On Anti Fuzzy Left h -ideals in Hemirings

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Abstract

In this paper we apply the Biswas's idea of anti fuzzy subgroups to left h -ideals of hemirings. We introduce the notion of anti fuzzy left h -ideals in hemirings and investigate some of related properties. Relationship between fuzzy left h -ideals and anti fuzzy left h -ideals of hemirings is also given.

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1 Introduction

Ideals of semirings play a central role in the structure theory and are useful for many purposes. However, they do not in general coincide with the usual ring ideals if R is a ring and, for this reason, their use is somewhat limited in trying to obtain analogues of ring theorems for semirings. Indeed, many results in rings apparently have no analogues in semirings using only ideals. Henriksen defined in [2] a more restricted class of ideals in semirings, which is called the class of k -ideals, with the property that if the semiring R is a ring

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then a complex in R is a k -ideal if and only if it is a ring ideal. Another more restricted class of ideals has been given in hemirings by Iizuka [1]. However, in an additively commutative semiring R , ideals of a semiring coincide with "ideals" of a ring, provided that the semiring is a hemiring. We now call this ideal an h -ideal of the hemiring R . The properties of h -ideals and also k -ideals of hemirings were thoroughly investigated by La Torre in [3] and by using the h -ideals and k -ideals, La Torre established some analogous ring theorems for hemirings. There have been many applications of semirings and hemirings in different areas, such as social networks analysis [7], theory of automata and formal languages [5], idempotent analysis, discrete dynamical systems, fuzzy set theory, optimization [10].

The notion of fuzzy sets was first introduced by Zadeh [15]. Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. There have been wide-ranging applications of the theory of fuzzy sets, from the design of robots and computer simulation to engineering and water resources planning. Rosenfeld [16] introduced the fuzzy sets in the realm of group theory. Since then many mathematicians have been involved in extending the concepts and results of abstract algebra to the broader framework of the fuzzy setting. The notion of fuzzy h -ideals in hemirings was introduced in [13]. In this paper we apply the Biswas's idea of anti fuzzy subgroups to left h -ideals of hemirings. We introduce the notion of anti fuzzy left h -ideals in hemirings, and investigate some of their properties. Relationship between fuzzy left h -ideals and anti fuzzy left h -ideals of hemirings is also given.

2 Preliminaries

In this section we review some elementary aspects that are necessary for this paper:

An algebra $(R; +, \cdot)$ is said to be a *semiring* if it satisfies the following conditions:

- $(R; +)$ is a semigroup,
- $(R; \cdot)$ is a semigroup,
- $(\forall a, b, c \in R) (a \cdot (b + c) = a \cdot b + a \cdot c \text{ and } (b + c) \cdot a = b \cdot a + c \cdot a)$.

A semiring $(R; +, \cdot)$ is called a *hemiring* if

(H1) '+' is commutative and

(H2) there exists an element $0 \in R$ such that 0 is the identity of $(R; +)$ and the zero element of $(R; \cdot)$. i.e., $0 \cdot a = a \cdot 0 = 0$, $\forall a \in R$.

A subset I of a semiring R is called a *left ideal* of R if I is closed under addition and $RI \subseteq I$. A left ideal of R is called a *left k -ideal* of R if $y, z \in I$ and $x \in R$, $x + y = z$ implies $x \in I$. A left h -ideal of a hemiring R is defined to be a left ideal A of R such that

$$(\forall x, z \in R)(\forall a, b \in A)(x + a + z = b + z \longrightarrow x \in A).$$

Right h -ideals are defined similarly. A mapping $f : R_1 \rightarrow R_2$ is said to be hemiring homomorphism of R_1 into R_2 if $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in R_1$. A mapping $\mu : X \rightarrow [0, 1]$, where X is an arbitrary nonempty set, is called a *fuzzy set* in X . For any fuzzy set μ in X and any $\alpha \in [0, 1]$ we define the set

$$L(\mu; \alpha) = \{x \in X \mid \mu(x) \leq \alpha\},$$

which is called *lower level cut* of μ . A fuzzy subset μ of a semiring R is said to be a *fuzzy left ideal* of R if

- $(\forall x, y \in R) (\mu(x + y) \geq \min\{\mu(x), \mu(y)\})$,
- $(\forall x, y \in R) (\mu(xy) \geq \mu(y))$.

