Bifuzzy Bi-Ideals with Operators in Semigroups

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Abstract

In this paper, we introduce the concept of a (generalized) Ω -bifuzzy bi-ideal in semigroups is introduced, and study some related properties. A condition for a generalized Ω -bifuzzy bi-ideal to be an Ω -bifuzzy bi-ideal is provided.

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1 Introduction

In [6, 7], Kuroki have studied several properties of fuzzy left (right) ideals, fuzzy bi-ideals and fuzzy interior ideals in semigroups. For more other study on the fuzzy theory in semigroups, we refer to papers [3, 8, 12, 14, 15]. Hur et al. [5] have discussed fuzzy subsemigroups, fuzzy bi-ideals and fuzzy interior ideals with operators in semigroups. Hur et al. [4] have studied the bifuzzy structure of subsemigroups with operators in semigroups. In this paper, we introduced the notion of bifuzzy bi-ideals, called Ω -bifuzzy bi-ideals, of semigroups, and investigate related properties. We give a condition for a generalized Ω -bifuzzy bi-ideal to be an Ω -bifuzzy bi-ideal. Using the notion of bifuzzy bi-ideals, we construct Ω -bifuzzy bi-ideals. Also, using an Ω -bifuzzy bi-ideal, we establish a bifuzzy bi-ideal. We show that every Ω -bifuzzy bi-ideal of a group S is constant.

2 Preliminaries

In this section, we give some notations and definitions on which our research in this paper is based.

A semigroup S is called *regular* if it satisfies:

$$(\forall a \in S) (\exists x \in S) (a = axa).$$

A semigroup S is called *completely regular* if it satisfies:

$$(\forall a \in S) (\exists x \in S) (a = axa, ax = xa).$$

By a *subsemigroup* of a semigroup S we mean a nonempty subset A of S such that $A^2 \subseteq A$, and by a *left (right) ideal* of S we mean a non-empty subset A of S such that $SA \subseteq A$ ($AS \subseteq A$). By an *ideal*, we mean a nonempty subset of a semigroup S which is both a left and a right ideal of S . A subsemigroup A of a semigroup S is called a *bi-ideal* of S if $ASA \subseteq A$. A nonempty subset A of a semigroup S is called a *generalized bi-ideal* of S if $ASA \subseteq A$ (see Lajos[11]).

Given a set X , a function μ from X to the unit interval $[0, 1]$ is called a fuzzy set of X (see Zadeh [16]). Let S be a semigroup.

A fuzzy set μ in S is called a *fuzzy subsemigroup* of S if it satisfies

$$(\forall x, y \in S) (\mu(xy) \geq \min\{\mu(x), \mu(y)\}), \quad (2.1)$$

and is called a *fuzzy left (right) ideal* of S if

$$(\forall x, y \in S) (\mu(xy) \geq \mu(y) \quad (\mu(xy) \geq \mu(x))). \quad (2.2)$$

If μ is both a fuzzy left and a fuzzy right ideal of S , we say that μ is a *fuzzy ideal* of S . A fuzzy subsemigroup μ of S is called a *fuzzy bi-ideal* of S if it satisfies

$$(\forall w, x, y \in S) (\mu(xwy) \geq \min\{\mu(x), \mu(y)\}). \quad (2.3)$$

Let X be a nonempty set. By the original Definition of Atanassov in [1], an *intuitionistic fuzzy set* is an object of the form: $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X\}$ where μ_A and γ_A be two functions from X to $[0, 1]$ such that

$$(\forall x \in X) (0 \leq \mu_A(x) + \gamma_A(x) \leq 1). \quad (2.4)$$

This object is also called a *bifuzzy set* (according to [2]). We consider it in a form of an ordered triple: $A = \langle X; \mu_A, \gamma_A \rangle$ where X , μ_A and γ_A are as above. A bifuzzy set $A = \langle S; \mu_A, \gamma_A \rangle$ in a semigroup S is called *bifuzzy subsemigroup* of S (see [9]) if it satisfies:

