

Intuitionistic (T, S) -Normed Fuzzy Ideals of Γ -Rings

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Abstract

We consider the generalization of the notion of fuzzy ideals of Γ -rings. In this paper, using t -norm T and s -norm S , we introduce the notion of intuitionistic (T, S) -normed fuzzy ideals in Γ -rings, and some related properties are investigated.

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1 Introduction

N. Nobusawa ([8]) introduced the notion of a Γ -ring, as more general than a ring. W. E. Barnes ([4]) weakened slightly the conditions in the definition of the Γ -ring in the sense of Nobusawa. After the introduction of the concept of fuzzy sets by Zadeh [12], several researches were conducted on the generalization of the notion of fuzzy sets. The idea of “intuitionistic fuzzy set” was first published by Atanassov [2, 3], as a generalization of the notion of fuzzy set. In this paper, using t -norm T and s -norm S , we introduce the notion of intuitionistic (T, S) -normed fuzzy ideals in Γ -ring, and some related properties are investigated.

2 Preliminaries

In this section we include some elementary aspects that are necessary for this paper.

If $M = \{x, y, z, \dots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \dots\}$ are additive abelian groups, and for all x, y, z in M and all α, β in Γ , the following conditions are satisfied

- $x\alpha y$ is an element of M ,
- $(x+y)\alpha z = x\alpha z + y\alpha z, x(\alpha+\beta)y = x\alpha y + x\beta y, x\alpha(y+z) = x\alpha y + x\alpha z,$
- $(x\alpha y)\beta z = x\alpha(y\beta z),$

then M is called a Γ -ring.

A subset A of the Γ -ring M is a *left* (resp. *right*) *ideal* of M if A is an additive subgroup of M and

$$M\Gamma A = \{x\alpha y \mid x \in M, \alpha \in \Gamma, y \in A\}(A\Gamma M)$$

is contained in A . If A is both a left and a right ideal, then A is a two-sided ideal, or simply an ideal of M .

A fuzzy set in M is a function $\mu : M \rightarrow [0, 1]$. Let μ be a fuzzy set in M . For $\alpha \in [0, 1]$, the set $U(\mu, \alpha) = \{x \in M \mid \mu(x) \geq \alpha\}$ is called *level set* of μ . A fuzzy set μ in a Γ -ring M is called a *left* (resp. *right*) *ideal* of M if it satisfies:

- $\mu(x-y) \geq \min\{\mu(x), \mu(y)\},$
- $\mu(x\alpha y) \geq \mu(y)$ (resp. $\mu(x\alpha y) \geq \mu(x)$), for all $x, y \in M$ and all $\alpha \in \Gamma$.

A fuzzy set μ in a Γ -ring M is called a *fuzzy ideal* of M if μ is a both a fuzzy left and a fuzzy right ideal of M .

Definition 2.1. [1] By a *t-norm* T , we mean a function $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfying the following conditions:

- (T1) $T(x, 1) = x,$
- (T2) $T(x, y) \leq T(x, z)$ if $y \leq z,$
- (T3) $T(x, y) = T(y, x),$
- (T4) $T(x, T(y, z)) = T(T(x, y), z),$

for all $x, y, z \in [0, 1]$.

Proposition 2.2. *Every t-norm T has a useful property:*

$$T(\alpha, \beta) \leq \min(\alpha, \beta)$$

for all $\alpha, \beta \in [0, 1]$.

Definition 2.3. [11] By a *s-norm* S , we mean a function $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfying the following conditions:

- (S1) $S(x, 0) = x,$

- (S2) $S(x, y) \leq S(x, z)$ if $y \leq z$,
 (S3) $S(x, y) = S(y, x)$,
 (S4) $S(x, S(y, z)) = S(S(x, y), z)$,
 for all $x, y, z \in [0, 1]$.

Proposition 2.4. *Every s -norm S has a useful property:*

$$\max(\alpha, \beta) \leq S(\alpha, \beta)$$

for all $\alpha, \beta \in [0, 1]$.

For a t -norm (or s -norm) P on $[0, 1]$, denote by Δ_P the set of element $\alpha \in [0, 1]$ such that $P(\alpha, \alpha) = \alpha$, i.e., $\Delta_P := \{\alpha \in [0, 1] \mid P(\alpha, \alpha) = \alpha\}$.

Definition 2.5. Let P be a t -norm (or s -norm). A fuzzy set μ in X is said to satisfy *idempotent property with respect to P* if $\text{Im}(\mu) \subseteq \Delta_P$.