Note that if μ is a fuzzy left ideal of a hemiring R , then $\mu(0) \geq \mu(x)$ for all $x \in R$. A fuzzy left ideal μ of a semiring R is called a *fuzzy left k -ideal* of R if for all $x, y, z \in R$, $x + y + z$ implies $\mu(x) \geq \min(\mu(y), \mu(z))$. A fuzzy left h -ideal of a hemiring R is defined to be a fuzzy left ideal μ of R such that

$$(\forall x, z, a, b \in R)(x + a + z = b + z \longrightarrow \mu(x) \geq \min(\mu(a), \mu(b))).$$

Fuzzy right h -ideals are defined similarly.

3 Anti fuzzy left h -ideals in hemirings

Definition 3.1. A fuzzy subset μ of a hemiring R is said to be an *anti fuzzy left h -ideal* of R if

- (a) $(\forall x, y \in R) (\mu(x + y) \leq \max(\mu(x), \mu(y)))$,
- (b) $(\forall x, y \in R) (\mu(xy) \leq \mu(y))$,
- (c) $(\forall x, z, a, b \in R)(x + a + z = b + z \longrightarrow \mu(x) \leq \max(\mu(a), \mu(b)))$.

Example 3.2. Let $R = \{0, 1, 2, 3, 4\}$ be a hemiring with zero multiplication and addition defined by the following table.

	0	1	2	3	4
0	0	1	2	3	4
1	1	1	4	4	4
2	2	4	4	4	4
3	3	4	4	4	4
4	4	4	4	4	4

We define a fuzzy set $\mu : R \rightarrow [0, 1]$ by letting $\mu(0) = t_1$ and $\mu(x) = t_2$ for all $x \neq 0$, $t_1 < t_2$. By routine computations, we can also easily check that μ is an anti fuzzy h -ideal of hemiring R .

Proposition 3.3. *Let R be a hemiring and μ be a fuzzy set in R . Then μ is an anti fuzzy h -ideal in R if and only μ^c is a fuzzy left h -ideal in R .*

Proof. Let μ be an anti fuzzy left h -ideal in R . For $x, y \in R$, we have

$$\begin{aligned}\mu^c(x+y) &= 1 - \mu(x+y) \geq 1 - \max(\mu(x), \mu(y)) \\ &= \min(1 - \mu(x), 1 - \mu(y)) = \min(\mu^c(x), \mu^c(y)),\end{aligned}$$

$$\mu^c(xy) = 1 - \mu(xy) \geq 1 - \mu(y) = \mu^c(y).$$

Let $x, z, a, b \in R$ be such that $x + a + z = b + z$. Then

$$\begin{aligned}\mu^c(x) &= 1 - \mu(x) \geq 1 - \max(\mu(a), \mu(b)) \\ &= \min(1 - \mu(a), 1 - \mu(b)) = \min(\mu^c(a), \mu^c(b)).\end{aligned}$$

Hence μ^c is a fuzzy left h -ideal of R .

Conversely, let μ^c be a fuzzy left h -ideal in R . For $x, y \in R$, we have

$$\begin{aligned}\mu(x+y) &= 1 - \mu^c(x+y) \leq 1 - \min(\mu^c(x), \mu^c(y)) \\ &= \max(\mu(x), \mu(y)),\end{aligned}$$

$$\mu(xy) = 1 - \mu^c(xy) \leq 1 - \mu^c(y) = \mu(y).$$

Let $x, z, a, b \in R$ be such that $x + a + z = b + z$. Then

$$\begin{aligned}\mu(x) &= 1 - \mu^c(x) \leq 1 - \min(\mu^c(a), \mu^c(b)) \\ &= \max(\mu(a), \mu(b)).\end{aligned}$$

Hence μ is an anti fuzzy left h -ideal of R . □

Theorem 3.4. *Let μ be an anti fuzzy left h -ideal in a hemiring R such that $L(\mu; \alpha)$ is a left h -ideal of R for each $\alpha \in \text{Im}(\mu)$, $\alpha \in [0, 1]$. Then μ is an anti fuzzy left h -ideal of R .*

Proof. Let $x, y \in R$ be such that $\mu(x) = \alpha_1$ and $\mu(y) = \alpha_2$. Then $x + y \in L(\mu; \alpha)$. Without loss of generality, we may assume that $\alpha_1 \geq \alpha_2$. It follows that $L(\mu; \alpha_2) \subseteq L(\mu; \alpha_1)$ so that $x \in L(\mu; \alpha_1)$ and $y \in L(\mu; \alpha_2)$. Since $L(\mu; \alpha_1)$ is a left h -ideal of R , we have $x + y \in L(\mu; \alpha_1)$. Thus

$$\mu(x + y) \leq \alpha_1 = \max(\mu(x), \mu(y)),$$

$$\mu(xy) \leq \alpha_1 = \mu(y).$$

Let $x, z, a, b \in R$ be such that $x + a + z = b + z$. Then $\mu(x) \leq \alpha_1 = \max\{\mu(a), \mu(b)\}$. This shows that μ is an anti fuzzy left h -ideal of R . $\square$

