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On Intuitionistic Q-Fuzzy R-Subgroups of Near-Rings

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Abstract

In this paper, we introduce the notion of intuitionistic Q-fuzzification of R-subgroups (subnear-rings) in a near-ring and investigate some related properties. Characterization of intuitionistic Q-fuzzy R-subgroups (subnear-rings) is given.

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1 Introduction

The theory of fuzzy sets which was introduced by Zadeh [18] is applied to many mathematical branches. Abou-Zaid [1], introduced the notion of a fuzzy subnear-ring and studied fuzzy ideals of a near-ring. This concept discussed further by many researchers, among whom Cho, Davvaz, Dudek, Jun, Kim [6,7-8,9,10-11,12-14].

The notion of intuitionistic fuzzy sets was introduced by Atanassov [2-4] as a generalization of the notion of fuzzy sets. In [5], Biswas applied the concept of intuitionistic fuzzy set to the theory of groups and studied intuitionistic fuzzy subgroups of a group. The notion of an intuitionistic fuzzy ideal of a near-ring is given by Jun in [11]. In [16], Yon et al. considered the intuitionistic fuzzification of a right(resp.left) R-subgroup of a near-ring. Also Cho at al., in [6] the notion of a normal intuitionistic fuzzy R-subgroup in a near-ring is introduced and related properties are investigated. Recently, Davvaz at al. [8] considered the intuitionistic fuzzification of the concept of the H_ϑ-submodules in a H_ϑ- module and Dudex at al. [9] considered the intuitionistic fuzzification of the concept of sub-hyperquasigroups in a hyperquasigroup. They investigated some properties of such hyperquasigroups. The notion of intuitionistic Q-fuzzy semiprimality in a semigroup is given by Kim[14]. Also Roh at al. [17] considered the intuitionistic Q-fuzzification of BCK/BCI-algebras. In this paper, we introduce the notion of intuitionistic Q-fuzzification of R-subgroups (subnear-rings) in a near-ring and investigate some related properties. Characterization of intuitionistic Q-fuzzy R-subgroups (subnear-rings) is given.

2 Definition and Preliminaries

Definition 2.1 (15) . *A non empty-set R with two binary operations "+" and "·" is called a near-ring if it satisfies the following axioms.*

(i) $(R, +)$ *is a group,*

(ii) $(R, \cdot)$ *is a semigroup,*

(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ *for all* $x, y, z \in R$.

Precisely speaking it is a left near-ring. Because it satisfies the left distributive law. We will use the word "near-ring" to mean "left near-ring". We denote xy instead of $x \cdot y$. Note that x0 =0 and x(-y) =-xy for all $x, y \in R$, but in general, $0x \neq 0$ for some $x \in R$.

An R-subgroup of a near-ring R is a subset H of R such that

(i) $(H, +)$ is a subgroup of (R, +),

(ii) $RH \subseteq H$,

(iii) $HR \subseteq H$.

If H satisfies (i) and (ii), then it is called a left R-subgroup of R, and if H satisfies (i) and(iii) then it is called a right R-subgroup of R. A map f from a near-ring R into a near-ring S is called a homomorphism if f(x + y)= f(x)+f(y) and f(xy)=f(x)f(y) for all $x, y \in R$.

Let R be a near-ring. A fuzzy set μ in R is called a fuzzy subnear-ring in R if [1]

(i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$,

(ii) $\mu(xy) \geq \mu(x) \wedge \mu(y)$ for all $x, y \in R$.

A fuzzy set μ in R is called a fuzzy R-subgroup of R if

(i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$,

(ii) $\mu(rx) \geq \mu(x)$,

(iii) $\mu(xr) \geq \mu(x)$ for all $x, y, r \in R$.

