International Mathematical Forum, 3, 2008, no. 29, 1451 - 1455

Ranking Units in Data Envelopment Analysis by Pessimistic Weights

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Abstract

In this paper, we present a new concept of the efficiency score. Whereas pessimistic input and output weights scenario, we evaluate the efficiency score of DMUs, and this is unlike the technical efficiency that use of optimistic weights to evaluate them. On the other hand, we shown that the measure of efficiency together with technical efficiency can give us a good insight respect to the efficiency of DMUs. And by evaluation these efficiency, we present a new method for DMUs ranking.

Mathematics Subject Classification: Operations Research, No. 90

Keywords: Data envelopment analysis (DEA), Efficiency, Ranking

1 Introduction

The underpinnings of efficiency measurement date back to the work of Debreu (1951) and Koopmans (1957). Debreu provided the first measure of efficiency, which was called 'coefficient of resource utilization', and Koopmans was the first to define the concept of technical efficiency. Farell (1957) extended their work in a seminal paper whose key developments. Farrell's initial analysis, that was multiple inputs and one output case generalized by charnes, cooper and Rhodes (1978) to multiple inputs and multiple outputs, and presented as CCR paper. In 1994, Thompson attained CCR model by measuring the relative efficiency of DMUs with the best input and output weights.

In this paper, we obtain efficiency with pessimistic input and output weights, and by determination this efficiencies together with the technical efficiency, we present a new method for DMUs ranking.

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The structure of this paper as follows, section two, describes the technical efficiency away weighted input and output variables. In section three, we had described the efficiency with pessimistic input and output weights.

2 Farrell's efficiency

The Farrell decomposition is a fundamental cornerstone of the theory of efficiency measurement. Farrell (1957) explicitly decomposed overall *economic efficiency* into components of *technical efficiency* and *allocative efficiency.*

The production possibility set is defined as:

$$T = \{(X, Y) \mid \text{The nonnegative vector X can produce the nonnegative vector Y}\}$$

and corresponding to each Y, we define the set $L(Y)$ following as:

$$L(Y) = \{X \mid (X, Y) \in T\}.$$

Actually, $L(Y)$ is a function that carry Y to a subset of inputs as these inputs can produce Y, where, $X = (x_1, ..., x_m)$ and $Y = (y_1, ..., y_s)$.

By noting to Farrell's technical efficiency concept, the efficiency score of DMU_o, $o \in \{1, ..., n\}$, is defined as

$$TE = Min\{\theta \mid \theta X_o \in L(Y_o)\}$$

TE is the technical efficiency of under evaluation DMU_o.

Cooper, Charnes and Rhoths (1987) defined the production possibility set to consider some statute axioms following as

$$T_c = \{(X, Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j,\;\; Y \leq \sum_{j=1}^{n} \lambda_j Y_j,\;\; \lambda_j \geq 0,\;\; j = 1, ..., n\}$$

and with due attention to the above definition of the production possibility set, the technical efficiency is attained by following famous model CCR as

$$\begin{array}{lll} \theta_o^* = & \text{Min} & \theta_o \\ & \text{s.t.} & \sum_{j=1}^{n} \lambda_j X_j \leq \theta_o X_o \\ & & \sum_{j=1}^{n} \lambda_j Y_j \geq Y_o \\ & & \lambda_j \geq 0, \qquad \text{j=1,...,n} \end{array} \tag{1}$$

We continue our discussion with to present a efficiency measure that was introduced by Thomson (1994). Therefore, consider following model

$$RE = \max_{V,U \geq 0} \{ \frac{\frac{U^tY_o}{V^tX_o}}{\max_{1 \leq j \leq n} \{ \frac{U^tY_j}{V^tX_j} \}} \} \tag{2}$$

above fraction is called *relative measure TDT*, that U and V are output and input weights vectors, respectively.

The concept of expression (2) is concluded of the relative efficiency definition, thus we allocate some weights for inputs and outputs, and the relative efficiency is computed for all U and V, such as maximize (2). we can linearize it following as

$$\begin{array}{lll} RE_o = & \text{Max} & U^tY_o \\ & \text{s.t.} & V^tX_o = 1, \\ & & U^tY_j - V^tX_j \leq 0, \quad j = 1, ..., n, \\ & & U, V \geq 0. \end{array} \tag{3}$$

that is the dual problem of model (2) and is called *multiplier CCR* model.

3 The efficiency with pessimistic input and output weights

According to the definition of the efficiency with pessimistic input and output weights, we say a DMU is efficient if its performance be better than other DMUs for all input and output weights. That is, for each $U, V \geq 0$, least performance this DMU be better than other DMUs. Therefore

$$OE = \min_{V,U \geq 0} \{ \frac{\frac{U^tY_o}{V^tX_o}}{\max_{1 \leq j \leq n} \{ \frac{U^tY_j}{V^tX_j} \}} \} \tag{4}$$

In base of definition (4), OE is pessimistic relative efficiency that the under evaluation DMU can be had. Here, we want to linearize this model, for this purpose we first consider following theorem.