Corollary 3.5. *Let μ be an anti-fuzzy left h -ideal of R . Then μ is an anti-fuzzy left h -ideal of R if and only if $L(\mu; \alpha)$ is left h -ideal of R for every $\alpha \in [\mu(0), 1]$ with $\alpha \in [0, 1]$.*

Theorem 3.6. *Let μ be an anti fuzzy set in a hemiring R . Then two lower level subsets $L(\mu; t_1)$ and $L(\mu; t_2)$ ($t_1 < t_2$) are equal if and only if there is no $x \in R$ such that $t_1 < \mu(x) \leq t_2$.*

Proof. From definition of $L(\mu; \alpha)$, it follows that $L(\mu; t) = \mu^{-1}([\mu(0), t])$ for $t \in [0, 1]$.

Let $t_1, t_2 \in [0, 1]$ be such that $t_1 < t_2$. Then

$$\begin{aligned} L(\mu; t_1) &= L(\mu; t_2) \\ \Leftrightarrow \mu^{-1}([\mu(0); t_1]) &= \mu^{-1}([\mu(0); t_2]) \\ \Leftrightarrow \mu^{-1}(t_1, t_2) &= \emptyset \\ \Leftrightarrow \text{there is no } x \in R \text{ such that } t_1 < \mu(x) &\leq t_2. \end{aligned}$$

This complete the proof. $\square$

Definition 3.7. A left h -ideal A of hemiring R is said to be *characteristic* if $f(A) = A$, for all $f \in \text{Aut}(R)$, where $\text{Aut}(R)$ is the set of all automorphisms of R . Anti fuzzy left h -ideal μ of hemiring R is said to be *anti fuzzy characteristic* if $\mu^f(x) = \mu(x)$, for all $x \in R$ and $f \in \text{Aut}(R)$.

Lemma 3.8. *Let μ be an anti fuzzy left h -ideal of a hemiring R and let $x \in R$. Then $\mu(x) = s$ if and only if $x \in L(\mu; s)$ and $x \notin L(\mu; t)$, for all $s > t$.*

Proof. Straightforward. $\square$

Theorem 3.9. *Let μ be an anti fuzzy h -ideal of a hemiring R . Then each level left h -ideal of μ is characteristic if and only if μ is an anti fuzzy characteristic.*

Proof. Suppose that μ is anti fuzzy characteristic and let $s \in \text{Im}(\mu)$, $f \in \text{Aut}(R)$ and $x \in L(\mu; s)$. Then

$$\mu^f(x) = \mu(x) \leq s \Rightarrow \mu(f(x)) \leq s \Rightarrow f(x) \in L(\mu; s).$$

Thus

$$f(L(\mu; s)) \subseteq L(\mu; s).$$

Let $x \in L(\mu; s)$ and $y \in R$ such that $f(y) = x$. Then

$$\begin{aligned} \mu(y) &= \mu^f(y) = \mu(f(y)) = \mu(x) \leq s \\ \Rightarrow y &\in L(\mu; s) \text{ so that } x = f(y) \in L(\mu; s). \end{aligned}$$

Consequently, $L(\mu; s) \subseteq f(L(\mu; s))$. Hence $f(L(\mu; s)) = L(\mu; s)$, i.e., $L(\mu; s)$ is characteristic. Conversely, suppose that each level h -ideal of μ is characteristic and let $x \in R$, $f \in \text{Aut}(R)$ and $\mu(x) = s$. Then, by virtue of Lemma 3.8, $x \in L(\mu; s)$ and $x \notin L(\mu; t)$, for all $s > t$. It follows from the assumption that $f(x) \in f(L(\mu; s)) = L(\mu; s)$, so that $\mu^f(x) = \mu(f(x)) \leq s$. Let $t = \mu^f(x)$ and assume that $s > t$. Then $f(x) \in L(\mu; t) = f(L(\mu; t))$, which implies from the injectivity of f that $x \in L(\mu; t)$, a contradiction. Hence $\mu^f(x) = \mu(f(x)) = s = \mu(x)$ showing that μ is an anti fuzzy characteristic. $\square$

