

Comprehensive Congruences on U -Cyber Semigroups

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Abstract

An U -cyber semigroup S is an idempotent-connected U -abundant semigroup whose subset U forms a subsemigroup of S . In this paper, we consider an admissible relation “ σ ” defined on such a semigroup. In fact, an U -cyber semigroup is a special U -semiabundant semigroup which is a generalization of an orthodox semigroup and also a type W semigroup. We will prove that the admissible relation “ σ ” defined on an U -cyber semigroup is a comprehensive congruence and consequently, a related result previously obtained by X. J. Guo on IC quasi-adequate semigroups in 2001 is extended to U -cyber semigroups.

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1 Introduction

Let $E(S)$ be the set of idempotents of a semigroup S and U a subset of $E(S)$. In order to give a generalization of an abundant semigroup, Lawson [9] first introduced the relations $\tilde{\mathcal{L}}^U$ and $\tilde{\mathcal{R}}^U$ on S by

$$(a, b) \in \tilde{\mathcal{L}}^U \quad \text{if and only if} \quad (\forall e \in U) \quad (ae = a \Leftrightarrow be = b);$$

$$(a, b) \in \tilde{\mathcal{R}}^U \quad \text{if and only if} \quad (\forall e \in U) \quad (ea = a \Leftrightarrow eb = b).$$

Clearly, $\tilde{\mathcal{L}}^U$ and $\tilde{\mathcal{R}}^U$ are equivalent relations on S . It is easy to see that $\mathcal{L} \subseteq \mathcal{L}^* \subseteq \tilde{\mathcal{L}}^U$ and $\mathcal{R} \subseteq \mathcal{R}^* \subseteq \tilde{\mathcal{R}}^U$.

A semigroup S is called U -semiabundant if every $\tilde{\mathcal{L}}^U$ -class and every $\tilde{\mathcal{R}}^U$ -class of S contains idempotents in U . Since the structure of U -semiabundant semigroup S depends heavily on the subset U of S , we usually denote this semigroup by $S(U)$. It is clear that an abundant semigroup S is U -semiabundant when $U = E(S)$. In recent years, U -semiabundant semigroups and their subclasses have been studied by many authors (for instance, see [3], [4], [9]-[13]).

Recall in [12] that an U -semiabundant semigroup $S(U)$ is called U -abundant if $S(U)$ satisfies the congruence condition, that is, $\tilde{\mathcal{L}}^U$ is a right congruence and $\tilde{\mathcal{R}}^U$ is a left congruence. According to G. M. S. Gomes and V. Gould [4], an U -abundant semigroup $S(U)$ is named Ehresmann if U forms a semilattice. Clearly, an U -abundant semigroup is a generalization of an abundant semigroup while an Ehresmann semigroup is a generalization of an adequate semigroup (or a type A semigroup). We here consider a common generalization of an orthodox semigroup and a type W semigroup within the class of U -abundant semigroups, namely, the U -cyber semigroup. We will also introduce an admissible relation “ σ ” on an U -semiabundant semigroup $S(U)$ and prove that such a relation σ is a comprehensive congruence on an U -cyber semigroup. In studying the structure of type W semigroups within the class of abundant semigroups, A. El-Qallali and J. B. Fountain [2] introduced an equivalent relation “ δ ” on an IC quasi-adequate semigroup S and asked whether or not δ is a congruence, that is, whether or not every IC quasi-adequate semigroup S is a type W semigroup? In fact if the relation “ δ ” is a congruence, then the structure of the type W semigroup S can be described by using the above congruence “ δ ”. The above question was then answered by X. J. Guo in 2001 by proving that the above relation “ δ ” is always a congruence on an IC quasi-adequate semigroup S (see [5]). Our result in this paper is to further extend the result of X. J. Guo (see Theorem 2.6 in [5]) to U -cyber semigroups.

For notations and terminologies not given in this paper, the reader is referred to the monographs of J. M. Howie ([6] and [7]).