$$\begin{aligned} \mu_A(xy) &\geq \min\{\mu_A(x), \mu_A(y)\}, \\ \gamma_A(xy) &\leq \max\{\gamma_A(x), \gamma_A(y)\}, \end{aligned} \quad (2.5)$$

and is called a *bifuzzy left* (resp. *right*) *ideal* of S (see [9]) if it satisfies:

$$\begin{aligned}\mu_A(xy) &\geq \mu_A(y) \text{ (resp. } \mu_A(xy) \geq \mu_A(x)), \\ \gamma_A(xy) &\leq \gamma_A(y) \text{ (resp. } \gamma_A(xy) \leq \gamma_A(x))\end{aligned}\quad (2.6)$$

for all $x, y \in S$. A bifuzzy subsemigroup $A = \langle S; \mu_A, \gamma_A \rangle$ of S is called a *bifuzzy bi-ideal* of S (see [9]) if it satisfies:

$$\begin{aligned}\mu_A(xwy) &\geq \min\{\mu_A(x), \mu_A(y)\}, \\ \gamma_A(xwy) &\leq \max\{\gamma_A(x), \gamma_A(y)\}\end{aligned}\quad (2.7)$$

for all $w, x, y \in S$.

3 Ω -bifuzzy bi-ideals

In what follows let S and Ω denote a semigroup and a nonempty set, respectively, unless otherwise specified.

An Ω -bifuzzy set A_Ω in S is defined to be an object having the form

$$A_\Omega := \{ \langle (x, \alpha); \mu_{A_\Omega}(x, \alpha), \gamma_{A_\Omega}(x, \alpha) \rangle \mid (x, \alpha) \in S \times \Omega \}$$

where the function $\mu_{A_\Omega} : S \times \Omega \rightarrow [0, 1]$ and $\gamma_{A_\Omega} : S \times \Omega \rightarrow [0, 1]$ define the degree of membership and the degree of nonmembership of the element $(x, \alpha) \in S \times \Omega$ to $A_\Omega \subset S \times \Omega$, respectively, such that

$$(\forall (x, \alpha) \in S \times \Omega) (0 \leq \mu_{A_\Omega}(x, \alpha) + \gamma_{A_\Omega}(x, \alpha) \leq 1).$$

We shall use the symbol $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ for the Ω -bifuzzy set

$$A_\Omega := \{ \langle (x, \alpha); \mu_{A_\Omega}(x, \alpha), \gamma_{A_\Omega}(x, \alpha) \rangle \mid (x, \alpha) \in S \times \Omega \}.$$

Definition 3.1. [4] An Ω -bifuzzy set $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ in S is called an Ω -bifuzzy subsemigroup of S if it satisfies

$$\begin{aligned}\mu_{A_\Omega}(xy, \alpha) &\geq \min\{\mu_{A_\Omega}(x, \alpha), \mu_{A_\Omega}(y, \alpha)\}, \\ \gamma_{A_\Omega}(xy, \alpha) &\leq \max\{\gamma_{A_\Omega}(x, \alpha), \gamma_{A_\Omega}(y, \alpha)\}.\end{aligned}\quad (3.1)$$

for all $\alpha \in \Omega$ and $x, y \in S$.

Definition 3.2. An Ω -bifuzzy set A_Ω in S is called an Ω -bifuzzy left (resp. right) ideal of S if

$$\begin{aligned}\mu_{A_\Omega}(xy, \alpha) &\geq \mu_{A_\Omega}(y, \alpha) \text{ (resp. } \mu_{A_\Omega}(xy, \alpha) \geq \mu_{A_\Omega}(x, \alpha)), \\ \gamma_{A_\Omega}(xy, \alpha) &\leq \gamma_{A_\Omega}(y, \alpha) \text{ (resp. } \gamma_{A_\Omega}(xy, \alpha) \leq \gamma_{A_\Omega}(x, \alpha)).\end{aligned}\quad (3.2)$$

for all $\alpha \in \Omega$ and $x, y \in S$.

If A_Ω is both an Ω -bifuzzy left and an Ω -bifuzzy right ideal of S , we say that A_Ω is an Ω -bifuzzy ideal of S .