Let X be a non-empty set. A mapping $\mu : X \to [0, 1]$ is called a fuzzy set in X. The complement of a fuzzy set μ in X, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

In what follows, let Q and R denote a set and a near-ring, respectively, unless otherwise specified. A mapping $\mu : R \times Q \to [0, 1]$ is called a Q-fuzzy set in R. For any Q-fuzzy set μ in R and any $t \in [0, 1]$ we define two sets

$$U(\mu; t) = \{x \in X \mid \mu(x, q) \geq t, q \in Q\}$$

and

$$L(\mu; t) = \{x \in X \mid \mu(x, q) \leq t, q \in Q\}$$

which are called an upper and lower t-level cut of μ and can be used to the characterization of μ. An intuitionistic Q-fuzzy set (IQFS for short) A is an object having the form $A = \{((x, q), \mu_A(x, q), \lambda_A(x, q)) \mid x \in X, q \in Q\}$, where the functions $\mu_A : X \to [0, 1]$ and $\lambda_A : X \times Q \to [0, 1]$ denote the degree of membership (namely, $\mu_A(x, q)$) and the degree of nonmembership (namely, $\lambda_A(x, q)$) of each element $(x, q) \in X \times Q$ to the set A, respectively, and $0 \leq \mu_A(x, q) + \lambda_A(x, q) \leq 1$ for all $x \in X$ and $q \in Q$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the IQFS $A = \{(x, \mu_A(x, q), \lambda_A(x, q)) \mid x \in X, q \in Q\}$.

Definition 2.2 . *A Q-fuzzy set μ is called a fuzzy R-subnear-ring of R over Q (shortly, Q-fuzzy R-subnear-ring of R) if*

$(QF1)$ $\mu(x - y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$,

$(QF2)$ $\mu(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$

for all $x, y \in R$ and $q \in Q$.

Definition 2.3 . *A Q-fuzzy set μ is called a fuzzy R-subgroup of R over Q (shortly, Q-fuzzy R-subgroup of R) if μ satisfies (QF1) and*

$(QFR3)$ $\mu(rx, q) \geq \mu_A(x, q)$,

$(QFR4)$ $\mu(xr, q) \geq \mu_A(x, q)$

for all $x, r \in R$ and $q \in Q$.

Definition 2.4 (16) . *Let θ be a mapping from X to Y. If $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are intuitionistic Q-fuzzy sets in X and Y,respectively, then the inverse image of B under θ denoted by $\theta^{-1}(B)$, is an intuitionistic Q-fuzzy set in X defined by $\theta^{-1}(B) = (\mu_{\theta^{-1}(B)}, (\lambda_{\theta^{-1}(B)})$, where $\mu_{\theta^{-1}(B)}(x, q) = \mu_B(\theta(x), q)$ and $\lambda_{\theta^{-1}(B)}(x, q) = \lambda_B(\theta(x), q)$ for all $x \in X, q \in Q$, and the image of A under f denoted by $\theta(A) = (\mu_{\theta(A)}, \lambda_{\theta(A)})$ where*

$$\mu_{\theta(A)}(y, q) = \begin{cases} \bigvee_{x \in \theta^{-1}(y)} \mu_A(x, q) & \text{if} \quad \theta^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}$$

$$\lambda_{\theta(A)}(y, q) = \begin{cases} \bigwedge_{x \in \theta^{-1}(y)} \lambda_A(x, q) & \text{if} \quad \theta^{-1}(y) \neq \emptyset, \\ 1, & \text{otherwise.} \end{cases}$$

for all $y \in Y, q \in Q$.

3 Intuitionistic Q-fuzzy R-subgroups of near-rings

In what follows, let Q and R denote a set and a near-ring, respectively, unless otherwise specified.

Definition 3.1 . *An IQFS $A = (\mu_A, \lambda_A)$ in R is called an intuitionistic Q-fuzzy subnear-ring of R if*

$(IQF1)$ $\mu_A(x - y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$ *and* $\lambda_A(x - y, q) \leq \lambda_A(x, q) \vee \lambda_A(y, q)$,

$(IQF2)$ $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$ *and* $\lambda_A(xy, q) \leq \lambda_A(x, q) \vee \lambda_A(y, q)$

for all $x, y \in R$ and $q \in Q$.

Definition 3.2 . *An IQFS $A = (\mu_A, \lambda_A)$ in R is called an intuitionistic Q-fuzzy R-subgroup of R if A satisfies (IQF1) and*

$(IQF3)$ $\mu_A(rx, q) \geq \mu_A(x, q)$ *and* $\lambda_A(rx, q) \leq \lambda_A(x, q)$,

$(IQF4)$ $\mu_A(xr, q) \geq \mu_A(x, q)$ *and* $\lambda_A(xr, q) \leq \lambda_A(x, q)$

for all $x, r \in R$ and $q \in Q$.

