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An Addendum to: Special Stable Vector Bundles with High Degree on a Smooth Curve

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Abstract. Let C be a smooth curve of genus $g \geq 2$. Here we study stable vector bundles E on C such that $\mathrm{rank}(E) = r \geq 2$, $h^1(C, E) > 0$ and $\deg(E)$ is very near to $r(2g-2)$.

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Let C be a smooth and connected projective curve of genus $g \geq 2$ defined over an algebraically closed field $\mathbb{K}$. For all integer r, d such that $r > 0$ let $U(r, d)$ denote the moduli space of stable vector bundles on C with rank r and degree d. $U(r, d)$ is a non-empty integral variety of dimension $r^2(g-1)+1$. If $r \geq 2$ and $d \geq r(2g-2)$, then $h^1(X, E) = 0$ for every $E \in U(r, d)$, because the definition of stability implies the non-existence of a non-zero morphism $E \to \omega_C$ and then we may apply Serre duality. For all integers r, d, k such that $r \geq 2$ and $d < r(g-2)$ set $A[r, d, k] := \{E \in U(r, d) : h^1(C, E) \geq k\}$ and $B[r, d, k] := \{E \in U(r, d) : h^1(C, E) = k\}$. This note is an addeendum to [1]. Here we prove the following result.

Theorem 1. *Fix integers r, d, k such that $1 \leq k \leq r-1$.*

(a) *If $d \leq r(2g-2) - k - 1$, then there iare $E \in A[r, d, k]$ and a surjection $u : E \to \omega_C^{\oplus k}$.*
(b) *If $r(g-1) + k \leq d \leq r(2g-2) - k - 1$, then there are $E \in B[r, d, k]$ such and a surjection $u : E \to \omega_C^{\oplus k}$.*
(c) *There is a stable vector bundle E on C fitting in an exact sequence*

$$0 \to F \to E \to \omega_C^{\oplus k} \to 0 \tag{1}$$

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if $d < r(2g-2)$ and $r(2g-2) - d + (r-k)g \geq r$. If $d < r(2g-2)$ and $r(2g-2) - d + (r-k)g \geq r$, then the set Γ of all such vector bundles is irreducible of dimension $r^2(g-1) + 1 - k(k+d-r(g-1))$ and a general $E \in \Gamma$ fits in an exact sequence (1) in which F is a general element of $U(r-k, d-k(2g-2))$.

Proof. Take $r-k$ paiwise non-isomorphic line bundles $L_i \in \mathrm{Pic}^{2g-2}(C)$ and set $A := \omega_C^{\oplus k} \oplus \bigoplus_{i=1}^{r-k} L_i$. Notice that $h^1(C, A) = k$. Let E be a general vector bundle obtained from A making $r(2g-2) - d$ general negative elementary transformations. $h^1(C, E) = \max\{k, d - r(g-1)\}$ ([4], p. 101)). The dual of [1], Theorem 2.9, proves parts (a) and (b). Part (c) follows from [3], Theorems A and B; for the assertion on F for a general $E \in \Gamma$ we also need to quote their proofs, not just the statements. □

It would be nice to extend [2] and the present note to torsion free sheaves on singular curves.

References

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