Let $S^\Omega := \{u \mid u : \Omega \rightarrow S\}$. For any $u, v \in S^\Omega$, we define $(uv)(\alpha) = u(\alpha)v(\alpha)$ for all $\alpha \in \Omega$. Then S^Ω is a semigroup (see [5]).

Lemma 3.3. [4] Let $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ be an Ω -bifuzzy subsemigroup of S and let $\Phi = \langle S^\Omega; \mu_\Phi, \gamma_\Phi \rangle$ be a bifuzzy set in S^Ω defined by

$$\begin{aligned}\mu_\Phi(u) &= \inf\{\mu_{A_\Omega}(u(\alpha), \alpha) \mid \alpha \in \Omega\}, \\ \gamma_\Phi(u) &= \sup\{\gamma_{A_\Omega}(u(\alpha), \alpha) \mid \alpha \in \Omega\}\end{aligned}\tag{3.3}$$

for all $u \in S^\Omega$. Then $\Phi = \langle S^\Omega; \mu_\Phi, \gamma_\Phi \rangle$ is a bifuzzy subsemigroup of S^Ω .

Let A_Ω be an Ω -bifuzzy left (right) ideal of S and let Φ be the bifuzzy set in S^Ω given in Lemma 3.3. For any $u, v \in S^\Omega$, we have

$$\begin{aligned}\mu_\Phi(uv) &= \inf\{\mu_{A_\Omega}((uv)(\alpha), \alpha) \mid \alpha \in \Omega\} \\ &= \inf\{\mu_{A_\Omega}(u(\alpha)v(\alpha), \alpha) \mid \alpha \in \Omega\} \\ &\geq \inf\{\mu_{A_\Omega}(v(\alpha), \alpha) \mid \alpha \in \Omega\} \\ &= \mu_\Phi(v),\end{aligned}$$

$$\begin{aligned}\gamma_\Phi(uv) &= \sup\{\gamma_{A_\Omega}((uv)(\alpha), \alpha) \mid \alpha \in \Omega\} \\ &= \sup\{\gamma_{A_\Omega}(u(\alpha)v(\alpha), \alpha) \mid \alpha \in \Omega\} \\ &\leq \sup\{\gamma_{A_\Omega}(v(\alpha), \alpha) \mid \alpha \in \Omega\} \\ &= \gamma_\Phi(v).\end{aligned}$$

Similarly $\mu_\Phi(uv) \geq \mu_\Phi(u)$ and $\gamma_\Phi(uv) \leq \gamma_\Phi(u)$. Hence $\Phi = \langle S^\Omega; \mu_\Phi, \gamma_\Phi \rangle$ is a bifuzzy ideal of S^Ω .

Definition 3.4. An Ω -bifuzzy set A_Ω in S is called an Ω -bifuzzy bi-ideal of S if it is an Ω -bifuzzy subsemigroup of S satisfying the following assertions:

$$\begin{aligned}\mu_{A_\Omega}(xyz, \alpha) &\geq \min\{\mu_{A_\Omega}(x, \alpha), \mu_{A_\Omega}(z, \alpha)\}, \\ \gamma_{A_\Omega}(xyz, \alpha) &\leq \max\{\gamma_{A_\Omega}(x, \alpha), \gamma_{A_\Omega}(z, \alpha)\}\end{aligned}\tag{3.4}$$

for all $\alpha \in \Omega$ and $x, y, z \in S$.

If A_Ω satisfies the condition (3.4) only, we say that A_Ω is a *generalized Ω -bifuzzy bi-ideal* of S .

Example 3.5. Let $A = \langle S; \mu_A, \gamma_A \rangle$ be a bifuzzy subsemigroup of S and let $A_\Omega = \langle S^\Omega \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ be an Ω -bifuzzy set in S^Ω defined by $\mu_{A_\Omega}(u, \alpha) = \mu_A(u(\alpha))$ and $\gamma_{A_\Omega}(u, \alpha) = \gamma_A(u(\alpha))$ for all $u \in S^\Omega$ and $\alpha \in \Omega$. Then $A_\Omega = \langle S^\Omega \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ is an Ω -bifuzzy subsemigroup of S^Ω (see [4, Example 3.3]). If $A = \langle S; \mu_A, \gamma_A \rangle$ is a bifuzzy bi-ideal of S , then $A_\Omega = \langle S^\Omega \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ is an Ω -bifuzzy bi-ideal of S^Ω .

