International Mathematical Forum, 3, 2008, no. 21, 1029 - 1038

Intuitionistic (T, S)-Normed Fuzzy Closed Ideals of BCH-Algebra

Kyung Ho Kim

Department of Mathematics, Chungju National University
Chungju 380-702, Korea
ghkim@cjnu.ac.kr

Abstract

We consider the generalization of the notion of fuzzy subalgebras and closed ideal in BCH-algebras. In this paper, using t-norm T and s-norm S, we introduce the notion of intuitionistic (T, S)-normed fuzzy subalgebra and intuitionistic (T, S)-normed fuzzy closed ideal in BCH-algebras, and some related properties are investigated.

Mathematics Subject Classification: 06F35, 03G25, 03E72

Keywords: BCH-algebra, t-norm (resp. s-norm), intuitionistic (T, S)-normed fuzzy subalgebra, intuitionistic (T, S)-normed fuzzy closed ideal

1 Introduction

In 1966, Y. Imai and K. Iséki ([10]) and K. Iséki ([11]) introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In 1983, Q. P. Hu and X. Li ([8, 9]) introduced a wide class of abstract algebras: BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. They have studied some properties of these algebras. Certain other properties have been studied by B. Ahmad ([2]), M. A. Chaudhry ([5]), W. A. Dudek and J. Thomys([7]). After the introduction of the concept of fuzzy sets by Zadeh [18], several researches were conducted on the generalization of the notion of fuzzy sets. The idea of "intuitionistic fuzzy set" was first published by Atanassov [3, 4], as a generalization of the notion of fuzzy set. In this paper, using t-norm T and s-norm S, we introduce the notion of intuitionistic (T, S)-normed fuzzy subalgebra and intuitionistic (T, S)-normed fuzzy closed ideal in BCH-algebras, and some related properties are investigated.

2 Preliminaries

In this section we include some elementary aspects that are necessary for this paper.

By a *BCH-algebra* we mean an algebra $(X, *, 0)$ of type (2, 0) satisfying the following axioms:

(H1) $x * x = 0$,
(H2) $x * y = 0$ and $y * x = 0$ imply $x = y$,
(H3) $(x * y) * z = (x * z) * y$, for every $x, y, z \in X$.

In a BCH-algebra X, the following statements hold:

(P1) $x * 0 = x$.
(P2) $x * 0 = 0$ implies $x = 0$.
(P3) $x * (x * y) = (0 * x) * (0 * y)$.

A non-empty subset A of a BCH-algebra X is called a *subalgebra* of X if $x * y \in A$ whenever $x, y \in A$. A nonempty subset A of a BCH-algebra X is called a *closed ideal* of X if

(i) $0 * x \in A$ for all $x \in A$,
(ii) $x * y \in A$ and $y \in A$ imply that $x \in A$.

In what follows , let X denote a BCH-algebra unless otherwise specified. A *fuzzy set* in X is a function $\mu : X \rightarrow [0, 1]$.

A fuzzy set μ in X is called a fuzzy subalgebra of X if

$$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in X,$$

and the complement of μ, denoted by $\bar{\mu}$, is the fuzzy set in X given by $\bar{\mu}(x) = 1 - \mu(x)$ for all $x \in X$.

A mapping $f : X \rightarrow Y$ of BCH-algebras is called a *homomorphism* if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

Definition 2.1. [1] By a *t-norm* T, we mean a function $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfying the following conditions:

(T1) $T(x, 1) = x$,
(T2) $T(x, y) \leq T(x, z)$ if $y \leq z$,
(T3) $T(x, y) = T(y, x)$,
(T4) $T(x, T(y, z)) = T(T(x, y), z)$,

for all $x, y, z \in [0, 1]$.

Proposition 2.2. *Every t-norm T has a useful property:*

$$T(\alpha, \beta) \leq \min(\alpha, \beta)$$

for all $\alpha, \beta \in [0, 1]$.

Definition 2.3. [17] By a *s-norm* S, we mean a function $S:[0,1]\times[0,1]\to[0,1]$ satisfying the following conditions:

(S1) $S(x,0)=x$,
(S2) $S(x,y)\le S(x,z)$ if $y\le z$,
(S3) $S(x,y)=S(y,x)$,
(S4) $S(x,S(y,z))=S(S(x,y),z)$,

for all $x,y,z\in[0,1]$.