2 Preliminaries

Throughout this paper, we assume that $S(U)$ is always an U -semiabundant semigroup. The $\tilde{\mathcal{L}}^U$ -class (resp. $\tilde{\mathcal{R}}^U$ -class) containing the element a is denoted by \tilde{L}_a^U (resp. \tilde{R}_a^U). Moreover, we denote a typical idempotent of $\tilde{L}_a^U \cap U$ by a^* and a typical idempotent of $\tilde{R}_a^U \cap U$ by a^+ .

The following lemmas give some basic properties of $\tilde{\mathcal{L}}^U$ and $\tilde{\mathcal{R}}^U$ on $S(U)$.

Lemma 2.1 [10] *Let $a \in S(U)$ and $e \in U$. Then the following statements hold:*

- (i) $a\tilde{\mathcal{L}}^U e$ if and only if $ae = a$ and for all $f \in U$, $af = a$ implies $ef = e$;
- (ii) $a\tilde{\mathcal{R}}^U e$ if and only if $ea = a$ and for all $f \in U$, $fa = a$ implies $fe = e$.

It can be easily seen that $aa^* = a$ and $a^+a = a$, for some $a^* \in \tilde{L}_a^U$ and $a^+ \in \tilde{R}_a^U$.

Lemma 2.2 [10] *For any $e, f \in U$, the following statements hold:*

- (i) $e\tilde{\mathcal{L}}^U f$ if and only if $e\mathcal{L}f$;
- (ii) $e\tilde{\mathcal{R}}^U f$ if and only if $e\mathcal{R}f$.

Next we cite the definition of good congruences on an U -semiabundant semigroup $S(U)$.

Definition 2.3 [13] *We call a semigroup homomorphism $\varphi : S(U) \rightarrow T$ good if $a\tilde{\mathcal{L}}^U b$ implies $(a\varphi)\tilde{\mathcal{L}}^{\bar{U}}(T)(b\varphi)$ and $a\tilde{\mathcal{R}}^U b$ implies $(a\varphi)\tilde{\mathcal{R}}^{\bar{U}}(T)(b\varphi)$ for all $a, b \in S(U)$, where $\bar{U} = U\varphi$.*

A congruence ρ on $S(U)$ is said to be good if the natural homomorphism $\rho^\natural : S(U) \rightarrow S(U)/\rho$ is good.

We have the following lemma for good homomorphisms on $S(U)$.

Lemma 2.4 [13] *Let $\phi : S(U) \rightarrow T$ be a semigroup homomorphism. Then ϕ is good if and only if for any $a \in S(U)$ there exist idempotents $e \in \tilde{L}_a^U \cap U$ and $f \in \tilde{R}_a^U \cap U$ such that $(a\phi)\tilde{\mathcal{L}}^{\bar{U}}(T)(e\phi)$ and $(a\phi)\tilde{\mathcal{R}}^{\bar{U}}(T)(f\phi)$, where $\bar{U} = U\phi$.*

By Lemma 2.1 and Lemma 2.4, there is the following description for good congruences on an U -semiabundant semigroup $S(U)$.

Lemma 2.5 [13] *If ρ is a congruence on $S(U)$, then the following statements are equivalent:*

- (i) ρ is good;
- (ii) For any $a \in S(U)$, there exist idempotents $e \in \tilde{L}_a^U \cap U$ and $f \in \tilde{R}_a^U \cap U$ such that for any $g \in U$,
 - (a) $(ag, a) \in \rho$ implies $(eg, e) \in \rho$,
 - (b) $(ga, a) \in \rho$ implies $(gf, f) \in \rho$.

We now have a description of the homomorphic image $S(U)/\rho$ of $S(U)$ induced by a good congruence ρ on $S(U)$.

Lemma 2.6 [13] *If ρ is a good congruence on $S(U)$, then $S(U)/\rho$ is an \bar{U} -semiabundant semigroup such that $\bar{U} = U/\rho$.*

3 Comprehensive congruences

Let E be the set of idempotents of an U -semiabundant semigroup $S(U)$ and $U \subseteq E$. Let $B(U)$ be the semiband generated by U . Then, following [1], $\langle e \rangle$ denotes the subsemigroup of $eB(U)e$ generated by the idempotents of $eB(U)e$ and $\omega(e) = \{f \in U : f \leq e\}$, for any idempotent $e \in U$.