Example 3.6. Let

$$\Omega := \{A \mid A \text{ is a bifuzzy bi-ideal of } S\}$$

and let $\Psi = \langle S \times \Omega; \mu_\Psi, \gamma_\Psi \rangle$ be an Ω -bifuzzy set in S defined by $\mu_\Psi(x, \mu_A) = \mu_A(x)$ and $\gamma_\Psi(x, \gamma_A) = \gamma_A(x)$ for all $x \in S$ and $A = \langle S; \mu_A, \gamma_A \rangle \in \Omega$. Then Ψ is an Ω -bifuzzy bi-ideal of S .

Example 3.7. Consider a semigroup $S = \{a, b\}$ with the following Cayley table:

	a	b
a	a	b
b	b	a

Let $\Omega = \{1, 2\}$ and let $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ be an Ω -bifuzzy set in S defined by

$$A_\Omega = \langle S \times \Omega; \left(\frac{(a,1)}{1}, \frac{(a,2)}{1}, \frac{(b,1)}{0.8}, \frac{(b,2)}{0.5} \right), \left(\frac{(a,1)}{0}, \frac{(a,2)}{0}, \frac{(b,1)}{0.1}, \frac{(b,2)}{0.3} \right) \rangle.$$

Then $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ is an Ω -bifuzzy subsemigroup of S (see [4, Example 3.2]). But it is not an Ω -bifuzzy bi-ideal of S since

$$\mu_{A_\Omega}(aba, 1) = \mu_{A_\Omega}(b, 1) = 0.8 < 1 = \mu_{A_\Omega}(a, 1) = \min\{\mu_{A_\Omega}(a, 1), \mu_{A_\Omega}(a, 1)\}$$

and/or

$$\gamma_{A_\Omega}(aba, 1) = \gamma_{A_\Omega}(b, 1) = 0.1 > 0 = \gamma_{A_\Omega}(a, 1) = \max\{\gamma_{A_\Omega}(a, 1), \gamma_{A_\Omega}(a, 1)\}.$$

It is clear that every Ω -bifuzzy bi-ideal of S is a generalized Ω -bifuzzy bi-ideal of S , but not conversely as seen in the following Example.

Example 3.8. Let $S = \{a, b, c, d\}$ be a semigroup with the following multiplication table:

	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

Let $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ be an Ω -bifuzzy set in $S \times \Omega$ defined by

$$A_\Omega = \langle S \times \Omega; \left(\frac{(a,\alpha)}{0.5}, \frac{(b,\alpha)}{0}, \frac{(c,\alpha)}{0.2}, \frac{(d,\alpha)}{0} \right), \left(\frac{(a,\alpha)}{0.4}, \frac{(b,\alpha)}{0.8}, \frac{(c,\alpha)}{0.6}, \frac{(d,\alpha)}{0.8} \right) \rangle.$$

Then A_Ω is a generalized Ω -bifuzzy bi-ideal of S , which is not an Ω -bifuzzy bi-ideal of S since

$$\mu_{A_\Omega}(cc, \alpha) = \mu_{A_\Omega}(b, \alpha) = 0 < 0.2 = \min\{\mu_{A_\Omega}(c, \alpha), \mu_{A_\Omega}(c, \alpha)\}$$

and/or

$$\gamma_{A_\Omega}(cc, \alpha) = \gamma_{A_\Omega}(b, \alpha) = 0.8 > 0.6 = \max\{\gamma_{A_\Omega}(c, \alpha), \gamma_{A_\Omega}(c, \alpha)\}.$$

Theorem 3.9. For each $\alpha \in \Omega$, let $A_\Omega^\alpha = \langle S; \mu_{A_\Omega^\alpha}, \gamma_{A_\Omega^\alpha} \rangle$ be a bifuzzy bi-ideal of S . Then the Ω -bifuzzy set $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ in S given by $\mu_{A_\Omega}(x, \alpha) = \mu_{A_\Omega^\alpha}(x)$ and $\gamma_{A_\Omega}(x, \alpha) = \gamma_{A_\Omega^\alpha}(x)$ is an Ω -bifuzzy bi-ideal of S .