Proposition 2.4. *Every s-norm S has a useful property:*

$$\max(\alpha,\beta)\le S(\alpha,\beta)$$

for all $\alpha,\beta\in[0,1]$.

For a t-norm (or s-norm) P on $[0,1]$, denote by Δ_P the set of element $\alpha\in[0,1]$ such that $P(\alpha,\alpha)=\alpha$, i.e., $\Delta_P:=\{\alpha\in[0,1]\mid P(\alpha,\alpha)=\alpha\}$.

Definition 2.5. Let P be a t-norm (or s-norm). A fuzzy set μ in X is said to satisfy *idempotent property with respect to P* if $\mathrm{Im}(\mu)\subseteq\Delta_P$.

Let X denote a BCH-algebra. An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A=\{(x,\mu_A(x),\gamma_A(x)):x\in X\}$$

where the functions $\mu_A:X\to[0,1]$ and $\gamma_A:X\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for all $x\in X$.

For the sake of simplicity, we shall use the symbol $A=(\mu_A,\gamma_A)$ for the IFS $A=\{(x,\mu_A(x),\gamma_A(x)):x\in X\}$.

3 Intuitionistic (T,S)-normed fuzzy closed ideals

Definition 3.1. Let T be a t-norm and S be a s-norm on $[0,1]$. An IFS $A=(\mu_A,\gamma_A)$ in X is called an *intuitionistic (T,S)-normed fuzzy subalgebra* of BCH-algebra X if

(F1) $\mu_A(x*y)\ge T(\mu_A(x),\mu_A(y))$,
(F2) $\gamma_A(x*y)\le S(\gamma_A(x),\gamma_A(y))$, for all $x,y\in X$.

Example 3.2. Let $X = \{0, a, b, c, d\}$ be a BCH-algebra with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	d
a	a	0	0	a	d
b	b	b	0	0	d
c	c	c	c	0	d
d	d	d	d	d	0

Let $T : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by

$$T(\alpha, \beta) = \max(\alpha + \beta - 1, 0)$$

for all $\alpha, \beta \in [0, 1]$ and $S : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by

$$S(\alpha, \beta) = \min(\alpha + \beta, 1)$$

for all $\alpha, \beta \in [0, 1]$. Then T is a t-norm and S is a s-norm. Define an intuitionistic fuzzy set IFS $A = (\mu_A, \gamma_A)$ by $\mu_A(0) = \mu_A(d) = 0.9, \mu_A(a) = \mu_A(b) = \mu_A(c) = 0.09$ and $\gamma_A(a) = \gamma_A(b) = \gamma_A(c) = 0.9, \gamma_A(0) = \gamma_A(d) = 0.09$. Then IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic (T, S)-normed fuzzy subalgebra of X.

Theorem 3.3. *IF $\{A_i\}$ is a family of intuitionistic (T, S)-normed fuzzy subalgebra of X, then $\bigcap_{i \in I} A_i$ is an intuitionistic (T, S)-normed fuzzy subalgebra of X, where $\bigcap_{i \in I} A_i = (\bigvee \mu_i, \bigwedge \gamma_i)$.*

Let χ_A denote the characteristic function of a non-empty subset A of an BCH-algebra X.

Theorem 3.4. *If A is a subalgebra of an BCH-algebra X, then the IFS $\bar{I} = (\chi_A, \bar{\chi}_A)$ is an intuitionistic (T, S)-normed fuzzy subalgebra of X.*

Proof. Let $x, y \in X$. If $x, y \in A$, then $x * y \in A$ since A is a subalgebra of X. Hence

$$\chi_A(x * y) = 1 \geq T(\chi_A(x), \chi_A(y)).$$

Also, we have

$$0 = 1 - \chi_A(x * y) = \bar{\chi}_A(x * y) \leq S(\bar{\chi}_A(x), \bar{\chi}_A(y)).$$

If $x \in A$ and $y \notin A$, (or, $x \notin A$ and $y \in A$), then $\chi_A(x) = 1$, or $\chi_A(y) = 0$. Thus we have

$$\chi_A(x * y) \geq T(\chi_A(x), \chi_A(y)) = T(1, 0) = T(0, 1) = 0.$$

Next we have

$$S(\bar{\chi}_A(x), \bar{\chi}_A(y)) = S(1 - \chi_A(x), 1 - \chi_A(y)) = S(0, 1) = 1 \geq \bar{\chi}_A(x * y)$$