Let M denote a Γ -ring. An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) : x \in M\}$$

where the functions $\mu_A : M \rightarrow [0, 1]$ and $\gamma_A : M \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in M$ to the set A , respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in M$.

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in M\}$.

3 Intuitionistic (T, S) -normed fuzzy ideals

In what follows, let M denote a Γ -ring-algebra unless otherwise specified. All proofs are going to proceed the only left cases, because the right cases are obtained from similar method. In what follows, the term “fuzzy ideal” means “fuzzy left ideal”.

Definition 3.1. Let T be a t -norm and S be a s -norm on $[0, 1]$. An IFS $A = (\mu_A, \gamma_A)$ in M is called an *intuitionistic (T, S) -normed fuzzy left (resp. right) ideal of Γ -ring M* if

(F1) $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$ and $\mu_A(x\alpha y) \geq \mu_A(y)$ (resp. $\mu_A(x\alpha y) \geq \mu_A(x)$),

(F2) $\gamma_A(x-y) \leq S(\gamma_A(x), \gamma_A(y))$ and $\gamma_A(x\alpha y) \leq \gamma_A(y)$ (resp. $\gamma_A(x\alpha y) \leq \gamma_A(x)$), for all $x, y \in M$ and all $\alpha \in \Gamma$.

Example 3.2. If G and H are additive abelian groups and $M = \text{Hom}(G, H)$, $\Gamma = \text{Hom}(H, G)$, then M is a Γ -ring with the operations pointwise addition and composition of homomorphisms. Define a fuzzy set $\mu : M \rightarrow [0, 1]$ by $\mu(0_M) = 0.5, \mu(f) = 0.3$ and $\gamma_A : M \rightarrow [0, 1]$ by $\gamma_A(0_M) = 0.3, \mu(f) = 0.5$ where f is any member of M with $f \neq 0_M$. Let $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ be a function defined by

$$T(\alpha, \beta) = \max(\alpha + \beta - 1, 0)$$

and $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$ be a function defined by

$$S(\alpha, \beta) = \min(\alpha + \beta, 1)$$

for all $\alpha, \beta \in [0, 1]$. Then T is a t -norm and S is a s -norm. By routine calculations, we know that IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic (T, S) -normed fuzzy ideal of Γ -ring M .

Theorem 3.3. *If I is an ideal of an Γ -ring M , then the IFS $\bar{I} = (X_I, \bar{X}_I)$ is an intuitionistic (T, S) -normed fuzzy ideal of M .*

Proof. Let $x, y \in M$. If $x, y \in I$ and $\alpha \in \Gamma$, then $x - y \in I$ and $x\alpha y \in I$ since I is an ideal of M . Hence

$$X_I(x - y) = 1 \geq T(X_I(x), X_I(y)) \text{ and } X_I(x\alpha y) = 1 \geq X_I(y).$$

Also, we have

$$0 = 1 - X_I(x - y) = \bar{X}_I(x - y) \leq S(\bar{X}_I(x), \bar{X}_I(y))$$

and

$$0 = 1 - X_I(x\alpha y) = \bar{X}_I(x\alpha y) \leq \bar{X}_I(y).$$

If $x \notin I$, or $y \notin I$, then $X_I(x) = 0$, or $X_I(y) = 0$. Thus we have

$$X_I(x - y) \geq T(X_I(x), X_I(y))$$

by Proposition 2.2 and $X_I(x\alpha y) \geq X_I(y)$ for all $\alpha \in \Gamma$. Next we have

$$\bar{X}_I(x - y) \leq S(\bar{X}_I(x), \bar{X}_I(y)) = S(1 - X_I(x), 1 - X_I(y)) = 1$$

by Proposition 2.4 and $\bar{X}_I(x\alpha y) = 1 - X_I(x\alpha y) \leq 1 - X_I(y) = \bar{X}_I(y)$. This proves the theorem. \square

Theorem 3.4. *Let I be a nonempty subset of M . If $\bar{I} = (X_I, \bar{X}_I)$ satisfies (F1) or (F2), then I is an ideal of an Γ -ring M .*

