International Mathematical Forum, 3, 2008, no. 2, 85 - 88

Modifying of Optimal Paths and Costs of Adjustment in Dynamic DEA

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Abstract

In this paper by considering the cost of non-discretionary inputs and influence of modifying of outputs, we modify the proposed method by Filadelfo and et al. [1] for determining optimal paths and costs of adjustment in dynamic Data Envelopment Analysis (DEA).

Keywords: Adjustment, Optimal path, Cost efficiency, DEA

1 Introduction

Model of dynamic Data Envelopment Analysis (DEA) suggested by Filadelfo and et. al [1] for determining optimal paths and costs of adjustment is as follows:

$$
\begin{array}{ll}
\text{Max} & \pi_k = \sum_{t=1}^{T} \Big[s_t(p_{kt}y_{kt} - w_{kt}x_{kt}) - s_{t-1}(w^+_{kt}x^+_{kt} + \bar{w}_{kt}\bar{x}_{kt}) \Big] \\
\text{s.t.} & w^+_{kt}x^+_{kt} + \bar{w}_{kt}\bar{x}_{kt} \leq b_{kt}, t = 1, \ldots, t_a \\
& x_{kt} = x_{k,t-1} + x^+_{kt} - x^-_{kt}, t = 1, \ldots, t_a \\
& q_{kt} = q_{k,t-1} + c_{kt} - \delta q_{k,t-1}, t = 1, \ldots, t_a \\
& Y\lambda_{kt} \geq y_{kt}, t = 1, \ldots, T \\
& X\lambda_{kt} \leq x_{kt}, t = 1, \ldots, T \\
& Q\lambda_{kt} \leq q_{kt}, t = 1, \ldots, T \\
& Z\lambda_{kt} = Z_k, t = 1, \ldots, T \\
& q_{kt} \leq \bar{q}_{kt}, t = 1, \ldots, t_a \\
& c_{kt} \leq \bar{c}_{kt}, t = 1, \ldots, t_a \\
& t_a \leq t^*_a, \\
& s_t = (1 + r/100)^{-t}, t = 1, \ldots, T.
\end{array} \tag{1}
$$

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The organization of this paper is as follows. Section 2 presents the optimal paths with considering all costs and discusses the optimal paths with adjustment of outputs. Section 3 concludes.

2 Modifying of optimal paths and costs of adjustment

For modeling it is assumed that all variables have constant values within each period, although they may change values from one period to the next. Suppose for DMU$_k$, X, Y, Z and Q are matrixes of inputs, outputs, non-discretionary inputs and constrain discretionary inputs, respectively. Let $x^+_{kt}, x^-_{kt}, w^+_{kt}$ and w^-_{kt} denote increases, decreases, the cost of increasing and decreasing inputs, respectively and y_{kt}, x_{kt}, q_{kt} and c_{kt} are vectors of outputs, inputs, constrained discretionary inputs and capital investment, respectively in the period t. s_t is the present value factor and δ is the rate of depreciation of assets. λ_{kt} and input quantity adjustment vectors, x^a_{kt}, are decision variables. The problem for DMU$_k$ is to choose the time sequence of input vectors, x^a_{kt}, $(t = 1, \ldots, t_a)$, which maximizes the adjustments of inputs transforms the initial input quantity vector, x_{ko}, into the target input quantity vector, x^*_k. The adjustment are preformed within the specified adjustment period, t^*_a.

2.1 Optimal paths with considering all costs

Model (1) consists non-discretionary inputs. Since these inputs have costs. So, they have influence in estimating optimal paths and cost efficiency. Therefore, with bringing them in objective function, we suggest the following model:

$$
\begin{aligned}
\text{Max} \quad & \pi_k = \sum_{t=1}^{T} \Big[s_t(p_{kt}y_{kt} - v_k z_k - w_{kt}x_{kt}) - s_{t-1}(w^+_{kt}x^+_{kt} + w^-_{kt}x^-_{kt}) \Big] \\
\text{s.t.} \quad & w^+_{kt}x^+_{kt} + w^-_{kt}x^-_{kt} \leq b_{kt}, t = 1, \ldots, t_a \\
& x_{kt} = x_{k,t-1} + x^+_{kt} - x^-_{kt}, t = 1, \ldots, t_a \\
& q_{kt} = q_{k,t-1} + c_{kt} - \delta q_{k,t-1}, t = 1, \ldots, t_a \\
& Y\lambda_{kt} \geq y_{kt}, t = 1, \ldots, T \\
& X\lambda_{kt} \leq x_{kt}, t = 1, \ldots, T \\
& Q\lambda_{kt} \leq q_{kt}, t = 1, \ldots, T \\
& Z\lambda_{kt} = Z_k, t = 1, \ldots, T \\
& q_{kt} \leq q^-_{kt}, t = 1, \ldots, t_a \\
& c_{kt} \leq c^-_{kt}, t = 1, \ldots, t_a \\
& t_a \leq t^*_a, \\
& s_t = (1 + r/100)^{-t}, t = 1, \ldots, T
\end{aligned}
\tag{2}
$$

where v_k is a vector of constant non-discretionary input prices.

2.2 Adjustment of outputs

To obtain the maximum profit in addition to adjustment of inputs we can adjustment the outputs. We denote the increasing and decreasing of outputs with y_{kt}^{+} and y_{kt}^{-}, respectively. Also the cost of increasing of outputs may be different with the cost of its decreasing, hence we show the costs of increasing and decreasing with p_{kt}^{+} and p_{kt}^{-}. Consequently, we present the following model:

$$\begin{array}{ll} \text{Max} & \pi_k = \sum_{t=1}^{T} \Big[s_t(p_{kt}y_{kt} - w_{kt}x_{kt}) + s_{t-1}(p_{kt}^{+}y_{kt}^{+} - p_{kt}^{-}y_{kt}^{-}) \\ & \qquad -s_{t-1}(w_{kt}^{+}x_{kt}^{+} + w_{kt}^{-}x_{kt}^{-}) \Big] \\ \text{s.t.} & w_t^{+}x_{kt}^{+} + w_t^{-}x_{kt}^{-} \leq b_{kt}, t = 1, \ldots, t_a \\ & x_{kt} = x_{k,t-1} + x_{kt}^{+} - x_{kt}^{-}, t = 1, \ldots, t_a \\ & q_{kt} = q_{k,t-1} + c_{kt} - \delta q_{k,t-1}, t = 1, \ldots, t_a \\ & Y\lambda_{kt} \geq y_{kt}, t = 1, \ldots, T \\ & X\lambda_{kt} \leq x_{kt}, t = 1, \ldots, T \\ & Q\lambda_{kt} \leq q_{kt}, t = 1, \ldots, T \\ & Z\lambda_{kt} = Z_k, t = 1, \ldots, T \\ & q_{kt} \leq q_{kt}^{-}, t = 1, \ldots, t_a \\ & c_{kt} \leq c_{kt}^{-}, t = 1, \ldots, t_a \\ & t_a \leq t_a^{*}, \\ & s_t = (1 + r/100)^{-t}, t = 1, \ldots, T \\ & y_{kt} = y_{k,t-1} + y_{kt}^{+} - y_{kt}^{-}, t = 1, \ldots, t_a. \end{array} \tag{3}$$

Also, we can state model (2) with adjustment of outputs. In this case, we propose the following model, where the constraints of this model is like constraints of model (2),

$$\begin{array}{ll} \text{Max} \quad \pi_k = & \sum_{t=1}^{T} \Big[s_t(p_{kt}y_{kt} - v_k z_k - w_{kt}x_{kt}) + s_{t-1}(p_{kt}^{+}y_{kt}^{-} \\ & -p_{kt}^{-}y_{kt}^{-}) - s_{t-1}(w_{kt}^{+}x_{kt}^{+} + w_{kt}^{-}x_{kt}^{-}) \Big] \end{array} \tag{4}$$

3 Conclusion

In this paper we modified the proposed method by Filadelfo and et al. [1] for determining optimal paths and costs of adjustment in dynamic DEA by considering the cost of non-discretionary inputs and influence of modifying of outputs.

References

[1] Filadelfo de Mateo, Tim Coelli, Cheis O'Donnell, *Optimal paths and costs of adjustment in dynamic DEA models: with applica-*

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Received: August 7, 2007