Synchronization in Arrays of Neural Networks with Distributed Delays and Coupling

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Abstract

In this paper, the global exponential synchronization in arrays of neural networks with distributed delays and coupling is investigated. Without assuming the inner coupling matrix to be diagonal, by employing a new Lyapunov-Krasovskii functional, applying the theory of Kronecker product of matrices and the linear matrix inequality (LMI) technique, a sufficient condition in LMI form is obtained for global exponential synchronization of such systems. Moreover, the exponential convergence index is estimated. The proposed LMI approach has the advantage of considering the difference of neuronal excitatory and inhibitory efforts, which is also computationally efficient as it can be solved numerically using efficient Matlab LMI toolbox, and no tuning of parameters is required. An example is given to show the effectiveness of the obtained results.

Keywords: Global exponential synchronization; coupled connected neural networks; distributed delays; Lyapunov-Krasovskii functional; linear matrix inequality

1 Introduction

Recently, arrays of coupled systems have attracted much attention of researchers in different research fields as they can exhibit some interesting phenomena [1]-[2]. Since chaos synchronization in an array of linearly coupled dynamical systems was investigated by Wu and Chua in [3], many results on local and global synchronization in various coupled systems have also been obtained, for example, see [4]-[11] and references therein. In [8], the authors have given some sufficient conditions for an array of coupled connected neural networks by defining a distance between any point and the synchronization
manifold. In [9], the authors have obtained a sufficient condition for global synchronization based on the Lyapunov functional methods and Hermitian matrix theory. The models considered in [8]-[9] are a simple uniform delayed neural networks without delayed coupling. However, in practice, there usually exist delays in spreading due to the finite speeds of transmission as well as traffic congestions. Therefore, it is important to investigate the synchronization of an arrays of delayed neural networks model with delayed coupling. In [11], the authors considered an arrays of delayed neural networks model with delayed coupling, and have given some sufficient conditions which are independent on the size of the time delays for the global exponential synchronization of such system. It is well known that neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, it is desired to model them by introducing continuously distributed delays over a certain duration of time such that the distant past has less influence compared to the recent behavior of the state [12]. To the best of our knowledge, few authors have considered an arrays of coupled connected neural networks model with distributed delays.

In this paper, we aim to tackle the global exponential synchronization of coupled connected neural networks with distributed delays.

2 Model description and preliminaries

In this paper, we consider the following model

\[
\frac{dx_i(t)}{dt} = -Cx_i(t) + Af(x_i(t)) + B \int_{t-\tau}^{t} f(x_i(s))ds + I(t) \\
+ \sum_{j=1}^{m} \alpha_{ij}Px_j(t) + \sum_{j=1}^{m} \beta_{ij}Q \int_{t-\tau}^{t} x_j(s)ds
\]

(1)

for \(i = 1, 2, \cdots, m\). Where \(x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in \mathbb{R}^n\) is the state vector of the \(i\)th network at time \(t\), \(n\) corresponds to the number of neurons. The scalar \(\tau > 0\) denotes the distributed time delay. \(C = \text{diag}(c_1, c_2, \cdots, c_n) > 0, c_k (k = 1, 2, \cdots, n)\) denotes the rate with which the \(k\)th neuron reset its potential to the resting state in isolation when disconnected from the network and external inputs. \(A = (a_{ij})_{n \times n}\) and \(B = (b_{ij})_{n \times n}\) represent the connection weight matrix, the delay connection weight matrix, respectively. \(f_k (k = 1, 2, \cdots, n)\) is the activation function, \(f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)), \cdots, f_n(x_{in}(t)))^T\). \(I(t) = (I_1(t), I_2(t), \cdots, I_n(t))^T \in \mathbb{R}^n\) is an external input vector. \(\alpha = (\alpha_{ij})_{m \times m}, \beta = (\beta_{ij})_{m \times m}\) are the coupling configuration matrices representing the coupling strength and topological structures of the networks satisfy the
diffusive coupling connection, and satisfy the following conditions [11]:

\[
\begin{align*}
\alpha_{ij} &= \alpha_{ji} \geq 0(i \neq j), \quad \alpha_{ii} = -\sum_{j=1, j \neq i}^{m} \alpha_{ij}, \quad i, j = 1, 2, \cdots m, \quad (2) \\
\beta_{ij} &= \beta_{ji} \geq 0(i \neq j), \quad \beta_{ii} = -\sum_{j=1, j \neq i}^{m} \beta_{ij}, \quad i, j = 1, 2, \cdots m, \quad (3)
\end{align*}
\]

\(P = (p_{ij})_{n \times n}\) and \(Q = (q_{ij})_{n \times n}\) are constant matrices, which describe the individual coupling between two systems.

The initial condition associated with (1) is

\[x_{ij}(s) = \varphi_{ij}(s) \in C([-\tau, 0], R), \quad i = 1, 2, \cdots m, j = 1, 2, \cdots n.\]

Throughout this paper, we make the following assumption:

\textbf{(H)} There exists a positive constant \(F = diag(F_1, F_2, \cdots, F_n)\) \((k = 1, 2, \cdots, n)\) such that

\[|f_k(u_1) - f_k(u_2)| \leq F_k|u_1 - u_2|\]

for all \(u_1, u_2 \in R\).

For convenience, we introduce some notations. \(A > 0\) means that \(A\) is a positive definite symmetric matrix. \(A < 0\) means that \(A\) is a negative definite symmetric matrix. \(I_n\) denotes the \(n \times n\) identity matrix, \(E_n\) denotes the \(n \times n\) matrix whose all elements are 1. For a vector \(x = (x_1, x_2, \cdots, x_n)^T \in R^n\), \(\|x\|\) is the Euclidean norm, i.e., \(\|x\| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}\). For matrix \(A = (a_{ij})_{m \times n} \in R^{m \times n}\) and matrix \(B = (b_{ij})_{p \times q} \in R^{p \times q}\), \(A \otimes B\) denotes the Kronecker product of \(A\) and \(B\), i.e., \(A \otimes B = (a_{ij}B)_{(m \times p) \times (n \times q)}\).

**Definition 1.** [8, 11] Model (1) is said to be globally exponentially synchronized, if there exist two constants \(\varepsilon > 0\) and \(M > 0\), such that for all \(\varphi_i(s) = (\varphi_{i1}(s), \varphi_{i2}(s), \cdots, \varphi_{im}(s))^T \in C([-\tau, 0], R^n)\), and sufficient large \(T > 0\), we have

\[\|x_i(t) - x_j(t)\| \leq Me^{-\varepsilon t}\]

for all \(t > T\) and \(i, j = 1, 2, \cdots m\).

**Lemma 1.** [11] Let \(A \in R^{m \times n}\), \(B \in R^{p \times q}\), \(C \in R^{n \times r}\), \(D \in R^{q \times s}\), \(F \in R^{m \times n}\), \(a\) is a constant, then

\[(i)\quad (aA) \otimes B = A \otimes (aB) = a(A \otimes B),\]

\[(ii)\quad (A + F) \otimes B = A \otimes B + F \otimes B\]

\[(iii)\quad (A \otimes B)(C \otimes D) = (AC) \otimes (BD).\]
Lemma 3. [13] For any constant matrix $W \in \mathbb{R}^{m \times m}$, $W^T = W > 0$, scalar $r > 0$, vector function $\omega : [0, r] \to \mathbb{R}^{m \times m}$ such that the integrations concerned are well defined, then

$$r \int_0^r \omega^T(s)W\omega(s)ds \geq \left( \int_0^r \omega(s)ds \right)^T W \left( \int_0^r \omega(s)ds \right).$$