Proof. Suppose that $\bar{I} = (X_I, \bar{X}_I)$ satisfy (F1). Let $x, y \in I$. Then it follows from (F1) that

$$X_I(x - y) \geq T(X_I(x), X_I(y)) = T(1, 1) = 1$$

so that $X_I(x - y) = 1$, i.e., $x - y \in I$. Now let $x \in M, y \in I$ and $\alpha \in \Gamma$. Then $X_I(x\alpha y) \geq X_I(y) = 1$, and so $\bar{X}_I(x\alpha y) = 1$, that is., $x\alpha y \in I$. Hence I is an ideal of M . Now suppose that $\bar{I} = (X_I, \bar{X}_I)$ satisfy (F2). Let $x, y \in I$. Then from (F2), we have

$$\bar{X}_I(x - y) \leq S(\bar{X}_I(x), \bar{X}_I(y)) = S(1 - X_I(x), 1 - X_I(y)) = S(0, 0) = 0,$$

and thus $\bar{X}_I(x - y) = 1 - X_I(x - y) = 0$, i.e., $X_I(x - y) = 1$. Hence $x - y \in I$. For all $x \in M, y \in I$ and $\alpha \in \Gamma$, we have $\bar{X}_I(x\alpha y) \leq \bar{X}_I(y) = 1$ and so $\bar{X}_I(x\alpha y) = 1$. Hence $x\alpha y \in I$. This proves the theorem. \square

Definition 3.5. Let T be a t -norm and S be a s -norm on $[0, 1]$. An intuitionistic (T, S) -normed fuzzy ideals $A = (\mu_A, \gamma_A)$ is called an *intuitionistic idempotent (T, S) -normed fuzzy ideal* of M if μ_A and γ_A satisfy the idempotent property with respect to T and S respectively.

Proposition 3.6. *Let T be a t -norm and S be a s -norm on $[0, 1]$. If IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic idempotent (T, S) -fuzzy ideal of M , then we have*

$$\mu_A(0) \geq \mu_A(x) \text{ and } \gamma_A(0) \leq \gamma_A(x)$$

for all $x \in M$.

Proof. For every $x \in M$, we have

$$\mu_A(0) = \mu_A(x - x) \geq T(\mu_A(x), \mu_A(x)) = \mu_A(x),$$

and

$$\gamma_A(0) = \gamma_A(x - x) \leq S(\gamma_A(x), \gamma_A(x)) = \gamma_A(x).$$

This completes the proof. \square

Proposition 3.7. *Let T be a t -norm and S be a s -norm. If IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic idempotent (T, S) -fuzzy ideal of M , then the set*

$$M_A = \{x \in M \mid \mu_A(x) = \mu_A(0), \gamma_A(x) = \gamma_A(0)\}$$

is an ideal of Γ -ring M .

Proof. Let T be a t -norm and S be a s -norm. Let $x, y \in M_A$. Then $\mu_A(x) = \mu_A(y) = \mu_A(0)$ and $\gamma_A(x) = \gamma_A(y) = \gamma_A(0)$. Since $A = (\mu_A, \gamma_A)$ is an intuitionistic idempotent (T, S) -fuzzy ideal of M , it follows that

$$\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y)) = T(\mu_A(0), \mu_A(0)) = \mu_A(0),$$

and

$$\gamma_A(x - y) \leq S(\gamma_A(x), \gamma_A(y)) = S(\gamma_A(0), \gamma_A(0)) = \gamma_A(0)$$

so that $\mu_A(x - y) = \mu_A(0)$ and $\gamma_A(x - y) = \gamma_A(0)$. Thus $x - y \in M_A$. Now let $x \in M, \alpha \in \Gamma$ and $y \in M_A$. Then we have

$$\mu_A(x\alpha y) \geq \mu_A(y) = \mu_A(0) \text{ and } \gamma_A(x\alpha y) \geq \gamma_A(y) = \gamma_A(0).$$

Hence $\mu_A(x\alpha y) = \mu_A(0)$ and $\gamma_A(x\alpha y) = \gamma_A(0)$ by Proposition 3.6, and so $x\alpha y \in M_A$. This completes the proof. \square

Let $A = (\mu_A, \gamma_A)$ be an IFS in M and let $\alpha \in [0, 1]$. Then the sets

$$U(\mu_A; \alpha) := \{x \in M : \mu_A(x) \geq \alpha\}$$

and

$$L(\gamma_A; \alpha) := \{x \in M : \gamma_A(x) \leq \alpha\}$$

are called a μ -level α -cut and a γ -level α -cut of A , respectively.