Now we give the definition of idempotent-connected U -semiabundant semigroups similar to idempotent-connected abundant semigroups in [1].

Definition 3.1 *An U -semiabundant semigroup $S(U)$ is called idempotent-connected (briefly, denoted by IC) if for each element $a \in S$ and for some $a^* \in \tilde{L}_a^U \cap U$, $a^+ \in \tilde{R}_a^U \cap U$, there is a bijection $\alpha : \langle a^+ \rangle \rightarrow \langle a^* \rangle$ satisfying $xa = a(x\alpha)$ for all $x \in \langle a^+ \rangle$.*

The following lemma gives another characterization of $\langle e \rangle$ in an U -semiabundant semigroup $S(U)$.

Lemma 3.2 *If $S(U)$ is an U -semiabundant semigroup in which $U \subseteq E$ forms a subsemigroup of $S(U)$, then $\langle e \rangle = \omega(e)$ for any idempotent $e \in U$.*

Proof: Suppose that $S(U)$ is an U -semiabundant semigroup in which the subset U of E forms a subsemigroup of $S(U)$. Then $B(U) = U$ and $eB(U)e = eUe \subseteq U$, for any idempotent $e \in U$. It can be easily seen that eUe forms a subsemigroup of U and so $\langle e \rangle = eUe$. We now prove that $eUe = \omega(e)$. To show that $eUe \subseteq \omega(e)$, we let $f \in eUe \subseteq U$. Then there exists an idempotent $g \in U$ such that $f = ege$. It is clear that $fe = f = ef$. Thus $f \leq e$, that is, $f \in \omega(e)$. Hence $eUe \subseteq \omega(e)$. Conversely, we let $g \in \omega(e)$. Then $g \leq e$ and so $g = eg = ge$. This leads to $g = eg = ege$ and whence, $g \in eUe$. Hence $\omega(e) \subseteq eUe$. Consequently, $eUe = \omega(e)$, and hence $\langle e \rangle = \omega(e)$. \square

By Definition 3.1 and Lemma 3.2, we can easily obtain the following description for idempotent-connected U -semiabundant semigroups.

Lemma 3.3 *If $S(U)$ is an U -semiabundant semigroup in which $U \subseteq E$ forms a subsemigroup of $S(U)$, then the following statements are equivalent:*

- (a) $S(U)$ is IC, that is, $S(U)$ is idempotent-connected;
- (b) For every element a of $S(U)$, the following two conditions hold:
 - (i) For some a^* and for all $e \in \omega(a^*)$, there exists an idempotent $f \in \omega(a^+)$ such that $ae = fa$;
 - (ii) For some a^+ and for all $h \in \omega(a^+)$, there exists an idempotent $g \in \omega(a^*)$ such that $ha = ag$.

For an U -semiabundant semigroup $S(U)$, if U forms a subsemigroup of $S(U)$, then U is clearly a band, and hence U is a semilattice Y of rectangular bands U_α such that $U = \bigcup_{\alpha \in Y} U_\alpha$, where the core U_α is a \mathcal{J} -class of U (see, [6]). Denote the \mathcal{J} -class U_α containing the idempotent e of U by $U(e)$, and hence $U(e)$ is a rectangular band. Now, we define a partial order on the set of \mathcal{J} -classes of U by $U(e) \leq U(f)$ if and only if $U(e)U(f) \subseteq U(e)$ for any $e, f \in U$.

By using the above fact, we formulate the following definition.

Definition 3.4 *Let $S(U)$ be an U -semiabundant semigroup $S(U)$ in which $U \subseteq E$ forms a subsemigroup of $S(U)$. Define a relation σ on $S(U)$ by*

$$a\sigma b \text{ if and only if } U(a^+)aU(a^*) = U(b^+)bU(b^*), \text{ for some } a^+, a^* \text{ and } b^+, b^*.$$

Then, it is obvious that σ is an equivalent relation on $S(U)$. We now call this relation σ an admissible relation on $S(U)$.