3 Main results

Theorem 1. Under assumptions (H1), model (1) is globally exponentially synchronized, if there exist five symmetric positive matrices $G_i > 0$ ($i = 1, 2, 3$), a positive diagonal matrix $S = \text{diag}(s_1, s_2, \cdots, s_n)$ and a constant $\varepsilon_i \in (0, 1)$ such that the following LMI holds:

$$\Omega = \begin{pmatrix}
\Pi_1 & G_1A & -m\beta_{ij}G_1 & G_1B \\
A^TG_1 & \tau G_2 - S & 0 & 0 \\
-m\beta_{ij}Q^TG_1 & 0 & -\frac{1-\varepsilon_i}{\tau}G_3 & 0 \\
B^TG_1 & 0 & 0 & -\frac{1-\varepsilon_i}{\tau}G_2
\end{pmatrix} < 0 \quad (4)$$

for $1 \leq i < j \leq m$, where $\Pi_1 = -G_1C - CG_1 - m\alpha_{ij}(G_1P + P^TG_1) + \tau G_3 + SF^2$.

Proof. Let $x(t) = (x_1^T(t), x_2^T(t), \cdots, x_m^T(t))^T$, $\tilde{f}(x(t)) = (f^T(x_1(t)), f^T(x_2(t)), \cdots, f^T(x_n(t)))^T$, $J(t) = (I^T(t), I^T(t), \cdots, I^T(t))^T$, then model (1) can be rewritten as

$$\frac{dx(t)}{dt} = -(I_m \otimes C)x(t) + (I_m \otimes A)\tilde{f}(x(t)) + (I_m \otimes B) \int_{t-\tau}^t \tilde{f}(x(s))ds$$

$$+ J(t) + (\alpha \otimes P)x(t) + (\beta \otimes Q) \int_{t-\tau}^t x(s)ds. \quad (5)$$

Let $U = mI_m - E_m$. Considering Lyapunov-Krasovskii functional candidate for model (5) as

$$V(t) = e^{zt} \sum_{j=1}^3 V_j(t),$$

where

$$V_1(t) = x^T(t)(U \otimes G_1)x(t), \quad (6)$$

$$V_2(t) = \int_0^t \int_{t-s}^t \tilde{f}^T(x(\xi))(U \otimes G_3)\tilde{f}(x(\xi))d\xi ds, \quad (7)$$

$$V_3(t) = \int_0^t \int_{t-s}^t x^T(\xi)(U \otimes G_5)x(\xi)d\xi ds, \quad (8)$$
we first calculate the time derivative of $V_1(t)$ along the trajectories of (5), using Lemma 1 and Lemma 2, and noting (2), (3) and $(U \otimes G_1)f(t) = 0$, we obtain
\[
\frac{dV_1(t)}{dt} = 2x^T(t)(U \otimes G_1)\left[-(I_m \otimes C)x(t) + (I_m \otimes A)f(x(t)) + (I_m \otimes B)\int_{t-\tau}^{t} \tilde{f}(x(s))ds + J(t) + (\alpha \otimes P)x(t) + (\beta \otimes Q)\int_{t-\tau}^{t} x(s)ds\right]
= -2x^T(t)(U \otimes (G_1C))x(t) + 2x^T(t)(U \otimes (G_1A))\tilde{f}(x(t)) + 2x^T(t)(U \otimes (G_1B))\int_{t-\tau}^{t} \tilde{f}(x(s))ds + 2x^T(t)((\alpha \otimes (G_1P))x(t)
+ 2x^T(t)((\beta \otimes (G_1Q)))\int_{t-\tau}^{t} x(s)ds
= -2x^T(t)(U \otimes (G_1C))x(t) + 2x^T(t)(U \otimes (G_1A))\tilde{f}(x(t)) + 2x^T(t)(U \otimes (G_1B))\int_{t-\tau}^{t} \tilde{f}(x(s))ds + 2x^T(t)((m\alpha \otimes (G_1P))x(t)
+ 2x^T(t)((m\beta \otimes (G_1Q)))\int_{t-\tau}^{t} x(s)ds
= 2 \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} (x_i(t) - x_j(t))^T\left[(-G_1C - m\alpha \otimes G_1)f(x_i(t) - x_j(t)) + G_1A(f(x_i(t)) - f(x_j(t))) + G_1B \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))ds - m\beta \otimes G_1Q \int_{t-\tau}^{t} (x_i(s) - x_j(s))ds\right].
\tag{9}
\]
Calculate the time derivative of $V_2(t)$ along the trajectories of (5), and by Lemma 3, we get
\[
\frac{dV_2(t)}{dt} = \tau \tilde{f}^T(x(t))(U \otimes G_2)\tilde{f}(x(t)) - \int_{t-\tau}^{t} \tilde{f}^T(x(t) - s)(U \otimes G_2)\tilde{f}(x(t) - s)ds
= \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left[\tau(f(x_i(t)) - f(x_j(t)))^T G_2(f(x_i(t)) - f(x_j(t)))\right.
- \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))^T G_2(f(x_i(s)) - f(x_j(s)))ds\right]
= \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left[\tau(f(x_i(t)) - f(x_j(t)))^T G_2(f(x_i(t)) - f(x_j(t)))\right.
- \varepsilon_1 \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))^T G_2(f(x_i(s)) - f(x_j(s)))ds
- \left(1 - \varepsilon_1\right) \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))^T G_2(f(x_i(s)) - f(x_j(s)))ds\right]
\leq \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left[\tau(f(x_i(t)) - f(x_j(t)))^T G_2(f(x_i(t)) - f(x_j(t)))\right.$}
Similarly, calculate the time derivatives of $V_3(t)$ along the trajectories of (5), we have