If $A = (\mu_A, \lambda_A)$ satisfies (IQF1) and (IQF3), then A is called an Q-fuzzy left R-subgroup of R, and if $A = (\mu_A, \lambda_A)$ satisfies (IQF1) and (IQF4), then A is called an Q-fuzzy right R-subgroup of R.

Example 1. Let $R = \{a, b, c, d\}$ be a set with two binary operations as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then $(R, +, \cdot)$ is a near ring. We define an IFQS $A = (\mu_A, \lambda_A)$ in R as follows: for every $q \in Q$,

$$\mu_A(a, q) = 1, \mu(b, q) = \frac{1}{3}, \mu(c, q) = 0 = \mu(d, q),$$

$$\lambda_A(a, q) = 0, \lambda_A(b, q) = \frac{1}{3}, \lambda_A(c, q) = 1 = \lambda_A(d, q).$$

By routine calculation, we can check that $A = (\mu_A, \lambda_A)$ is both an intuitionistic Q-fuzzy subnear-ring and an intuitionistic Q-fuzzy R-subgroup of R.

Example 2. Let $R = \{a, b, c, d\}$ be a set with two binary operations as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	b	c	d

Then $(R, +, \cdot)$ is a near ring. Let $Q = \{1, 2, 3, 4\}$ and let $A = (\mu_A, \lambda_A)$ be an intuitionistic Q-fuzzy set in R defined by

$$\begin{aligned}
\mu_A(a, 0) &= \mu_A(a, 1) = \mu_A(a, 2) = \mu_A(a, 3) = \mu_A(a, 4) = 1, \\
\mu_A(b, 0) &= \mu_A(b, 1) = \mu_A(b, 2) = \mu_A(b, 3) = \mu_A(b, 4) = \frac{2}{3}, \\
\mu_A(c, 0) &= \mu_A(c, 1) = \mu_A(c, 2) = \mu_A(c, 3) = \mu_A(c, 4) = \frac{1}{3}, \\
\mu_A(d, 0) &= \mu_A(d, 1) = \mu_A(d, 2) = \mu_A(d, 3) = \mu_A(d, 4) = \frac{1}{3},
\end{aligned}$$

and

$$\begin{aligned}
\lambda_A(a, 0) &= \lambda_A(a, 1) = \lambda_A(a, 2) = \lambda_A(a, 3) = \lambda_A(a, 4) = 0, \\
\lambda_A(b, 0) &= \lambda_A(b, 1) = \lambda_A(b, 2) = \lambda_A(b, 3) = \lambda_A(b, 4) = \frac{1}{4}, \\
\lambda_A(c, 0) &= \lambda_A(c, 1) = \lambda_A(c, 2) = \lambda_A(c, 3) = \lambda_A(d, 4) = \frac{1}{2}, \\
\lambda_A(d, 0) &= \lambda_A(d, 1) = \lambda_A(d, 2) = \lambda_A(d, 3) = \lambda_A(d, 4) = \frac{1}{2}.
\end{aligned}$$

We can check that $A = (\mu_A, \lambda_A)$ is both intuitionistic Q-fuzzy subnear-ring and intuitionistic Q-fuzzy R-subgroup of R.

Lemma 3.3 . *If an IQFS $A = (\mu_A, \lambda_A)$ in R satisfies the condition (IQF1), then*

(i) $\mu_A(0, q) \geq \mu_A(x, q)$ *and* $\lambda_A(0, q) \leq \lambda_A(x, q)$,

(ii) $\mu_A(-x, q) = \mu_A(x, q)$ *and* $\lambda_A(-x, q) = \lambda_A(x, q)$

for all $x \in R$ *and* $q \in Q$.

Proposition 3.1 . *If an IQFS $A = (\mu_A, \lambda_A)$ in R satisfies the condition (IQF1), then*

(i) $\mu_A(x - y, q) \geq \mu_A(0, q)$ *implies* $\mu_A(x, q) = \mu_A(y, q)$,

(ii) $\lambda_A(x - y, q) \leq \lambda_A(0, q)$ *implies* $\lambda_A(x, q) = \lambda_A(y, q)$

for all $x, y \in R$ *and* $q \in Q$.