Proof. Using [4, Proposition 3.5], $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ is an Ω -bifuzzy subsemigroup of S . Let $x, y, z \in S$ and $\alpha \in \Omega$. Then

$$\begin{aligned} \mu_{A_\Omega}(xyz, \alpha) &= \mu_{A_\Omega^\alpha}(xyz) \geq \min\{\mu_{A_\Omega^\alpha}(x), \mu_{A_\Omega^\alpha}(z)\} \\ &= \min\{\mu_{A_\Omega}(x, \alpha), \mu_{A_\Omega}(z, \alpha)\}, \end{aligned}$$

$$\begin{aligned} \gamma_{A_\Omega}(xyz, \alpha) &= \gamma_{A_\Omega^\alpha}(xyz) \leq \max\{\gamma_{A_\Omega^\alpha}(x), \gamma_{A_\Omega^\alpha}(z)\} \\ &= \max\{\gamma_{A_\Omega}(x, \alpha), \gamma_{A_\Omega}(z, \alpha)\}. \end{aligned}$$

This completes the proof. $\square$

Theorem 3.10. Let $A_\Omega = \langle S \times \Omega; \mu_{A_\Omega}, \gamma_{A_\Omega} \rangle$ be an Ω -bifuzzy bi-ideal of S . For each $\omega \in \Omega$, the bifuzzy set $A_\Omega^\omega := \langle S; \mu_{A_\Omega^\omega}, \gamma_{A_\Omega^\omega} \rangle$ in S given by $\mu_{A_\Omega^\omega}(x) = \mu_{A_\Omega}(x, \omega)$ and $\gamma_{A_\Omega^\omega}(x) = \gamma_{A_\Omega}(x, \omega)$ for all $x \in S$ is a bifuzzy bi-ideal of S .

Proof. Using [4, Proposition 3.4], $A_\Omega^\omega := \langle S; \mu_{A_\Omega^\omega}, \gamma_{A_\Omega^\omega} \rangle$ is a bifuzzy subsemigroup of S . For any $x, y, z \in S$ we have

$$\begin{aligned} \mu_{A_\Omega^\omega}(xyz) &= \mu_{A_\Omega}(xyz, \omega) \geq \min\{\mu_{A_\Omega}(x, \omega), \mu_{A_\Omega}(y, \omega)\} \\ &= \min\{\mu_{A_\Omega^\omega}(x), \mu_{A_\Omega^\omega}(y)\}, \end{aligned}$$

$$\begin{aligned} \gamma_{A_\Omega^\omega}(xyz) &= \gamma_{A_\Omega}(xyz, \omega) \leq \max\{\gamma_{A_\Omega}(x, \omega), \gamma_{A_\Omega}(y, \omega)\} \\ &= \max\{\gamma_{A_\Omega^\omega}(x), \gamma_{A_\Omega^\omega}(y)\}. \end{aligned}$$

This completes the proof. $\square$

Theorem 3.11. If S is a group, then every Ω -bifuzzy bi-ideal of S is a constant function.

Proof. Let A_Ω be an Ω -bifuzzy bi-ideal of S . For any $x \in S$ and $\alpha \in \Omega$, we get

$$\begin{aligned}\mu_{A_\Omega}(x, \alpha) &= \mu_{A_\Omega}(exe, \alpha) \geq \min\{\mu_{A_\Omega}(e, \alpha), \mu_{A_\Omega}(e, \alpha)\} = \mu_{A_\Omega}(e, \alpha) \\ &= \mu_{A_\Omega}(ee, \alpha) = \mu_{A_\Omega}((xx^{-1})(x^{-1}x), \alpha) = \mu_{A_\Omega}(x(x^{-1}x^{-1})x, \alpha) \\ &\geq \min\{\mu_{A_\Omega}(x, \alpha), \mu_{A_\Omega}(x, \alpha)\} = \mu_{A_\Omega}(x, \alpha),\end{aligned}$$