. This proves the theorem. □

Theorem 3.5. *Let A be a nonempty subset of a BCH-algebra X. If $\bar{A} = (\chi_A, \bar{\chi}_A)$ satisfies (F1) or (F2), then A is a subalgebra of a BCH-algebra X.*

Proof. Suppose that $\bar{A} = (\chi_A, \bar{\chi}_A)$ satisfy (F1). Let $x, y \in A$. Then it follows from (F1) that

$$\chi_A(x * y) \geq T(\chi_A(x), \chi_A(y)) = T(1, 1) = 1$$

so that $\chi_A(x * y) = 1$, i.e., $x * y \in A$. Hence A is a subalgebra of X. Now suppose that $\bar{A} = (\chi_A, \bar{\chi}_A)$ satisfy (F2). Let $x, y \in A$. Then from (F2), we have

$$\bar{\chi}_A(x * y) \leq S(\bar{\chi}_A(x), \bar{\chi}_A(y)) \leq S(1 - \chi_A(x), 1 - \chi_A(y)) = S(0, 0) = 0,$$

and thus $\bar{\chi}_A(x * y) = 1 - \chi_A(x * y) = 0$, i.e., $\chi_A(x * y) = 1$. This proves the theorem. □

Definition 3.6. Let T be a t-norm and S be a s-norm on $[0, 1]$. An intuitionistic (T, S)-normed fuzzy subalgebra $A = (\mu_A, \gamma_A)$ is called an *intuitionistic idempotent (T, S)-normed fuzzy subalgebra* of X if μ_A and γ_A satisfy the idempotent property with respect to T and S, respectively.

Example 3.7. In Example 3.2, let $T : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by

$$T(\alpha, \beta) = \max(\alpha + \beta - 1, 0)$$

for all $\alpha, \beta \in [0, 1]$ and $S : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by

$$S(\alpha, \beta) = \min(\alpha + \beta, 1)$$

for all $\alpha, \beta \in [0, 1]$. Define an intuitionistic fuzzy set IFS $A = (\mu_A, \gamma_A)$ by $\mu_A(0) = \mu_A(d) = 1, \mu_A(a) = \mu_A(b) = \mu_A(c) = 0$ and $\gamma_A(a) = \gamma_A(b) = \gamma_A(c) = 1, \gamma_A(0) = \gamma_A(d) = 0$. Then IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic idempotent (T, S)-normed fuzzy subalgebra of X.

Proposition 3.8. *Let T be a t-norm and S be a s-norm on $[0, 1]$. If IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic idempotent (T, S)-normed fuzzy subalgebra of BCH-algebra X, then $\mu_A(0 * x) \geq \mu_A(x)$ and $\gamma_A(0 * x) \leq \gamma_A(x)$ for all $x \in X$.*

Proof. For any $x \in X$, we have

$$\begin{aligned} \mu_A(0 * x) &\geq T((\mu_A(0), \mu_A(x)) \\ &\geq T(\mu_A(x * x), \mu_A(x)) \qquad \text{[by (H1)]} \\ &= T(T(\mu_A(x), \mu_A(x)), \mu_A(x)) \qquad \text{[by (T2)and (T3)]} \\ &= \mu_A(x) \qquad \text{[Since } \mu_A \text{ satisfies the idempotent property],} \end{aligned}$$

and

$$\begin{aligned}\gamma_A(0*x) &\leq S((\gamma_A(0),\gamma_A(x)) \\ &\leq S(\gamma_A(x*x),\gamma_A(x)) \qquad \text{[by (H1)]} \\ &= S(S(\gamma_A(x),\gamma_A(x)),\gamma_A(x)) \qquad \text{[by (S2)and (S3)]} \\ &= \gamma_A(x) \qquad \text{[Since } \gamma_A \text{ satisfies the idempotent property],}\end{aligned}$$

This completes the proof. □

Definition 3.9. Let T be a t-norm and S be a s-norm on $[0,1]$. An IFS $A=(\mu_A,\gamma_A)$ in X is called an *intuitionistic* (T,S)*-normed fuzzy closed ideal* of BCH-algebra X if

(F3) $\mu_A(0*x)\geq\mu_A(x)$ and $\gamma_A(0*x)\leq\gamma_A(x)$,

(F4) $\mu_A(x)\geq T(\mu_A(x*y),\mu_A(y))$ and $\gamma_A(x)\leq S(\gamma_A(x*y),\gamma_A(y))$ for all $x,y\in X$.