Theorem 3.4 . *If $\{A_i\}_{i\in\Lambda}$ is a family of intuitionistic Q-fuzzy R-subgroups (subnear-rings) of R. Then $\cap_{i\in\Lambda}A_i$ is an intuitionistic Q- fuzzy R-subgroup (subnear-ring) of R, where* $\cap_{i\in\Lambda}A_i = \{(x, \wedge\mu_{A_i}(x, q), \vee\lambda_{A_i}(x, q)) \mid x \in X, q \in Q\}$.

Proof. Let $x, y \in R, q \in Q$. Then

$$(\cap_{i\in\Lambda}\mu_{A_i})(x - y, q) = \bigwedge_{i\in\Lambda} \mu_{A_i}(x - y, q) \geq \bigwedge_{i\in\Lambda}(\mu_{A_i}(x, q) \wedge \mu_{A_i}(y, q))$$

$$= (\bigwedge_{i\in\Lambda} \mu_{A_i}(x, q)) \wedge (\bigwedge_{i\in\Lambda} \mu_{A_i}(y, q)) = (\cap_{i\in\Lambda}\mu_{A_i})(x, q) \wedge (\cap_{i\in\Lambda}\mu_{A_i})(y, q).$$

and

$$(\cup_{i\in\Lambda}\lambda_{A_i})(x - y, q) = \bigvee_{i\in\Lambda} \lambda_{A_i}(x - y, q) \leq \bigvee_{i\in\Lambda}(\lambda_{A_i}(x, q) \vee \lambda_{A_i}(y, q))$$

$$= (\bigvee_{i\in\Lambda} \lambda_{A_i}(x, q)) \vee (\bigvee_{i\in\Lambda} \lambda_{A_i}(y, q)) = (\cup_{i\in\Lambda}\lambda_{A_i})(x, q) \vee (\cup_{i\in\Lambda}\lambda_{A_i})(y, q).$$

Let $x, r \in R, q \in Q$. Then

$$(\cap_{i\in\Lambda}\mu_{A_i})(xr, q) = \bigwedge_{i\in\Lambda} \mu_{A_i}(xr, q) \geq \bigwedge_{i\in\Lambda} \mu_{A_i}(x, q) = (\cap_{i\in\Lambda}\mu_{A_i})(x, q).$$

and

$$(\cup_{i\in\Lambda}\lambda_{A_i})(xr, q) = \bigvee_{i\in\Lambda} \lambda_{A_i}(xr, q) \leq \bigvee_{i\in\Lambda} \lambda_{A_i}(x, q) = (\cup_{i\in\Lambda}\lambda_{A_i})(x, q).$$

Similarly, we get $(\cap_{i\in\Lambda}\mu_{A_i})(rx, q) \geq (\cap_{i\in\Lambda}\mu_{A_i})(x, q)$ and $(\cup_{i\in\Lambda}\lambda_{A_i})(rx, q) \leq (\cup_{i\in\Lambda}\lambda_{A_i})(x, q)$.

Hence $\cap_{i\in\Lambda}A_i$ is an intuitionistic Q-fuzzy R-subgroup of R. We can prove intuitionistic Q-fuzzy subnear-ring similarly. So we omit the proof.

Lemma 3.5 . *If $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup (resp. subnear-ring) of R, then so is $\circ A = (\mu_A, \mu_A^c)$.*

Proof. It is sufficient to show that μ_A^c satisfies the conditions (IQF1), (IQF3) and (IFQ4) of Definition 3.1 and 3.2. For any $x, y \in R, q \in Q$, we have

$$\begin{aligned}\mu_A^c(x-y,q) &= 1-\mu_A(x-y,q) \leq 1-(\mu_A(x,q)\wedge\mu_A(y,q)) \\ &= (1-\mu_A(x,q))\vee(1-\mu_A(y,q)) = \mu_A^c(x,q)\vee\mu_A^c(y,q)\end{aligned}$$

Hence the condition (IQF1) of Definition 3.1 is valid. For any $x, r \in R$, $q \in Q$, we have

$$\mu_A^c(rx,q) = 1-\mu_A(rx,q) \leq 1-(\mu_A(x,q)) = \mu_A^c(x,q).$$

Hence the condition (IQF3) of Definition 3.2 is valid.

$$\mu_A^c(xr,q) = 1-\mu_A(xr,q) \leq 1-(\mu_A(x,q)) = \mu_A^c(x,q).$$

Hence the condition (IQF4) of Definition 3.2 is valid.