Theorem 1.

$$\min_{X \geq 0} \{ \frac{a^tX}{b^tX} \} = \min \{ \frac{a_i}{b_i} \mid b_i \neq 0 \} \tag{5}$$

Proof. In (5) let $b^t = \frac{1}{t}$ hence we have

$$\min \{ ta^tX \mid tb^tX = 1, X \geq 0 \}$$

let $\overline{X} = tX$, therefore

$$\min\{a^t\overline{X} \mid b^t\overline{X} = 1, \overline{X} \geq 0\} = \min\{\frac{a_i}{b_i} \mid b_i \neq 0\}$$

and proof is complet.□

Similarly, we have

$$\max_{X\geq 0}\{\frac{a^tX}{b^tX}\} = \max\{\frac{a_i}{b_i} \mid b_i \neq 0\}$$

In (4), we convert maximum to minimum and next replace them, that is

$$OE = \min_{V,U\geq 0}\{\frac{\frac{U^tY_o}{V^tX_o}}{\max\limits_{1\leq j\leq n}\{\frac{U^tY_j}{V^tX_j}\}}\} = \min_{1\leq j\leq n}\{\min_{V,U\geq 0}\frac{\frac{U^tY_o}{V^tX_o}}{\{\frac{U^tY_j}{V^tX_j}\}}\}$$

$$= \min_{1\leq j\leq n}\{\min_{V,U\geq 0}\{\frac{U^tY_o}{U^tY_j} \times \frac{V^tX_j}{V^tX_o}\}\} = \min_{1\leq j\leq n}\{\min_{U\geq 0}\{\frac{U^tY_o}{U^tY_j}\} \times \min_{V\geq 0}\{\frac{V^tX_j}{V^tX_o}\}\}$$

in base of theorem 1, we have

$$OE = \min_{1\leq j\leq n}\{\min_{1\leq r\leq s}\{\frac{y_{ro}}{y_{rj}}\} \times \min_{1\leq i\leq m}\{\frac{x_{ij}}{x_{io}}\}\}$$

Clearly, in this type of efficiency a DMU is efficient if each its input level be lesser than or equal to other DMUs and each its output level be greater than other DMUs too. Then when DMUs are distinct both of them, at most, one DMU can be OE efficient.

4 DMUs ranking

By using of DEA models to evaluation of relative efficiency DMUs, usually more than a DMU have been efficient. therefore ranking of these DMUs is important.

In this paper, we have used of OE and TE efficiency for DMUs ranking. Consider DMUs that they are TE efficient but OE inefficient. we rank them with comparison their OE in efficiency measure. It is considerable that the unique OE efficient DMU play important role in this DMUs ranking.

Example: Now, we employ above method to rank of 15 DMUs with there inputs and three outputs, that their information and results given in Table(1).

	I1	I2	I3	I4	O1	O2	O3	CCR	AP	AP RANK	OE	OE RANK
1	6.63	7.25	1.32	2.88	321	2.11	79	0.95	0.95		0.2	
2	6.63	7.75	1.35	2.250	297	2.04	197	0.94	0.94		0.46	
3	9.06	10.75	1.27	1.37	338	1.35	110	0.97	0.97		0.18	
4	7.56	2.50	1.18	2.27	503	2.08	81	1	1.56	2	0.15	2
5	6.71	5.50	1.22	2.92	215	2.60	306	1	2.18	1	0.30	1
6	7.29	12.50	1.34	3.80	337	2.84	69	1	1.03	6	0.10	4
7	5.89	7.25	1.28	1.38	173	1.77	38	1	1.01	7	0.09	5
8	5.89	7.50	1.22	1.73	134	2.40	11	1	1.26	3	0.04	6
9	10.18	7.50	1.38	1.84	322	1.72	72	0.92	0.92		0.16	
10	8.07	8.25	1.26	2.28	281	2.19	30	0.84	0.84		0.07	
11	6.80	7.75	1.21	2.35	331	2.55	57	1	1.06	4	0.13	3
12	6.98	7.50	1.53	2.65	267	2.29	19	0.87	0.87		0.05	
13	10.63	7.25	1.98	4.96	405	1.60	8	1	1.05	5	0.02	7
14	6.00	6.25	1.76	2.43	166	1.38	4	0.62	0.62		0.01	
15	6.71	5.25	1.08	1.78	94	1.39	4	0.64	0.64		0.01	

Table (1).The result attained of AP and OE ranking.

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Received: April 23, 2008