Lemma 3.5 *Let $S(U)$ be an U -semiabundant semigroup in which $U \subseteq E$ forms a subsemigroup of $S(U)$. If $a, b \in S(U)$ and $b = eaf$, where $e \in U(a^+)$, $f \in U(a^*)$, then the following statements hold:*

- (i) $U(a^*) = U(b^*)$, $U(a^+) = U(b^+)$;
- (ii) $U(a^+)aU(a^*) = U(b^+)bU(b^*)$.

Proof: (i) Suppose that a, b are elements of $S(U)$ such that $b = eaf$ for some $e \in U(a^+)$ and $f \in U(a^*)$. Clearly, $bf = b$. Since $b\tilde{\mathcal{L}}^U b^*$ and $f \in U(a^*) \subseteq U$, it follows that $b^*f = b^*$ which implies that $U(b^*)U(f) \subseteq U(b^*)$. Consequently, $U(b^*) \leq U(f) = U(a^*)$. Similarly, we can prove that $U(b^+) \leq U(e) = U(a^+)$.

Thus $U(b^*)U(a^*) \subseteq U(b^*)$ and $U(a^+)U(b^+) \subseteq U(b^+)$. This implies that $b^*a^* \in U(b^*)$ and $a^+b^+ \in U(b^+)$. Also we can deduce that

$$a = a^+aa^* = (a^+ea^+)a(a^*fa^*) = a^+eaf a^* = a^+ba^* = (a^+b^+)b(b^*a^*).$$

Thus, by the assertion above, $U(a^*) \leq U(b^*)$ and $U(a^+) \leq U(b^+)$. Hence $U(a^*) = U(b^*)$ and $U(a^+) = U(b^+)$.

(ii) To show that $U(b^+)bU(b^*) \subseteq U(a^+)aU(a^*)$, we let $c \in U(b^+)bU(b^*)$. Then there exist idempotents $g \in U(b^+) = U(a^+)$ and $h \in U(b^*) = U(a^*)$ such that $c = gbh = geafh = (ge)a(fh) \in U(a^+)aU(a^*)$. Thus $U(b^+)bU(b^*) \subseteq U(a^+)aU(a^*)$. On the other hand, we let $d \in U(a^+)aU(a^*)$. Then there exist idempotents $u \in U(a^+) = U(b^+)$ and $v \in U(a^*) = U(b^*)$ such that $d = uav$. We may choose idempotents $p \in U(a^+), q \in U(a^*)$ with $a^+\mathcal{R}p\mathcal{L}e$ and $a^*\mathcal{L}q\mathcal{R}f$ such that

$$a = a^+aa^* = pa^+aa^*q = pea^+aa^*fq = peafq = pbq.$$

Then $d = uav = upbqv = (up)b(qv) \in U(b^+)bU(b^*)$. Thus, $U(a^+)aU(a^*) \subseteq U(b^+)bU(b^*)$. Hence, $U(a^+)aU(a^*) = U(b^+)bU(b^*)$. \square

By the above lemma, we can immediately see that the admissible relation σ on $S(U)$ has the following properties.

Corollary 3.6 *Let $S(U)$ be an U -semiabundant semigroup in which $U \subseteq E$ forms a subsemigroup. If $a, b \in S(U)$, then*

- (i) $a\sigma = U(a^+)aU(a^*)$;
- (ii) $a\sigma b$ if and only if $b = eaf$ for some $e \in U(a^+)$ and $f \in U(a^*)$.

We now give the definition of U -cyber semigroups which was studied in [12]

Definition 3.7 *An U -abundant semigroup $S(U)$ is called U -cyber if $S(U)$ is IC and the subset U of E forms a subsemigroup of $S(U)$.*

Clearly, an U -cyber semigroup is a common generalization of an orthodox semigroup and a type W semigroup.

Definition 3.8 *A congruence ρ on an U -abundant semigroup $S(U)$ is called an Ehresmann congruence if $S(U)/\rho$ is an Ehresmann semigroup. The minimum Ehresmann good congruence on $S(U)$ is called a comprehensive congruence.*

We will prove the following main result for the admissible relation σ on an U -cyber semigroup $S(U)$.

