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A Logical Basis for the Standard Model and Quark Mass Ratios

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Abstract

We outline how an idempotent equation (2) relates logic to nuclear structure and a tetrahedral symmetry that leads naturally to the embeddings $E_6 \subset E_7 \subset E_8$ of 3 exceptional lie algebras which account for the 3 quark generations u,d; s,c; t,b. Then symmetry breaking of the algebras accounts beautifully for the ratios of quark masses to within experimental limits.Thus there is an intimate relationship between quark masses and the orders of the Weyl group of the permutations and reflections of the subalgebras of the Lie algebras E_7, E_8. In other words organizational or entropy production leads to energies governed by the Second Law of Thermodynamics $\Delta E \leq T\Delta S$ without any appeal to a Higgs field. In fact we may even be looking at the same particles in excited states! In the last section a decomposition of E_8 is related to the W,Z bosons.

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1 Introduction

Equation (2) below is a minimal left ideal in the center of the Dirac ring and is Lorentz invariant [3].It has been used to model the nucleon because E_{23}, E_{05} are rotation operators of spin and isospin while E_{14} is a parity operator in accordance with triality. $iE_4\psi_1$ is the identity operator corresponding to rotations(mod 2π). In Section 2 the irreducible representation $\Psi/4$ is also shown to be idempotent and therefore provides a logical basis for a realistic description of the natural World according to Boole[2].

Moreover Barth and Nieto[1] have related E_{23}, E_{05}, E_{14} to 3 pairs of 'fix-lines' lying in the $\pm i$ eigenspaces defined by $E_{\mu\nu}^2 = -1$ The 6 fix-lines are the edges of a fundamental tetrahedron that may be inscribed in a cube so (2) leads naturally to tetrahedral and cubic symmetry governed by the binary tetrahedral and octahedral quaternion groups T_d, O and their Lie algebras E_6, E_7 which are isomorphic by the Mackay correspondence[12].

In this way the tetrahedral symmetry imposes gauge invariance on the fundamental relation (2) and the theory of elementary particles is reduced to the study of the exceptional Lie algebras $E_6 \subset E_7 \subset E_8$. The 27 lines or weights of E_6 have been beautifully related to the 3 quarks u,d,s and the leptons e, μ, ν by Slansky [16].Details of how the charges carried by the fix-lines determine those of the up and down quarks have been described by de Wet[7] in a recent paper that also utilizes E_7, E_8 to model the remaining quarks c,t,b.

Moreover the properties of the up and down quarks are not changed by the huge energies needed to produce the quark flavors or generations, strange and charm (s,c),and top and bottom (t,b). This suggests that we are looking at stable up and down quarks lifted by high energy collisions into the symmetries E_7, E_8 which have greater dimensions dictated by the Weyl groups W(E) that preserve the permutations and reflections of the 72 roots of E_6 ,the 126 roots of E_7 and the 240 roots of E_8 (cf.[9]) .Manivel [14] has calculated the orders $|W(E)|$ of the maximal subalgebras of the Weyl groups arising from symmetry breaking of E_8 to E_7, E_6 of E_7 to E_6 and E_6 to $su_3 \times su_3 \times su_3$ which is the basis of Slansky's model [16].

In Section 3 a remarkable agreement is found between the ratios of the latest estimates of quark masses [13] and the ratios of the orders $|W(E)|$)of subalgebras if an average of $(m_u+m_d)/2$=6.5 Mev is adopted. Thus masses are intimately related to the number of permutations and reflections that preserve the subalgebras. In other words to their organization or entropy S which is related to energy changes by the Second Law of Thermodynamics namely $\Delta E \leq T\Delta S$¿. If this is the case then it is not necessary to appeal to a Higgs field to explain mass differences between quarks.

2 A Primitive Idempotent

According to George Boole[2] the symbols of logic are subject to the special law

$$x^2 = x \tag{1}$$

which is a law of thought and a fundamental axiom of all philosophy(cf.ChIII,prop, IV). This is because (1) written x(1-x)=0 expresses the fact that it is impossible for anything to possess a quality and at the same time not to possess that quality.Therefore Aristotle believed (1) to be the source of all the axioms of logic.