Let T be a t-norm and S be a s-norm on $[0,1]$. An intuitionistic (T,S)-normed fuzzy closed ideal $A=(\mu_A,\gamma_A)$ is called an *intuitionistic idempotent* (T,S)*-normed fuzzy closed ideal* of X if μ_A and γ_A satisfy the idempotent property with respect to T and S, respectively.

Example 3.10. Let $X=\{0,a,b,c\}$ be a BCH-algebra with the following Cayley table:

$*$	0	a	b	c
0	0	c	0	c
a	a	0	c	b
b	b	c	0	a
c	c	0	c	0

Define an intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ by

$$\mu_A(x)=\begin{cases}0.8 & \text{if } x\in\{0,c\},\\ 0.3 & \text{otherwise,}\end{cases} \quad \text{and} \quad \gamma_A(x):=\begin{cases}0.3 & \text{if } x\in\{0,c\}\\ 0.8 & \text{otherwise.}\end{cases}$$

let $T:[0,1]\times[0,1]\to[0,1]$ be a function defined by

$$T(\alpha,\beta)=\max(\alpha+\beta-1,0)$$

and and $S:[0,1]\times[0,1]\to[0,1]$ be a function defined by

$$S(\alpha,\beta)=\min(\alpha+\beta,1)$$

for all $\alpha,\beta\in[0,1]$. Then $A=(\mu_A,\gamma_A)$ is an intuitionistic (T,S)-normed fuzzy closed ideal of X which is not idempotent.

Example 3.11. In Example 3.10, define an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ by

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in \{0, c\}, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad \gamma_A(x) := \begin{cases} 0 & \text{if } x \in \{0, c\} \\ 1 & \text{otherwise.} \end{cases}$$

Then $A = (\mu_A, \gamma_A)$ is an intuitionistic idempotent (T,S)-normed fuzzy closed ideal of X.

Theorem 3.12. *Every intuitionistic idempotent (T,S)-normed fuzzy subalgebra satisfying (F4) is an intuitionistic idempotent (T,S)-normed fuzzy closed ideal.*

Proof. Using Proposition 3.8, it is straightforward. □

Proposition 3.13. *If IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic idempotent (T,S)-normed fuzzy closed ideal of BCH-algebra X, then $\mu_A(0) \geq \mu_A(x)$ and $\gamma_A(x) \leq \gamma_A(0)$ for all $x \in X$.*

Proof. Using (F3), (F4), (T2) and (S2), we have

$$\mu_A(0) \geq T(\mu_A(0 * x), \mu_A(x)) \geq T(\mu_A(x), \mu_A(x)) = \mu_A(x)$$

and

$$\gamma_A(0) \leq S(\gamma_A(0 * x), \gamma_A(x)) \leq S(\gamma_A(x), \gamma_A(x)) = \gamma_A(x)$$

for all $x \in X$, completing the proof. □

Theorem 3.14. *Every intuitionistic (T,S)-normed fuzzy closed ideal is an intuitionistic (T,S)-normed fuzzy subalgebra.*

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic (T,S)-normed fuzzy closed ideal of X and let $x, y \in X$. Then

$$\begin{aligned} \mu_A(x * y) &\geq T(\mu_A((x * y) * x), \mu_A(x)) && \text{[by (F(4)]} \\ &\geq T(\mu_A((x * x) * y), \mu_A(x)) && \text{[by (H3)]} \\ &= T(\mu_A(0 * y), \mu_A(x)) && \text{[by (H1)]} \\ &\geq T(\mu_A(x), \mu_A(y)), && \text{[by (F3), (T2) and (T3)]} \end{aligned}$$

and

$$\begin{aligned} \gamma_A(x * y) &\leq S(\gamma_A((x * y) * x), \gamma_A(x)) && \text{[by (F(4)]} \\ &\leq S(\gamma_A((x * x) * y), \gamma_A(x)) && \text{[by (H3)]} \\ &= S(\gamma_A(0 * y), \gamma_A(x)) && \text{[by (H1)]} \\ &\leq S(\gamma_A(x), \gamma_A(y)), && \text{[by (F3), (S2) and (S3)]} \end{aligned}$$