$$\frac{dV_3(t)}{dt} \leq \sum_{j=1}^{m-1} \sum_{i=j+1}^{m} \left[ \tau(x_i(t) - x_j(t))^T G_3(x_i(t) - x_j(t)) 
- \varepsilon_1 \int_{t-\tau}^{t} (x_i(s) - x_j(s))^T G_3(x_i(s) - x_j(s))ds 
- \frac{1 - \varepsilon_1}{\tau} \left( \int_{t-\tau}^{t} (x_i(s) - x_j(s))ds \right)^T G_3 \int_{t-\tau}^{t} (x_i(s) - x_j(s))ds \right].$$

Let $y_{ij}(t) = \left( (x_i(t) - x_j(t))^T, (f(x_i(t)) - f(x_j(t)))^T, (f_{t-\tau}(x_i(s) - x_j(s))ds)^T \right)^T$, $\Pi_2 = -G_1C - CG_1 - m\alpha_{ij}(G_1P + PTG_1) + \tau G_3$ and

$$\Omega^* = \begin{pmatrix}
\Pi_2 & G_1A & -m\beta_{ij}G_1Q & G_1B \\
A^T G_1 & \tau G_2 & 0 & 0 \\
-m\beta_{ij}Q^TG_1 & 0 & -\frac{1 - \varepsilon_1}{\tau} G_3 & 0 \\
B^T G_1 & 0 & 0 & -\frac{1 - \varepsilon_1}{\tau} G_2
\end{pmatrix}$$

for $1 \leq i < j \leq m$. From (9), (10) and (11), we have

$$\frac{d(\sum_{j=1}^{3}V_i(t))}{dt} \leq \sum_{j=1}^{m-1} \sum_{i=j+1}^{m} \left[ y_{ij}^T(t)\Omega^* y_{ij}(t) - \varepsilon_1 \int_{t-\tau}^{t} (x_i(s) - x_j(s))^T G_3(x_i(s) - x_j(s))ds 
- \varepsilon_1 \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))^T G_2(f(x_i(s)) - f(x_j(s)))ds \right].$$