$$\begin{aligned}\gamma_{A_\Omega}(x, \alpha) &= \gamma_{A_\Omega}(exe, \alpha) \leq \max\{\gamma_{A_\Omega}(e, \alpha), \gamma_{A_\Omega}(e, \alpha)\} = \gamma_{A_\Omega}(e, \alpha) \\ &= \gamma_{A_\Omega}(ee, \alpha) = \gamma_{A_\Omega}((xx^{-1})(x^{-1}x), \alpha) = \gamma_{A_\Omega}(x(x^{-1}x^{-1})x, \alpha) \\ &\leq \max\{\gamma_{A_\Omega}(x, \alpha), \gamma_{A_\Omega}(x, \alpha)\} = \gamma_{A_\Omega}(x, \alpha),\end{aligned}$$

where e is the identity element of S . Hence $\mu_{A_\Omega}(x, \alpha) = \mu_{A_\Omega}(e, \alpha)$ and $\gamma_{A_\Omega}(x, \alpha) = \gamma_{A_\Omega}(e, \alpha)$, and hence A_Ω is a constant function. $\square$

We give a condition for a generalized Ω -bifuzzy bi-ideal to be an Ω -bifuzzy bi-ideal.

Theorem 3.12. *If S is regular, then every generalized Ω -bifuzzy bi-ideal of S is an Ω -bifuzzy bi-ideal of S .*

Proof. Let A_Ω be a generalized Ω -bifuzzy bi-ideal of a regular semigroup S and let $\alpha \in \Omega$ and $x, y \in S$. Since S is regular, it follows that there exists $a \in S$ such that $y = yay$ so that

$$\begin{aligned}\mu_{A_\Omega}(xy, \alpha) &= \mu_{A_\Omega}(x(yay), \alpha) = \mu_{A_\Omega}(x(ya)y, \alpha) \\ &\geq \min\{\mu_{A_\Omega}(x, \alpha), \mu_{A_\Omega}(y, \alpha)\},\end{aligned}$$

$$\begin{aligned}\gamma_{A_\Omega}(xy, \alpha) &= \gamma_{A_\Omega}(x(yay), \alpha) = \gamma_{A_\Omega}(x(ya)y, \alpha) \\ &\leq \max\{\gamma_{A_\Omega}(x, \alpha), \gamma_{A_\Omega}(y, \alpha)\}.\end{aligned}$$

This means that A_Ω is an Ω -bifuzzy subsemigroup of S , and so it is an Ω -bifuzzy bi-ideal of S . $\square$

Proposition 3.13. *Let A_Ω be an Ω -bifuzzy bi-ideal of S . If S is completely regular, then*

$$\mu_{A_\Omega}(x, \alpha) = \mu_{A_\Omega}(x^2, \alpha), \gamma_{A_\Omega}(x, \alpha) = \gamma_{A_\Omega}(x^2, \alpha) \quad (3.5)$$

for all $x \in S$ and $\alpha \in \Omega$.

Proof. Let $\alpha \in \Omega$ and let x be any element of S . Then there exists $a \in S$ such that $x = x^2ax^2$, which implies that

$$\begin{aligned}\mu_{A_\Omega}(x, \alpha) &= \mu_{A_\Omega}(x^2ax^2, \alpha) \geq \min\{\mu_{A_\Omega}(x^2, \alpha), \mu_{A_\Omega}(x^2, \alpha)\} \\ &= \mu_{A_\Omega}(x^2, \alpha) \geq \min\{\mu_{A_\Omega}(x, \alpha), \mu_{A_\Omega}(x, \alpha)\} = \mu_{A_\Omega}(x, \alpha),\end{aligned}$$

$$\begin{aligned}\gamma_{A_\Omega}(x, \alpha) &= \gamma_{A_\Omega}(x^2ax^2, \alpha) \leq \max\{\gamma_{A_\Omega}(x^2, \alpha), \gamma_{A_\Omega}(x^2, \alpha)\} \\ &= \gamma_{A_\Omega}(x^2, \alpha) \leq \max\{\gamma_{A_\Omega}(x, \alpha), \gamma_{A_\Omega}(x, \alpha)\} = \gamma_{A_\Omega}(x, \alpha),\end{aligned}$$

so that $\mu_{A_\Omega}(x, \alpha) = \mu_{A_\Omega}(x^2, \alpha)$ and $\gamma_{A_\Omega}(x, \alpha) = \gamma_{A_\Omega}(x^2, \alpha)$. This completes the proof. $\square$