Therefore $\circ A$ is an intuitionistic Q-fuzzy R-subgroup of R. We can prove intuitionistic Q-fuzzy subnear-ring similarly. So we omit the proof.

Lemma 3.6 . *If $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R, then so is $\bullet A = (\lambda_A^c, \lambda_A)$.*

Proof. The proof is similar to the proof of Lemma 3.5. Combining the above two Lemmas it is not difficult to verify that the following Theorem is valid.

Theorem 3.7 . *$A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R if and only if $\circ A$ and $\bullet A$ are intuitionistic Q-fuzzy R-subgroups (subnear-rings) of R.*

Corollary 3.8 . *$A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R if and only if μ_A and λ_A^c are Q-fuzzy R-subgroups (subnear-rings) of R.*

Theorem 3.9 . *If an IQFS $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R. Then the sets*

$$R_{\mu_A} = \{x \in M \mid \mu_A(x,q) = \mu_A(0,q)\} \quad \textit{and} \quad R_{\lambda_A} = \{x \in M \mid \lambda_A(x,q) = \lambda_A(0,q)\}$$

are R-subgroups (subnear-rings) of R for all $q \in Q$.

Proof. Let $x, y \in R_{\mu_A}$ and $q \in Q$. Then $\mu_A(x,q) = \mu_A(0,q)$, $\mu_A(y,q) = \mu_A(0,q)$. Since $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup of R, we get $\mu_A(x-y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q) = \mu_A(0,q)$. By using Proposition 3.1 we get $\mu_A(x-y,q) = \mu_A(0,q)$. Hence $x - y \in R_{\mu_A}$.

For every $r \in R$, $x \in R_{\mu_A}$, we have $\mu_A(rx,q) \geq \mu_A(x,q) = \mu_A(0,q)$. By using Proposition 3.1 we get $\mu_A(rx,q) = \mu_A(0,q)$. Hence $rx \in R_{\mu_A}$. Similarly $xr \in R_{\mu_A}$. Therefore R_{μ_A} is a R-subgroup of R. We can prove subnear-ring similarly. So we omit the proof. Similarly R_{λ_A} is a R-subgroup (subnear-ring) of R.

Theorem 3.10 . *If $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R, then the sets $U(\mu_A; t)$ and $L(\lambda_A; t)$ are R-subgroups (subnear-rings) of R for all $q \in Q$, $t \in Im(\mu_A) \cap Im(\lambda_A)$.*

Proof. Let $t \in Im(\mu_A) \cap Im(\lambda_A) \subseteq [0,1]$ and let $x, y \in U(\mu_A; t)$ and $q \in Q$. Then $\mu_A(x,q) \geq t$, $\mu_A(y,q) \geq t$. Since $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroups of R, we have $\mu_A(x-y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q) \geq t$. Hence $x - y \in U(\mu_A; t)$. Let $x \in U(\mu_A; t)$ and $q \in Q$. Then $\mu_A(x,q) \geq t$. Since $A = (\mu_A, \lambda_A)$ is an an intuitionistic Q-fuzzy R-subgroup of R, we have $\mu_A(rx,q) \geq \mu_A(x,q) \geq t$ for all $r \in R$, which implies that $rx \in U(\mu_A; t)$. Similarly $xr \in R_{\mu_A}$. Therefore $U(\mu_A; t)$ is a R-subgroup of R. We can prove subnear-ring similarly. So we omit the proof. Similarly $L(\lambda_A; t)$ is a R-subgroup (subnear-ring) of R.

Theorem 3.11 . *If $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy set in R such that all non-empty level sets $U(\mu_A; t)$ and $L(\lambda_A; t)$ are R-subgroups (subnear-rings) of R, then $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R.*