Theorem 3.8. *Let T be a t -norm and S be a s -norm and let IFS $A = (\mu_A, \gamma_A)$ be an intuitionistic (T, S) -normed fuzzy ideal of M and $\alpha \in [0, 1]$. then we have*

(i) *if $\alpha = 1$, then the upper level set $U(\mu_A; \alpha)$ is either empty or an ideal of M .*

(ii) *if $\alpha = 0$, then the lower level set $L(\gamma_A; \alpha)$ is either empty or an ideal of M .*

(iii) *if $T = \min$, then the upper level set $U(\mu_A; \alpha)$ is either empty or an ideal of M .*

(iv) *if $S = \max$, then the lower level set $L(\gamma_A; \alpha)$ is either empty or an ideal of M .*

Proof. (i) Suppose that $\alpha = 1$ and let $x, y \in U(\mu_A; \alpha)$. Then $\mu_A(x) \geq \alpha = 1$ and $\mu_A(y) \geq \alpha = 1$. It follows that $\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y)) \geq T(1, 1) = 1$ so that $x - y \in U(\mu_A; \alpha)$. Now let $x \in M, \beta \in \Gamma$ and $y \in U(\mu_A; \alpha)$. Then we have $\mu_A(x\beta y) \geq \mu_A(y) \geq \alpha = 1$, and so $x\beta y \in U(\mu_A; \alpha)$. Hence $U(\mu_A; \alpha)$ is an ideal of M when $\alpha = 1$.

(ii) Suppose that $\alpha = 0$ and let $x, y \in L(\gamma_A; \alpha)$. Then $\gamma_A(x) \leq \alpha = 0$ and $\gamma_A(y) \leq \alpha = 0$. It follows that $\gamma_A(x - y) \leq S(\gamma_A(x), \gamma_A(y)) \leq S(0, 0) = 0$, so that $x - y \in L(\gamma_A; \alpha)$. Now let $x \in M, \beta \in \Gamma$ and $y \in L(\gamma_A; \alpha)$. Then we get

$\gamma_A(x\beta y) \leq \gamma_A(y) \leq \alpha = 0$, and so $x\beta y \in L(\gamma_A; \alpha)$. Hence $L(\gamma_A; \alpha)$ is an ideal of M when $\alpha = 0$.

(iii) Assume that $T = \min$ and let $x, y \in U(\mu_A; \alpha)$. Then

$$\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y)) = \min(\mu_A(x), \mu_A(y)) \geq \min(\alpha, \alpha) = \alpha$$

for all $\alpha \in [0, 1]$. Hence $x - y \in U(\mu_A; \alpha)$. Now let $x \in M, \beta \in \Gamma$ and $y \in U(\mu_A; \alpha)$. Then $\mu_A(x\beta y) \geq \mu_A(y) \geq \alpha$, and so $x\beta y \in U(\mu_A; \alpha)$. Hence $U(\mu_A; \alpha)$ is an ideal of M .

(iv) Assume that $S = \max$ and let $x, y \in L(\gamma_A; \alpha)$. Then

$$\gamma_A(x - y) \leq S(\gamma_A(x), \gamma_A(y)) = \max(\gamma_A(x), \gamma_A(y)) \leq \max(\alpha, \alpha) = \alpha$$

for all $\alpha \in [0, 1]$. Hence $x - y \in L(\gamma_A; \alpha)$. Now let $x \in M, \beta \in \Gamma$ and $y \in L(\gamma_A; \alpha)$. Then $\gamma_A(x\beta y) \leq \gamma_A(y) \leq \alpha$, and so $x\beta y \in L(\gamma_A; \alpha)$. Therefore $L(\gamma_A; \alpha)$ is an ideal of M . \square

If μ is a fuzzy set in M and θ is a mapping from M into itself, we define a mapping $\mu[\theta] : M \rightarrow [0, 1]$ by $\mu[\theta](x) = \mu(\theta(x))$ for all $x \in M$.