Lemma 3.14. [13, p.105] *For a semigroup S the following conditions are equivalent:*

- (i) S is completely regular.
- (ii) S is a union of groups.
- (iii) $a \in a^2Sa^2$ for all $a \in S$.

Lemma 3.15. [10, Theorem 1] *A semigroup S is a semilattice of groups if and only if the set of all bi-ideals of S is a semilattice under the multiplication of subsets.*

Proposition 3.16. *Let A_Ω be an Ω -bifuzzy bi-ideal of S . If S is a semilattice of groups, then*

$$\begin{aligned}\mu_{A_\Omega}(x, \alpha) &= \mu_{A_\Omega}(x^2, \alpha), \mu_{A_\Omega}(xy, \alpha) = \mu_{A_\Omega}(yx, \alpha), \\ \gamma_{A_\Omega}(x, \alpha) &= \gamma_{A_\Omega}(x^2, \alpha), \gamma_{A_\Omega}(xy, \alpha) = \gamma_{A_\Omega}(yx, \alpha)\end{aligned}\quad (3.6)$$

for all $x, y \in S$ and $\alpha \in \Omega$.

Proof. If S is a semilattice of groups, then S is a union of groups and so S is completely regular by Lemma 3.14. Hence $\mu_{A_\Omega}(x, \alpha) = \mu_{A_\Omega}(x^2, \alpha)$ and $\gamma_{A_\Omega}(x, \alpha) = \gamma_{A_\Omega}(x^2, \alpha)$ for all $x \in S$ and $\alpha \in \Omega$ by Proposition 3.13. Using Lemma 3.15, we have

$$\begin{aligned}(xy)^3 &= (xyx)(yxy) \in B[xyx]B[yxy] = B[yxy](B[xyx])^2 \\ &\subseteq B[yxy]SB[xyx] \subseteq yxySxyx \subseteq yxSyx\end{aligned}$$

for all $x, y \in S$, where $B[a]$ means the principal bi-ideal of S generated by $a \in S$. This implies that there exists $a \in S$ such that $(xy)^3 = (yx)a(yx)$. Thus for any $\alpha \in \Omega$, we get

$$\begin{aligned}\mu_{A_\Omega}(xy, \alpha) &= \mu_{A_\Omega}((xy)^3, \alpha) = \mu_{A_\Omega}((yx)a(yx), \alpha) \\ &\geq \min\{\mu_{A_\Omega}(yx, \alpha), \mu_{A_\Omega}(yx, \alpha)\} = \mu_{A_\Omega}(yx, \alpha),\end{aligned}$$

$$\begin{aligned}\gamma_{A_\Omega}(xy, \alpha) &= \gamma_{A_\Omega}((xy)^3, \alpha) = \gamma_{A_\Omega}((yx)a(yx), \alpha) \\ &\leq \max\{\gamma_{A_\Omega}(yx, \alpha), \gamma_{A_\Omega}(yx, \alpha)\} = \gamma_{A_\Omega}(yx, \alpha).\end{aligned}$$

Similarly, we obtain $\mu_{A_\Omega}(yx, \alpha) \geq \mu_{A_\Omega}(xy, \alpha)$ and $\gamma_{A_\Omega}(yx, \alpha) \leq \gamma_{A_\Omega}(xy, \alpha)$. Hence $\mu_{A_\Omega}(xy, \alpha) = \mu_{A_\Omega}(yx, \alpha)$ and $\gamma_{A_\Omega}(xy, \alpha) = \gamma_{A_\Omega}(yx, \alpha)$, completing the proof. $\square$

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