Proof. Assume that all non-empty level sets $U(\mu_A; t)$ and $L(\lambda_A; t)$ are R-subgroups of R. If $t_0 = \mu_A(x,q) \wedge \mu_A(y,q)$ and $t_1 = \lambda_A(x,q) \vee \lambda_A(y,q)$ for $x, y \in R, q \in Q$, then $x, y \in U(\mu_A; t_0)$ and $x, y \in L(\lambda_A; t_1)$. So $x-y \in U(\mu_A; t_0)$ and $x - y \in L(\lambda_A; t_1)$. Hence $\mu_A(x-y,q) \geq t_0 = \mu_A(x,q) \wedge \mu_A(y,q)$ and $\lambda_A(x-y,q) \leq t_1 = \lambda_A(x,q) \vee \lambda_A(y,q)$ which implies that the condition (IQF1) is valid. Now $t_2 = \mu_A(x,q)$ and $t_3 = \lambda_A(x,q)$ for some $x \in R, q \in Q$, then $x \in U(\mu_A; t_2)$ and $x \in L(\lambda_A; t_3)$. Since $U(\mu_A; t_2)$, $L(\lambda_A; t_3)$ are R-subgroups of R, we get $rx \in U(\mu_A; t_2)$ and $rx \in L(\lambda_A; t_3)$ for all $r \in R$. Therefore $\mu_A(rx) \geq t_2 = \mu_A(x,q)$ and $\lambda_A(rx) \leq t_3 = \lambda_A(x,q)$ which verify the condition (IQF2). Similarly the condition (IQF3) is valid. Therefore $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup of R. We can prove subnear-ring similarly. So we omit the proof.

Corollary 3.12 . *Let I be a R-subgroup (subnear-ring) of R. If Q-fuzzy sets μ_A, λ_A in R are defined by*

$$\mu_A(x,q) = \begin{cases} p & \text{if} \quad x \in I, \\ s & \text{otherwise.} \end{cases} \qquad \lambda_A(x,q) = \begin{cases} u & \text{if} \quad x \in I, \\ v & \text{otherwise.} \end{cases}$$

for all $x \in R, q \in Q$ where $0 \le s < p$, $0 \le u < v$ and $p + u \le 1$, $s + v \le 1$, then $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R and $U(\mu; p) = I = L(\lambda; u)$.

Proof. Let $x, y \in R, q \in Q$. If at least one of x and y does not belong to I, then $\mu_A(x-y,q) \ge s = \mu_A(x,q) \wedge \mu_A(y,q)$, $\lambda_A(x-y,q) \le v = \lambda_A(x,q) \vee \lambda_A(y,q)$. If $x, y \in I$, then $x-y \in I$ and so $\mu_A(x-y,q) = p = \mu_A(x,q) \wedge \mu_A(y,q)$, $\lambda_A(x-y,q) = v = \lambda_A(x,q) \vee \lambda_A(y,q)$.

If $x \in I$, then $rx, xr \in I$ for all $r \in R$ and so $\mu_A(rx) = \mu_A(xr) = p = \mu_A(x,q)$, $\lambda_A(rx) = \lambda_A(xr) = u = \lambda_A(x,q)$. If $x \notin I$, then $\mu_A(rx) = \mu_A(xr) = s = \mu_A(x,q)$ and $\lambda_A(rx) = \lambda_A(xr) = v = \lambda_A(x,q)$ for all $r \in R, q \in Q$.

Therefore $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy R-subgroup of R. We can prove subnear-ring similarly. So we omit the proof. Obviously $U(\mu; p) = I = L(\lambda; u)$.

Corollary 3.13 . *Let χ_I be the characteristic function of a R-subgroup (subnear-ring) I of a near-ring R. Then $\tilde{I} = (\chi_I, \chi_I^c)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R.*

Theorem 3.14 . *Let R and S be two near-rings and $\theta : R \to S$ a homomorphism. If $B = (\mu_B, \lambda_B)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of S, then the preimage $\theta^{-1}(B) = (\mu_{\theta^{-1}(B)}, \lambda_{\theta^{-1}(B)})$ of B under θ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R.*

Proof. Assume that $B = (\mu_B, \lambda_B)$ is an intuitionistic Q-fuzzy R-subgroup of S and let $x, y, r \in R, q \in Q$. Then

$$\begin{aligned} \mu_{\theta^{-1}(B)}(x-y,q) &= \mu_B(\theta(x-y),q) = \mu_B(\theta(x)-\theta(y),q) \\ &\ge \mu_B(\theta(x),q) \wedge \mu_B(\theta(y),q) = \mu_{\theta^{-1}(B)}(x,q) \wedge \mu_{\theta^{-1}(B)}(y,q). \end{aligned}$$

and

$$\begin{aligned} \lambda_{\theta^{-1}(B)}(x-y,q) &= \lambda_B(\theta(x-y),q) = \lambda_B(\theta(x)-\theta(y),q) \\ &\le \lambda_B(\theta(x),q) \vee \lambda_B(\theta(y),q) = \lambda_{\theta^{-1}(B)}(x,q) \vee \lambda_{\theta^{-1}(B)}(y,q). \end{aligned}$$