Theorem 3.9 *If $S(U)$ is an U -cyber semigroup, then the admissible relation σ is a comprehensive congruence on $S(U)$.*

In proving Theorem 3.9, we first prove the following lemmas.

Lemma 3.10 *The admissible relation σ on an U -cyber semigroup $S(U)$ is a congruence.*

Proof: We only need to prove that σ is left compatible with the semigroup multiplication because we can easily show that σ is right compatible in the dual way. Now, let $a, b \in S(U)$ such that $a\sigma b$. Then there exist idempotents $e \in U(a^+)$ and $f \in U(a^*)$ such that $b = eaf$. Hence for any element $c \in S(U)$, by Lemma 3.3, we can deduce that

$$\begin{aligned} cb &= ceaf = cc^*ea^+af \\ &= c(c^*ea^+)af \\ &= c(c^*ea^+)(c^*a^+)(c^*ea^+)af \quad (c^*ea^+ \in U(c^*a^+)) \\ &= c(c^*ea^+c^*)(a^+c^*ea^+)af \quad (c^*ea^+c^* \in \omega(c^*), a^+c^*ea^+ \in \omega(a^+)) \\ &= gcahf \quad (\text{where } g \in \omega(c^+) \text{ and } h \in \omega(a^*)) \\ &= g(ca)^+(ca)(ca)^*hf. \end{aligned}$$

Clearly, $cb = g(ca)^+cb$. From $cb \tilde{\mathcal{R}}^U (cb)^+$, we have $g(ca)^+(cb)^+ = (cb)^+$. This leads to $U((cb)^+) \leq U(g(ca)^+) \leq U((ca)^+)$. By using the symmetry of σ , we have $b\sigma a$. Applying a similar argument to $b\sigma a$, we obtain $U((ca)^+) \leq U((cb)^+)$ and hence $U((cb)^+) = U((ca)^+)$. This implies that $U(g(ca)^+) = U((ca)^+)$. By using dual arguments, we also obtain that $U((ca)^*hf) = U((ca)^*)$. Hence, $(ca)\sigma(cb)$ and so σ is clearly left compatible with semigroup multiplication. Thus, σ is a congruence on $S(U)$. \square

Lemma 3.11 *Let $S(U)$ be an U -cyber semigroup. Then the admissible relation σ is a good congruence on $S(U)$.*

Proof: From Lemma 3.10, the admissible relation σ is a congruence on $S(U)$. To show that σ is a good congruence on $S(U)$, we let $a \in S(U)$, $e \in \tilde{\mathcal{L}}_a^U \cap U$ and $f \in \tilde{\mathcal{R}}_a^U \cap U$. Then, by Lemma 2.5, we only need to show that for any $g \in U$, $(ag, a) \in \sigma$ implies $(eg, e) \in \sigma$ and $(ga, a) \in \sigma$ implies $(gf, f) \in \sigma$. If $(ag, a) \in \sigma$, then there exist idempotents $p \in U((ag)^+)$ and $q \in U((ag)^*)$ such that $a = pagq$. We may choose an idempotent $u \in U$ such that $u\mathcal{L}p$. Observe that $e\tilde{\mathcal{L}}^U a$ and $a = (pa)\tilde{\mathcal{L}}^U (ua)$, since $p\tilde{\mathcal{L}}^U u$ and $\tilde{\mathcal{L}}^U$ is a right congruence on $S(U)$. It hence follows that $e\tilde{\mathcal{L}}^U (pa)$. From $pa = pagq$, we can easily deduce that

$$e = egq = (eg)^+(eg)(eg)^*q \tag{3.1}$$

Since $e\tilde{\mathcal{L}}^U a$ and $\tilde{\mathcal{L}}^U$ is a right congruence on $S(U)$, $(eg)\tilde{\mathcal{L}}^U(ag)$. Thus, $U((ag)^*) = U((eg)^*)$ and so $(eg)^*q \in U((eg)^*)$. It now follows from equation (3.1) that $(eg, e) \in \sigma$. This shows that $(ag, a) \in \sigma$ implies $(eg, e) \in \sigma$. By using similar arguments, we can show that for any $g \in U$, $(ga, a) \in \sigma$ implies $(gf, f) \in \sigma$ and hence σ is a good congruence on $S(U)$. The proof is completed. \square