So this Section will be devoted to proving that the structure of the nucleus itself is described by a primitive idempotent.The fundamental irreducible representation describing spin about x_3 is

$$\frac{1}{4}\Psi = (iE_4\psi_1 + E_{23}\psi_2 + E_{14}\psi_3 + E_{05}\psi_4)e \tag{2}$$

which is a minimal left ideal of the center D of the Dirac ring[4].Here

$$\begin{aligned} e_1 &= -\tfrac{i}{4}(E_{03} + E_{12} + E_{45} + E_{16}) \\ e_2 &= -\tfrac{i}{4}(-E_{03} - E_{12} + E_{45} + E_{16}) \\ e_3 &= -\tfrac{i}{4}(-E_{03} + E_{12} - E_{45} + E_{16}) \\ e_4 &= -\tfrac{i}{4}(E_{05} - E_{12} - E_{45} + E_{16}) \end{aligned} \tag{3}$$

are primitive idempotents satisfying $e_ie_j = 0$ and Eddingtons E-numbers are mapped into the 4×4 Dirac matrices by

$$\begin{gathered} \gamma_\nu = iE_{0\nu}, E_{\mu\nu} = E_{\rho\mu}E_{\rho\nu} = -E_{\nu\mu}, \\ E_{\mu\nu}^2 = 1, E_{\mu\nu}E_{\sigma\tau} = E_{\sigma\tau}E_{\mu\nu} = iE_{\lambda\rho}, \mu < \nu = 1, \ldots, 5 \end{gathered} \tag{4}$$

E_4 is the unit matrix while $\psi_2, \psi_3 and \psi_4$ are half-angles of rotation. Two more equivalent representations may be obtained by a cyclic interchange of the indices 1,2,3 that express rotations about x_2, x_3

To prove that (2) is also idempotent we set $e = e_1$ and note that $e_1E_{23} = E_{23}e_2$ so that $e_1E_{23}e_1 = E_{23}e_2e_1 = 0$.Similarly $e_1E_{14}e_1 = E_{14}e_4e_1 = 0 = e_1E_{05}e_1 = E_{05}e_3e_1$. Therefore from (2)

$$(\Psi/4)^2 = (\Psi/4)iE_4\psi_1 \tag{5}$$

which is idempotent if the identity $iE_4\psi_1 = E_{16}\psi_1 = 1$. This view is supported by the calculation of state labels $[\lambda]$ for the many nucleon case derived from tensor products of (2) with itself [5].

3 Quark Mass Ratios

We turn now to the structure of the exceptional Lie algebras $E_6 \subset E_7 \subset E_8$ because the binary tetrahedral group T_d is characterized by the famous 27 lines that correspond to the 27 fundamental weights of E_6 and Slansky [16] has related these weights to elements of half the Standard Model in Table 21. De Wet[7] has associated the remainder with E_7 isomorphic to the binary octahedral group O .

The number of elements of the Weyl group $W(E_6)$,which includes all of the permutations of the 27 lines that preserve their intersection behavior is $|W(E_6)|$

=51840 [9].However the Standard Model makes use only of the subalgebra $su_3 \times su_3 \times su_3$ of order 216 (cf.[14]).But following Manivel [14] and Dynkin [8]E_6 is a subalgebra of E_7 which contains the charm quark c. Then the ratio 51840/216 =240 is close to the ratio m_c/m_d of the masses of c and the average $m_d \approx 6.5$ Mev of the up and down quarks[13]. So quark masses are strongly dependent on the order of the algebras where they live.

In the case of $E_8, |W(E_8)|= 696729600$ and there are two relevant subalgebras, $sl_2 \times e_7, sl_3 \times e_6$ of orders 5806080 and 6×51840 respectively. The ratio 5806080/216 =26880 is very close to the current experimental value m_t/m_d of the top and down quarks which is additional proof of quark masses generated by symmetry breaking.

The final section will discuss the relevance of $sl_3 \times e_6$ to the search for the Higgs boson soon to be conducted at CERN.

4 W,Z Bosons

Manivel gives the branching

$$e_8 = sl_3 \times e_6 \oplus (B \otimes J) \oplus (B \otimes J)* . \tag{6}$$

Here J is a 27-dimensional Jordan algebra represented by the Weyl group W(J)of order 25920(cf.[10],section 5.1). B is a natural representation of sl_3 so $B \otimes J$ would have half the dimension of $sl_3 \times e_6$. In fact the ratio 6×51840/216=1440 is close to double the experimental value m_b/m_d=720 which indicates that the pair of bottom quarks dictated by(6) could be produced by the decay of W,Z particles associated with the Standard Model Higgs boson .

Here it is worth mentioning that long ago de Wet [4] found boson representations of the Clifford algebra Cl(6)known to be Z_2 graded and isomorphic to E_8.

Finally Dynkin[8] lists in Tables 10,11 all the subalgebras of highest rank of the semisimple Lie algebras from which it is possible to calculate the orders utilizing the table of Humphreys [9]. However a careful scrutiny of the subalgebras of E_8 does not yield an order that could match the Higgs.

5 Conclusion

Entropy is another way of seeing the connection between the orders of the subalgebras of E_6, E_7, E_8 and quark masses, because the entropy of a system is a measure of its organization and therefore is a function of the number of possible states or orders of the relevant subalgebras which are a consequence of symmetry breaking to smaller states until ultimately the stable states m_u, m_d are reached.

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