Theorem 3.10. *Let $f : R_1 \rightarrow R_2$ be an epimorphism of hemirings. If ν is an anti-fuzzy left h -ideal of R_2 and μ is the pre-image of ν under f . Then μ is an anti fuzzy left h -ideal of R_1 .*

Proof. For any $x, y \in R_1$, we have

$$\begin{aligned} \mu(x + y) &= \nu(f(x + y)) = \nu(f(x) + f(y)) \\ &\leq \max(\nu(f(x)), \nu(f(y))) = \max(\mu(x), \mu(y)), \end{aligned}$$

$$\mu(xy) = \nu(f(xy)) = \nu(f(x)f(y)) \leq \nu(f(y)) = \mu(y).$$

Let $x, z, a, b \in R_1$ be such that $x + a + z = b + z$. Then, we have

$$\mu(x) = \nu(f(x)) \leq \max(\nu(f(a)), \nu(f(b))) = \max(\mu(a), \mu(b)).$$

Hence μ is an anti fuzzy left h -ideal of R_1 . $\square$

Definition 3.11. Let R_1 and R_2 be two hemirings and f be a function of R_1 into R_2 . If μ is a fuzzy set in R_2 , then the preimage of μ under f is the fuzzy set in R_1 defined by

$$f^{-1}(\mu)(x) = \mu(f(x)) \quad \forall x \in R_1.$$

Theorem 3.12. *Let $f : R_1 \rightarrow R_2$ be an onto homomorphism of hemirings. If μ is an anti fuzzy left h -ideal of R_2 , then $f^{-1}(\mu)$ is an anti fuzzy left h -ideal of R_1 .*

Proof. Let $x_1, x_2 \in R_1$, then we have

$$\begin{aligned} f^{-1}(\mu)(x_1 + x_2) &= \mu(f(x_1) + f(x_2)) \leq \max(\mu(f(x_1)), \mu(f(x_2))) \\ &= \max(f^{-1}(\mu)(x_1), f^{-1}(\mu)(x_2)), \end{aligned}$$

$$\begin{aligned} f^{-1}(\mu)(x_1 x_2) &= \mu(f(x_1) f(x_2)) \leq \mu(f(x_2)) \\ &= f^{-1}(\mu)(x_2). \end{aligned}$$

Let $x, z, a, b \in R_1$ be such that $x + a + z = b + z$. Then, we have

$$\begin{aligned} f^{-1}(\mu)(x) &= \mu(f(x)) \leq \max(\mu(f(a)), \mu(f(b))) \\ &= \max(f^{-1}(\mu)(a), f^{-1}(\mu)(b)). \end{aligned}$$

Hence $f^{-1}(\mu)$ is an anti fuzzy left h -ideal of R_1 . □

Definition 3.13. Let R_1 and R_2 be any sets and let $f : R_1 \rightarrow R_2$ be any function. A fuzzy subset μ of R_1 is called f -invariant if $f(x) = f(y)$ implies $\mu(x) = \mu(y)$, $x, y \in R_1$.

Theorem 3.14. *Let $f : R_1 \rightarrow R_2$ be an epimorphism of hemirings. Let μ be an f -invariant anti fuzzy left h -ideal of R_1 . Then $f(\mu)$ is an anti fuzzy left h -ideal of R_2 .*

Proof. Let $x, y \in R_2$. Then there exist $a, b \in R_1$ such that $f(a) = x$ and $f(b) = y$, then $x + y = f(a + b)$ and $xy = f(ab)$. Since μ is f -invariant,

$$\begin{aligned} f(\mu)(x + y) &= \mu(x + y) \leq \max(\mu(a), \mu(b)) \\ &= \max(f(\mu)(x), f(\mu)(y)), \end{aligned}$$

$$f(\mu)(xy) = \mu(ab) \leq \mu(b) = f(\mu)(y).$$

Let $x, z, a, b \in R_2$ be such that $x + a + z = b + z$, then there exist $\acute{x}, \acute{z}, \acute{a}, \acute{b}$ such that $f(\acute{x}) = x, f(\acute{y}) = y, f(\acute{a}) = a$ and $f(\acute{b}) = b$. Since μ is f -invariant,

$$\begin{aligned} f(\mu)(x) &= \mu(x) \leq \max(\mu(\acute{a}), \mu(\acute{b})) \\ &= \max(f(\mu)(a), f(\mu)(b)). \end{aligned}$$

Hence $f(\mu)$ is an anti fuzzy left h -ideal of R_2 . □

Definition 3.15. A fuzzy left h -ideal μ of a hemiring R is said to be *normal* if there exist $x \in R$ such that $\mu(x) = 1$. Note that if μ is a normal anti fuzzy left h -ideal of R , then $\mu(0) = 1$, and hence μ is normal if and only if $\mu(0) = 1$.

