b\},\{c,d\},\{b,c,d\}\}$. Then $\{a,b,d\}$ and $\{b,c\}$ are both 21-g.∧-sets but not 21-λ-closed. Also $\{a,c,d\}$ and $\{a\}$ are both 21-λ-closed sets but not 21-g.∧-sets.

Propositon 4.3. For a subset A of a bitopological space (X,τ_1,τ_2) , the following are equivalent :

(1) A is ij-λ-closed

(2) $A=L\cap j\text{-}cl(A)$, where L is $\wedge_i$-set

(3) $A = A^{\wedge_i} \cap j-cl(A)$

Proposition 4.4. In a bitopological space (X,τ_1,τ_2) ,

(1) every ij-locally closed set is ij-λ-closed (A is ij-locally closed if $A=U\cap F$, where U is a τ_i-open set and F is τ_j-closed [3]) .

(2) every $\wedge_i$-set is ij-λ-closed .

Generally, ij-locally closed sets and $\wedge_i$-sets are independent concepts, and so an ij-λ-closed set need not be ij-locally closed or a $\wedge_i$-set. However, infinite spaces the concept of ij-locally closed sets coincides with the concept of ij-λ-closed sets.

Proposition 4.5. A subset A of a bitopological space (X,τ_1,τ_2) is (τ_i,τ_j)-g-closed if and only if $j-cl(A) \subset A^{\wedge_i}$.

Theorem 4.6. For a subset A of a bitopological space (X,τ_1,τ_2) the following are equivalent

(1) A is τ_j-closed

(2) A is (τ_i,τ_j)-g-closed and ij-locally closed

(3) A is (τ_i,τ_j)-g-closed and ij-λ-closed.

Proof:: (1)⇒(2) : Every τ_j-closed set is both (τ_i,τ_j)-g-closed and ij-locally closed .

(2)⇒(3): That is Proposition 4.4(1).

(3)⇒(1): A is (τ_i,τ_j)-g-closed, so by Proposition 4.5, $j-cl(A) \subset A^{\wedge_i}$. A is ij-λ-closed so by Proposition 4.3, $A = A^{\wedge_i} \cap j-cl(A)$. Hence, $A= j\text{-}cl(A)$ and A is j-closed.

Theorem 4.7. A bitopological space (X,τ_1,τ_2) is pairwise T_0 if and only if every singleton subset of X is ij-λ-closed .

Proof : Clear

Definition 4.8 [2]. A bitopological space (X,τ_1,τ_2) is called $ij-T_{1/2}$ if any (τ_i,τ_j) -g-closed set is τ_j -closed. (X,τ_1,τ_2) is called strongly pairwise $ij-T_{1/2}$ if it is $12-T_{1/2}$ and $21-T_{1/2}$.

Theorem 4.9 [2]. A bitopological space (X,τ_1,τ_2) is $ij-T_{1/2}$ if and only if $\{x\}$ is τ_j -open or τ_i -closed for each $x \in X$.

Theorem 4.10. A bitopological space (X,τ_1,τ_2) is $ij-T_{1/2}$ if and only if every subset of X is ij-λ-closed.

Proof: Clear

Definition 4.11. A bitopological space (X,τ_1,τ_2) is called pairwise $T_{1/4}$ if for every finite subset F of X and every $y \notin F$ there exists a set A_y containing F and disjoint from $\{y\}$ such that A_y either τ_i -open or τ_j -closed.

Theorem 4.12. A bitopological space (X,τ_1,τ_2) is pairwise $T_{1/4}$ if and only if every finite subset of X is ij-λ-closed.

Proof: Let $F \subset X$ be a finite subset . Since X is pairwise $T_{1/4}$, for every point $y \notin F$, there exists a set A_y containing F and disjoint from $\{y\}$ such that A_y is τ_i -open or τ_j -closed. Let L be the intersection of all τ_i -open sets A_y and C be the intersection of all τ_j -closed sets A_y. Clearly, L is $\wedge_i$-set and C is τ_j -closed. Note that $F = L \cap C$. This shows that F is ij-λ-closed. Conversely, Let F be a finite subset of X and $y \in X \backslash F$. Then $F = L \cap C$ where L is $\wedge_i$-set and C is τ_j -closed. If C does not contain y, then $X \backslash C$ is a τ_j -open set containing y. If C contain y, then $y \notin L$ and thus for some τ_i -open set U containing F we have $y \notin U$. Hence, X is pairwise $T_{1/4}$.