$$\begin{aligned} \mu_{\theta^{-1}(B)}(rx,q) &= \mu_B(\theta(rx),q) = \mu_B(\theta(r)\theta(x),q) \\ &\ge \mu_B(\theta(x),q) = \mu_{\theta^{-1}(B)}(x,q). \end{aligned}$$

and

$$\begin{aligned}\lambda_{\theta^{-1}(B)}(rx,q) &= \lambda_B(\theta(rx),q)=\lambda_B(\theta(r)\theta(x),q)\\ &\leq \lambda_B(\theta(x),q)=\lambda_{\theta^{-1}(B)}(x,q).\end{aligned}$$

Similarly $\mu_{\theta^{-1}(B)}(xr,q) \geq \mu_{\theta^{-1}(B)}(x,q)$ and $\lambda_{\theta^{-1}(B)}(xr,q) \leq \lambda_{\theta^{-1}(B)}(x,q)$. Therefore $\theta^{-1}(B) = (\mu_{\theta^{-1}(B)}, \lambda_{\theta^{-1}(B)})$ is an intuitionistic Q-fuzzy R-subgroup of R. We can prove subnear-ring similarly. So we omit the proof.

If we strengthen the condition of θ, then we can construct the converse of Theorem 3.14 as follows.

Theorem 3.15 . *Let $\theta : R \to S$ be an epimorphism and let $B = (\mu_B, \lambda_B)$ is an intuitionistic Q-fuzzy set in S. If $\theta^{-1}(B) = (\mu_{\theta^{-1}(B)}, \lambda_{\theta^{-1}(B)})$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of R, then $B = (\mu_B, \lambda_B)$ is an intuitionistic Q-fuzzy R-subgroup (subnear-ring) of S.*

Proof. Let $x, y, s \in S, q \in Q$. Then there exist $a, b, r \in R$ such that $\theta(a) = x$, $\theta(b) = y$, $\theta(r) = s$. It follows that

$$\begin{aligned}\mu_B(x-y,q) &= \mu_B(\theta(a)-\theta(b),q)=\mu_B(\theta(a-b),q)\\ &= \mu_{\theta^{-1}(B)}(a-b,q)\geq \mu_{\theta^{-1}(B)}(a,q)\wedge\mu_{\theta^{-1}(B)}(b,q)\\ &= \mu_B(\theta(a),q)\wedge\mu_B(\theta(b),q)=\mu_B(x,q)\wedge\mu_B(y,q).\end{aligned}$$

and

$$\begin{aligned}\lambda_B(x-y,q) &= \lambda_B(\theta(a)-\theta(b),q)=\lambda_B(\theta(a-b),q)\\ &= \lambda_{\theta^{-1}(B)}(a-b,q)\leq \lambda_{\theta^{-1}(B)}(a,q)\vee\lambda_{\theta^{-1}(B)}(b,q)\\ &= \lambda_B(\theta(a),q)\vee\lambda_B(\theta(b),q)=\lambda_B(x,q)\vee\lambda_B(y,q).\end{aligned}$$

$$\begin{aligned}\mu_B(rx,q) &= \mu_B(\theta(c)\theta(a),q)=\mu_B(\theta(ca),q)=\mu_{\theta^{-1}(B)}(ca,q)\\ &\geq \mu_{\theta^{-1}(B)}(a,q)=\mu_B(\theta(a),q)=\mu_B(x,q).\end{aligned}$$

and

$$\begin{aligned}\lambda_B(sx,q) &= \lambda_B(\theta(r)\theta(a),q)=\lambda_B(\theta(ra),q)=\lambda_{\theta^{-1}(B)}(ra,q)\\ &\leq \lambda_{\theta^{-1}(B)}(a,q)=\lambda_B(\theta(a),q)=\lambda_B(x,q).\end{aligned}$$

Similarly $\mu_B(xs,q) \geq \mu_B(x,q)$ and $\lambda_B(sx,q) \leq \lambda_B(x,q)$. Therefore $B = (\mu_B, \lambda_B)$ is an intuitionistic Q-fuzzy R-subgroup of S. We can prove subnear-ring similarly. So we omit the proof.

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