Hence $A = (\mu_A, \gamma_A)$ is an intuitionistic (T,S)-normed fuzzy subalgebra of X □

The converse of Theorem 3.14 may not be true. For example, the intuitionistic (T, S)-normed fuzzy subalgebra in Example 3.2 is not a intuitionistic (T, S)-normed fuzzy closed ideal since

$$\mu_A(a) = 0.09 < 0.9 = T(\mu_A(a * d), \mu_A(d)).$$

We give a condition for an intuitionistic (T, S)-normed fuzzy subalgebra to be an intuitionistic (T, S)-normed fuzzy closed ideal.

Theorem 3.15. *Let $A = (\mu_A, \gamma_A)$ be an intuitionistic (T, S)-normed fuzzy subalgebra of X. If $A = (\mu_A, \gamma_A)$ satisfies the idempotent property and inequalities $\mu_A(x * y) \leq \mu_A(y * x)$ and $\gamma_A(x * y) \geq \gamma_A(y * x)$ for all $x, y \in X$, then $A = (\mu_A, \gamma_A)$ is an intuitionistic (T, S)-normed fuzzy closed ideal of X.*

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic (T, S)-normed fuzzy subalgebra of X which satisfies the inequalities

$$\mu_A(x * y) \leq \mu_A(y * x) \text{ and } \gamma_A(x * y) \geq \gamma_A(y * x)$$

for all $x, y \in X$. It follows from Proposition 3.8 that $\mu_A(0 * x) \geq \mu_A(x)$ and $\gamma_A(0 * x) \leq \gamma_A(x)$ for all $x, y \in X$. Then

$$\begin{aligned} \mu_A(x) &= \mu_A(x * 0) \geq \mu_A(0 * x) = \mu_A((y * y) * x) \\ &= \mu_A((y * x) * y) \geq T(\mu_A(y * x), \mu_A(y)) \geq T(\mu_A(x * y), \mu_A(y)), \end{aligned}$$

and

$$\begin{aligned} \gamma_A(x) &= \gamma_A(x * 0) \leq \gamma_A(0 * x) = \gamma_A((y * y) * x) \\ &= \gamma_A((y * x) * y) \leq S(\gamma_A(y * x), \gamma_A(y)) \leq S(\gamma_A(x * y), \gamma_A(y)), \end{aligned}$$

Hence $A = (\mu_A, \gamma_A)$ is an intuitionistic (T, S)-normed fuzzy closed ideal of X □

Let $A = (\mu_A, \gamma_A)$ be an IFS in X and let $\alpha \in [0, 1]$. Then the sets

$$U(\mu_A; \alpha) := \{x \in X : \mu_A(x) \geq \alpha\}$$

and

$$L(\gamma_A; \alpha) := \{x \in X : \gamma_A(x) \leq \alpha\}$$

are called a *μ-level α-cut* and a *γ-level α-cut* of A, respectively.

Theorem 3.16. *Let T be a t-norm and S be a s-norm let $A = (\mu_A, \gamma_A)$ be an* IFS *in X such that the non-empty sets $U(\mu_A; \alpha)$ and $L(\gamma_A; \alpha)$ are closed ideals of X. Then $A = (\mu_A, \gamma_A)$ is an intuitioinistic (T, S)-normed fuzzy closed ideal of X.*

Proof. Suppose that there exists $x_0, y_0 \in X$ such that

$$\mu_A(x_0) < T(\mu_A(x_0 * y_0), \mu_A(y_0)).$$

Taking $\alpha_0 := \frac{1}{2}(\mu_A(x_0) + T(\mu_A(x_0 * y_0), \mu_A(y_0)))$, then

$$\begin{aligned}\min(\mu_A(x_0 * y_0), \mu_A(y_0)) &\geq T(\mu_A(x_0 * y_0), \mu_A(y_0)) \\ &\geq \alpha_0 > \mu_A(x_0).\end{aligned}$$