From assumption (H), we get

$$\left( (f_k(x_i(t)) - f_k(x_j(t)))(x_i(t) - x_j(t)) \right) \leq 0$$

for all $i, j = 1, 2, \ldots, m; k = 1, 2, \ldots, n$, which is equivalent to

$$\left( \begin{array}{c}
x_i(t) - x_j(t) \\
f(x_i(t)) - f(x_j(t))
\end{array} \right)^T \left( -F^2_k e_k e_k^T 0 0 \right) \left( \begin{array}{c}
x_i(t) - x_j(t) \\
f(x_i(t)) - f(x_j(t))
\end{array} \right) \leq 0$$

for all $i, j = 1, 2, \ldots, m; k = 1, 2, \ldots, n$, where $e_k$ denotes the unit column vector having 1 element on its $k$th row and zeros elsewhere. Hence

$$y_{ij}^T(t)\Omega^* y_{ij}(t) \leq y_{ij}^T(t)\Omega^* y_{ij}(t) - \sum_{k=1}^{n} s_k \left( \begin{array}{c}
x_i(t) - x_j(t) \\
f(x_i(t)) - f(x_j(t))
\end{array} \right)^T$$
\[ \frac{d}{dt} \left( \sum_{j=1}^{3} V_i(t) \right) \leq \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left[ y_{ij}^T(t)\Omega_1 y_{ij}(t) - \varepsilon_1 \int_{t-\tau}^{t} (x_i(s) - x_j(s))^T G_3(x_i(s) - x_j(s))ds \right. \\
\left. - \varepsilon_1 \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))^T G_2(f(x_i(s)) - f(x_j(s)))ds \right] \\
\leq \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left[ -\lambda_{\text{min}}(-\Omega_1)\|y_{ij}(t)\|^2 - \varepsilon_1 \int_{t-\tau}^{t} (x_i(s) - x_j(s))^T G_3(x_i(s) - x_j(s))ds \right. \\
\left. - \varepsilon_1 \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))^T G_2(f(x_i(s)) - f(x_j(s)))ds \right]. \tag{14} \]

From (12), (13) and \( \Omega_1 < 0 \), we get

\[ V_1(t) \leq \lambda_{\text{max}}(G_1) \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)), \tag{15} \]

\[ V_2(t) \leq \tau \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))^T G_2(f(x_i(s)) - f(x_j(s)))ds, \tag{16} \]

\[ V_3(t) \leq \tau \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \int_{t-\tau}^{t} (x_i(s) - x_j(s))^T G_3(x_i(s) - x_j(s))ds, \tag{17} \]

From (14) to (17), and noting that \( \|x_i(t) - x_j(t)\|^2 \leq \|y_{ij}(t)\|^2 \), we get

\[ \frac{dV(t)}{dt} = e^{st} \left[ \varepsilon \sum_{j=1}^{3} V_i(t) + \frac{d}{dt} \left( \sum_{j=1}^{3} V_i(t) \right) \right] \leq e^{st} \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left[ -\lambda_{\text{min}}(-\Omega_1) - \varepsilon \lambda_{\text{max}}(G_1) \|x_i(t) - x_j(t)\|^2 \right. \\
\left. - (\varepsilon_1 - \varepsilon \tau) \int_{t-\tau}^{t} (x_i(s) - x_j(s))^T G_3(x_i(s) - x_j(s))ds \right] \]
\[-(\varepsilon - \varepsilon \tau) \int_{t-\tau}^{t} (f(x_i(s)) - f(x_j(s)))^T G_3 (f(x_i(s)) - f(x_j(s))) ds \].

Taking
\[\varepsilon = \min \left\{ \frac{\lambda_{\min}(-\Omega_1)}{\lambda_{\max}(G_1)}, \varepsilon_1 \right\},\]
from (18), we get \(\frac{dV(t)}{dt} \leq 0\), which implies \(V(t) \leq V(0)\), namely, \(V(t)\) is a bounded function. Hence \(e^{\varepsilon t} x^T(t)(U \otimes G_1)x(t)\) is also bounded, it yields that
\[\lambda_{\min}(G_1) \| x_i(t) - x_j(t) \|^2 \leq \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} (x_i(t) - x_j(t))^T G_1 (x_i(t) - x_j(t)) = O(e^{-\varepsilon t})\]
for \(1 \leq i < j \leq m\). According to Definition 1, we can conclude that system (1) is globally exponentially synchronized and the exponential convergence index
\[\varepsilon = \min \left\{ \frac{\lambda_{\min}(-\Omega_1)}{\lambda_{\max}(G_1)}, \varepsilon_1 \right\}.\]
The proof is completed.