Theorem 3.9. *Let T be a t -norm and S be a s -norm. Let θ be an endomorphism of M . If $A = (\mu_A, \gamma_A)$ is an intuitionistic (T, S) -normed fuzzy ideal of M , then $B = (\mu_A[\theta], \gamma_A[\theta])$ is an intuitionistic (T, S) -normed fuzzy ideal of M .*

Proof. For any $x, y \in M$, we have

$$\begin{aligned} \mu_A[\theta](x - y) &= \mu_A(\theta(x - y)) = \mu_A(\theta(x) - \theta(y)) \\ &\geq T(\mu_A(\theta(x)), \mu_A(\theta(y))) = T(\mu_A[\theta](x), \mu_A[\theta](y)). \end{aligned}$$

Now let $x, y \in M, \alpha \in \Gamma$. Then we have

$$\mu_A[\theta](x\alpha y) = \mu_A(\theta(x\alpha y)) = \mu_A(\theta(x)\alpha\theta(y)) \geq \mu_A[\theta](y).$$

Similarly, we have for any $x, y \in M$, we have

$$\begin{aligned} \gamma_A[\theta](x - y) &= \gamma_A(\theta(x - y)) = \gamma_A(\theta(x) - \theta(y)) \\ &\leq S(\gamma_A(\theta(x)), \gamma_A(\theta(y))) = S(\gamma_A[\theta](x), \gamma_A[\theta](y)). \end{aligned}$$

Now let $x, y \in M, \alpha \in \Gamma$. Then we have

$$\gamma_A[\theta](x\alpha y) = \gamma_A(\theta(x\alpha y)) = \gamma_A(\theta(x)\alpha\theta(y)) \leq \gamma_A[\theta](y).$$

This completes the proof. \square

Theorem 3.10. *Let T be a t -norm and S be a s -norm let $A = (\mu_A, \gamma_A)$ be an IFS in M such that the non-empty sets $U(\mu_A; \alpha)$ and $L(\gamma_A; \alpha)$ are ideals of M for all $\alpha \in [0, 1]$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic (T, S) -normed fuzzy ideal of M .*

Proof. Suppose that there exists $x_0, y_0 \in M$ such that

$$\mu_A(x_0 - y_0) < T(\mu_A(x_0), \mu_A(y_0)).$$

Taking $\alpha_0 := \frac{1}{2}(\mu_A(x_0 - y_0) + T(\mu_A(x_0), \mu_A(y_0)))$, then

$$\begin{aligned} \min(\mu_A(x_0), \mu_A(y_0)) &\geq T(\mu_A(x_0), \mu_A(y_0)) \\ &\geq \alpha_0 > \mu_A(x_0 - y_0). \end{aligned}$$

It follows that $x_0, y_0 \in U(\mu_A; \alpha_0)$ and $x_0 - y_0 \notin U(\mu_A; \alpha_0)$. Hence μ_A satisfies the inequality $\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y))$ for all $x, y \in M$. Now let $x_0, y_0 \in M$ and $\alpha \in \Gamma$ such that $\mu_A(x_0 \alpha y_0) \leq \mu_A(y_0)$. Taking $\beta_0 := \frac{1}{2}(\mu_A(x_0 \alpha y_0) + \mu_A(y_0))$, then we obtain $\mu_A(x_0 \alpha y_0) \leq \beta_0 \leq \mu_A(y_0)$. It follows that $y_0 \in U(\mu_A; \beta_0)$ and $x_0 \alpha y_0 \notin U(\mu_A; \beta_0)$. This is a contraction. Similarly, suppose that there exists $x_0, y_0 \in X$ such that

$$\gamma_A(x_0 - y_0) > S(\gamma_A(x_0), \gamma_A(y_0)).$$

Taking $\beta_0 := \frac{1}{2}(\gamma_A(x_0 - y_0) + S(\gamma_A(x_0), \gamma_A(y_0)))$, then

$$\max(\gamma_A(x_0), \gamma_A(y_0)) \leq S(\gamma_A(x_0), \gamma_A(y_0)) \leq \beta_0 < \gamma_A(x_0 * y_0).$$

It follows that $x_0, y_0 \in L(\gamma_A; \beta_0)$ and $x_0 - y_0 \notin L(\gamma_A; \beta_0)$. This is a contradiction and hence γ_A satisfies the inequality $\gamma_A(x - y) \leq S(\gamma_A(x), \gamma_A(y))$ for all $x, y \in M$. Now let $x_0, y_0 \in M$ and $\alpha \in \Gamma$ such that $\gamma_A(x_0 \alpha y_0) \geq \gamma_A(y_0)$. Taking $\beta_0 := \frac{1}{2}(\gamma_A(x_0 \alpha y_0) + \gamma_A(y_0))$, then we obtain $\gamma_A(x_0 \alpha y_0) \geq \beta_0 \geq \gamma_A(y_0)$. It follows that $y_0 \in L(\gamma_A; \beta_0)$ and $x_0 \alpha y_0 \notin L(\gamma_A; \beta_0)$. This is a contraction. This proves the theorem. \square

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