It follows that $x_0 * y_0, y_0 \in U(\mu_A; \alpha_0)$ and $x_0 \notin U(\mu_A; \alpha_0)$. This is a contradiction and hence μ_A satisfies the inequality $\mu_A(x) \geq T(\mu_A(x * y), \mu_A(y))$ for all $x, y \in X$. Similarly, suppose that there exists $x_0, y_0 \in X$ such that

$$\gamma_A(x_0) > S(\gamma_A(x_0 * y_0), \gamma_A(y_0)).$$

Taking $\beta_0 := \frac{1}{2}(\gamma_A(x_0) + S(\gamma_A(x_0 * y_0), \gamma_A(y_0)))$, then

$$\max(\gamma_A(x_0 * y_0), \gamma_A(y_0)) \leq S(\gamma_A(x_0 * y_0), \gamma_A(y_0)) \leq \beta_0 < \gamma_A(x_0).$$

It follows that $x_0 * y_0, y_0 \in L(\gamma_A; \beta_0)$ and $x_0 \notin L(\gamma_A; \beta_0)$. This is a contradiction and hence γ_A satisfies the inequality $\gamma_A(x) \leq S(\gamma_A(x * y), \gamma_A(y))$ for all $x, y \in X$. Now assume that there exists $x_0 \in X$ such that $\mu_A(0 * x_0) < \mu_A(x_0)$. Taking

$$\alpha_0 := \frac{1}{2}(\mu_A(0 * x_0) + \mu_A(x_0))$$

then $\mu_A(0 * x_0) \leq \alpha_0$ and $\mu_A(x_0) \geq \alpha_0$. It follows that $x_0 \in U(\mu_A; \alpha_0)$ but $0 * x_0 \notin U(\mu_A; \alpha_0)$. This is a contraction. Hence $\mu_A(0 * x) \geq \mu_A(x)$ for all $x \in X$. Similarly, we get $\gamma_A(0 * x) \leq \gamma_A(x)$ for all $x \in X$. □

References

[1] M. T. Abu Osman, *On some product of fuzzy subgroups,* Fuzzy Sets and Systems **24** (1987), 79-86.

[2] B. Ahmad, *On classification of BCH-algebras,* Math. Japon **35** (No. 5) (1990), 801-804.

[3] K. Atanassov, *Intuitionistic fuzzy sets,* Fuzzy Sets and Systems **20** (No.1)(1986), 87-96.

[4] K. Atanassov, *New operations defined over the intuitionistic fuzzy sets,* Fuzzy Sets and Systems **61** (1994), 137-142.

[5] M. A. Chaudhry, *On BCH-algebras,* Math. Japon **36** (No.4) (1991), 665-676.

[6] M. A. Chaudhry and H. Fakhar-ud-din, *Ideals and filters in BCH-algebras,* Math. Japon **44** (No.1) (1996), 101-112.

[7] W. A. Dudek and J. Thomys, *On decomposition of BCH-algebras,* Math. Japon **35** (1990), 1131-1138.

[8] Q. P. Hu and X. Li, *On BCH-algebras,* Math. Seminar Notes **11** (1983), 313-320.

[9] Q. P. Hu and X. Li, *On proper BCH-algebras,* Math. Japon **30** (1985), 659-661.

[10] Y. Imai and K. Iséki, *On axiom systems of propositional calculi XIV,* Proc. Japan Academy **42** (1966), 19-22.

[11] Y. Imai and K. Iséki, *An algebra related with a propositional calculus,* Proc. Japan Academy **42** (1966), 26-29.

[12] Y. Imai, *On BCI-algebras,* Math. Seminar Notes **8** (1980), 125-130.

[13] Y. Imai and S. Tanaka, *Ideal theory of BCK-algebras,* Math. Japonica **21** (1976), 351-366.

[14] Y. Imai and S. Tanaka, *An introduction to the theory of BCK-algebras,* Math. Japonica **23** (1978), 1-26.

[15] B. Schweizer and A. Sklar, *Statistical metric spaces,* Pacific J. Math. **10** (1960), 313-334.

[16] Y. H. Yon and K. H. Kim, *On intuitionistic fuzzy filters and ideals of lattices,* Far East J Math. Sci.(FJMS) **1 (3)**, (1999), 429-442.

[17] Y. Yu, J. N. Mordeson and S. C. Cheng, *Elements of L-algebra,* Lecture Notes in Fuzzy Math. and Computer Sciences, Creighton Univ., Omaha, Nebraska 68178, USA (1994).

[18] L. A. Zadeh, *Fuzzy sets*, Inform. and Control, **8** (1965), 338-353.

Received: December 13, 2007