**Remark.** In \([8, 9]\), the inner coupling matrix was required to be diagonal. However, in this paper, the inner coupling matrices are not necessary to be diagonal. So our results improve and generalize the earlier works.

### 4 Example

**Example 1.** Consider the following model
\[
\frac{dx_i(t)}{dt} = -Cx_i(t) + Af(x_i(t)) + B \int_{t-\tau}^{t} f(x_i(s)) ds + I(t) + \sum_{j=1}^{3} \alpha_{ij} P x_j(t) + \sum_{j=1}^{3} \gamma_{ij} Q \int_{t-\tau}^{t} x_j(s) ds, \quad i = 1, 2, 3,
\]
where \(x_i(t) = (x_{i1}(t), x_{i2}(t))^T\) is the state vector of the networks, \(f(x_i) = (\frac{1}{2}(|x_{i1} + 1| + |x_{i1} - 1|, |x_{i2} + 1| - |x_{i2} - 1|)^T\) is the activation functions vector, the distributed time delay \(\tau = 0.3\), the external input vector \(I(t) = (2 \sin t, -3 \cos t)^T\), and the connection weight matrices are such matrices
\[
C = \begin{pmatrix} 19 & 0 \\ 0 & 27 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & -1 \\ 1.2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1.1 & 1 \\ 1.2 & 1.4 \end{pmatrix}, \quad P = \begin{pmatrix} 1 & 0 \\ 0 & 1.5 \end{pmatrix}, \quad Q = \begin{pmatrix} 1.3 & 0 \\ 0 & 1.2 \end{pmatrix}, \quad \alpha = \begin{pmatrix} -3 & 1 & 2 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}, \quad \beta = \begin{pmatrix} -1 & 0.8 & 0.2 \\ 0.8 & -2.1 & 1.3 \\ 0.2 & 1.3 & -1.5 \end{pmatrix}.
\]

Obviously, assumption (H) is satisfied, and \(F = \text{diag}(1, 2)\). Take \(\varepsilon_1 = 0.01\), by the Matlab LMI Control Toolbox, we find a solution to the LMI in (4) are as follows:
\[
G_1 = \begin{pmatrix} 0.3123 & -0.0012 \\ -0.0012 & 0.1321 \end{pmatrix}, \quad G_2 = \begin{pmatrix} 2.1721 & -0.0329 \\ -0.0329 & 3.556 \end{pmatrix}.
\]
\[ G_3 = \begin{pmatrix} 1.327 & 0.0174 \\ 0.0174 & 4.235 \end{pmatrix}, \quad S = \begin{pmatrix} 8.1245 & 0 \\ 0 & 8.1245 \end{pmatrix}. \]

Therefore, by Theorem 1, we know that model (19) is globally exponentially synchronized. Moreover, we can also get that the exponential convergence index \( \varepsilon = 0.0099 \).

## 5 Conclusions

In this paper, the global exponential synchronization of coupled connected neural networks with distributed delays have been discussed. A sufficient condition has been obtained for global exponential synchronization of such systems based on Lyapunov functional method and Kronecker product technique of matrices. Moreover, the exponential convergence index can be estimated. We do not assume the inner coupling matrices to be diagonal. The developed synchronization condition is in terms of LMIs, which can be checked easily by recently developed algorithms solving LMIs. The main significance of this paper is that we introduce the distributed time delay coupling terms, thus the results greatly extend and improve the earlier results. Finally, an example is provided to demonstrate the effectiveness of the obtained results.

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## References


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