Lemma 3.12 *Let $S(U)$ be an U -cyber semigroup. Then the admissible relation σ on $S(U)$ is an Ehresmann congruence.*

Proof: We will verify that $S(U)/\sigma$ is an Ehresman semigroup. By Lemma 2.6, $S(U)/\sigma$ is an U/σ -semiabundant semigroup. Thus, it suffices to show that U/σ is a semilattice and $S(U)/\sigma$ satisfies the congruence condition. It is easy to verify that $\sigma \cap (U \times U) = \mathcal{J}_U$, where \mathcal{J}_U is the usual Green relation on U . Thus, $U/\sigma = U/\mathcal{J}_U$ is a semilattice. In order to show that $S(U)/\sigma$ satisfies the congruence condition, we let $a\sigma, b\sigma$ and $c\sigma \in S(U)/\sigma$ such that $a\sigma\tilde{\mathcal{L}}^{\bar{U}}b\sigma$, where $\bar{U} = U/\sigma$. Since σ is a good congruence on $S(U)$, by Lemma 2.4, there exist idempotents $e \in \tilde{L}_a^U \cap U, f \in \tilde{L}_b^U \cap U$ and $g \in \tilde{L}_c^U \cap U$ such that $a\sigma\tilde{\mathcal{L}}^{\bar{U}}e\sigma, b\sigma\tilde{\mathcal{L}}^{\bar{U}}f\sigma$ and $c\sigma\tilde{\mathcal{L}}^{\bar{U}}g\sigma$. Hence, $e\sigma\tilde{\mathcal{L}}^{\bar{U}}f\sigma$ and so $e\sigma f\sigma = e\sigma$ and $f\sigma e\sigma = f\sigma$. Again since U/σ is a semilattice, $e\sigma = e\sigma f\sigma = f\sigma e\sigma = f\sigma$. Hence, $e\sigma f$ and so $(ec)\sigma(fc)$. Observe that $a\tilde{\mathcal{L}}^U e$ and $b\tilde{\mathcal{L}}^U f$, we hence derive that $ac\tilde{\mathcal{L}}^U ec$ and $bc\tilde{\mathcal{L}}^U fc$, since $\tilde{\mathcal{L}}^U$ is a right congruence on $S(U)$. Thus $(ac)\sigma\tilde{\mathcal{L}}^{\bar{U}}(ec)\sigma = (fc)\sigma\tilde{\mathcal{L}}^{\bar{U}}(bc)\sigma$, since σ is a good congruence on $S(U)$. This shows that $\tilde{\mathcal{L}}^{\bar{U}}$ is a right congruence on $S(U)/\sigma$. Dually, we can show that $\tilde{\mathcal{R}}^{\bar{U}}$ is a left congruence on $S(U)/\sigma$ and therefore $S(U)/\sigma$ satisfies the congruence condition. The proof is now completed. \square

We now return to prove Theorem 3.9. In view of Lemma 3.11 and Lemma 3.12, we have already known that σ is an Ehresmann good congruence on $S(U)$. We only need to show that σ is minimum. For this purpose, we let ρ be an arbitrary Ehresmann good congruence on $S(U)$ so that $S(U)/\rho$ is an Ehresmann semigroup with the subsemilattice U/ρ . It is clear that $\rho \cap (U \times U)$ is a semilattice congruence on U . Thus, $\mathcal{J}_U \subseteq \rho|_U$ since \mathcal{J}_U is the smallest semilattice congruence on U (see, [6]). Let $a, b \in S(U)$ and $a\sigma b$. Then there exist idempotents $e \in U(a^+)$ and $f \in U(a^*)$ such that $b = eaf$. Notice that $e\mathcal{J}_U a^+$ and $f\mathcal{J}_U a^*$. Clearly, $e\rho a^+$ and $f\rho a^*$. This leads to $(eaf)\rho(a^+aa^*)$ which implies that $a\rho b$. Thus $\sigma \subseteq \rho$, and hence σ is indeed a comprehensive congruence on $S(U)$.

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