A finite union of ij-λ-closed sets need not be ij-λ-closed. However since any intersection of $\wedge_i$ -sets is $\wedge_i$ –set, we have the following:

Proposition 4.13. In a bitopological space (X,τ_1,τ_2), an arbitrary intersection of ij-λ-closed sets is ij-λ-closed.

Now, one can ask the following question: for which spaces is the set of all ij-λ-open subsets is a topology? Call those spaces ij-λ-spaces.

Clearly a bitopological space (X,τ_1,τ_2) is an ij-λ-space if and only if the union of two ij-λ-closed sets is ij-λ-closed. From Theorem 4.10, we have that every an *ij*-$T_{1/2}$ space is an ij-λ-space. Also a pairwise T_0 ij-λ-space is pairwise $T_{1/4}$, since from Theorem 4.7 , every singleton is ij-λ-closed and in an ij-λ-space , finite union of ij-λ-closed sets is ij-λ-closed .

Definition 4.14. A function *f*: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is called

(1) ij-g.continuous if the inverse image of each σ_j-closed set in *Y* is (τ_i,τ_j)-g.closed in *X*. *f* is called pairwise g-continuous if it is 12-g-continuous and 21-g-continuous.

(2) ij-co-LC-continuous if $f^{-1}(V)$ is ij-locally closed in *X* for every σ_j-closed set *V* of *Y*. *f* is called pairwise co-LC-continuous if it is 12-co-LC-continuous and 21-co-LC-continuous.

(3) ij-λ-continuous if $f^{-1}(V)$ is ij-λ-closed in *X* for every σ_j-closed *V* of *Y*. *f* is called pairwise λ-continuous if it is 12-λ-continuous and 21-λ-continuous .

Every ij-co-LC-continuous function is ij-λ-continuous but the converse my not be true, in general, as can be shown by the following example

Example 4.15. Let R be the set of all real numbers , τ_1 = the usual topology on *R* , τ_2 = the indiscrete topology on *R*, σ_1 = the cofinite topology on *R* and σ_2 = {ϕ, *R*, *R*\{0}} . The identity function *f*: $(R,\tau_1,\tau_2) \rightarrow (R,\sigma_1,\sigma_2)$ is 12-λ-continuous but not 12-co-LC-continuous. Since {0} is the only proper σ_2-closed set and $f^{-1}(\{0\}) = \{0\}$ is 12-λ-closed because {0}={0}∩*R* , {0} is $\wedge_1$-set and *R* is τ_2-closed. But {0} is not 12-locally closed. Indeed, {0} neither τ_1-open nor τ_2-closed.

To see that ij-g-continuity and ij-λ-continuity are concepts totally independent from each other consider the following two examples

Example 4.16. Let *X*=*Y* ={*a*,*b*,*c*,*d*}, τ_1={*X*,ϕ,{*a*},{*b*,*c*},{*a*,*b*,*c*}}, τ_2= {*X* , ϕ, {*a*,*b*},{*c*,*d*}}, σ_1={*Y*,ϕ,{*b*},{*d*},{*b*,*d*}} and σ_2= {*Y*,ϕ,{*a*},{*c*},{*a*,*c*},{*b*,*c*,*d*}}. Let *f*: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ be the identity function. Then *f* is 21-g-continuous but not 21-λ-continuous.

Example 4.17. Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) as in Example 4.16 and let *f*: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ defined by *f*(*a*) = *f*(*b*) = *a* , *f*(*c*) = *f*(*d*) = *d*. Then *f* is 21-λ-continuous but not 21-g-continuous.

Finally we present the new decomposition of j-continuity and pairwise continuity.

Theorem 4.18. For a function $f : X \rightarrow Y$, the following are equivalent
(1) f is j-continuous.
(2) f is ij-g-continuous and ij-co-LC-continuous.
(3) f is ij-g-continuous and ij-λ-continuous .

Corollary 4.19. For a function f: $X \rightarrow Y$, the following are equivalent
(1) f is pairwise continuous.
(2) f is pairwise g-continuous and pairwise co-LC-continuous.
(3) f is pairwise g-continuous and pairwise λ-continuous .

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