Theorem 3.16. Let μ be an anti fuzzy left h ideal of a hemiring R . Let μ^+ be a fuzzy set in R defined by $\mu^+(x) = \mu(x) + 1 - \mu(0)$ for all $x \in R$. Then μ^+ is a normal anti fuzzy left h -ideal of R which contains μ .

Proof. For any $x, y \in R$, we have $\mu^+(x) = \mu(0) + 1 - \mu(0) = 1$ and

$$\begin{aligned}\mu^+(x+y) &= \mu(x+y) + 1 - \mu(0) \leq \max(\mu(x), \mu(y)) + 1 - \mu(0) \\ &= \max(\mu(x) + 1 - \mu(0), \mu(y) + 1 - \mu(0)) = \max(\mu^+(x), \mu^+(y)),\end{aligned}$$

$$\mu^+(xy) = \mu(xy) + 1 - \mu(0) \leq \mu(y) + 1 - \mu(0) = \mu^+(y).$$

This shows that μ^+ is an anti fuzzy left ideal of R . Let $a, b, x, z \in R$ be such that $x + a + z = b + z$. Then

$$\begin{aligned}\mu^+(x) &= \mu(x) + 1 - \mu(0) \leq \max(\mu(a), \mu(b)) + 1 - \mu(0) \\ &= \max(\mu(a) + 1 - \mu(0), \mu(b) + 1 - \mu(0)) = \max(\mu^+(a), \mu^+(b)).\end{aligned}$$

Hence μ^+ is a normal anti fuzzy left h -ideal of hemiring of R . Clearly, $\mu \subset \mu^+$. This ends the proof. $\square$

Definition 3.17. Let $\mathbb{N}(R)$ denote the set of all normal anti fuzzy left h -ideals of R . Note that $\mathbb{N}(R)$ is a poset under the set inclusion. A fuzzy set μ in a hemiring R is called a *maximal anti fuzzy left h -ideal* of R if it is non-constant and μ^+ is a maximal element of $(\mathbb{N}(R), \subseteq)$.

Theorem 3.18. Let $\mu \in \mathbb{N}(R)$ be non-constant such that it is a maximal element of $(\mathbb{N}(R), \subseteq)$. Then it takes only two values $\{0, 1\}$.

Proof. Since μ is normal, $\mu(0) = 1$. We claim that $\mu(x) = 0$. If not, then there exists $x_0 \in R$ such that $0 < \mu(x_0) < 1$. Define on R a fuzzy set ν by putting $\nu(x) = \frac{1}{2}(\mu(x) + \mu(x_0))$ for each $x \in R$. Then clearly ν is well-defined and for all $x, y \in R$, we have

$$\begin{aligned}\nu(x+y) &= \frac{1}{2}\mu(x+y) + \frac{1}{2}\mu(x_0) \leq \frac{1}{2}(\max\{\mu(x), \mu(y)\} + \mu(x_0)) \\ &= \max\left\{\frac{1}{2}(\mu(x) + \mu(x_0)), \frac{1}{2}(\mu(y) + \mu(x_0))\right\} = \max\{\nu(x), \nu(y)\},\end{aligned}$$

$$\begin{aligned}\nu(xy) &= \frac{1}{2}\mu(xy) + \frac{1}{2}\mu(x_0) \leq \frac{1}{2}(\mu(y) + \mu(x_0)) \\ &= \nu(y).\end{aligned}$$

Thus ν is an anti fuzzy left ideal of R . Let $a, b, x, z \in R$ be such that $x + a + z = b + z$. Then

$$\begin{aligned}\nu(x) &= \frac{1}{2}\mu(x) + \frac{1}{2}\mu(x_0) \leq \frac{1}{2}(\max\{\mu(a), \mu(b) + \mu(x_0)\}) \\ &= \max\left\{\frac{1}{2}(\mu(a) + \mu(x_0)), \frac{1}{2}(\mu(b) + \mu(x_0))\right\} = \max(\nu(a), \nu(b)).\end{aligned}$$

Hence ν is an anti fuzzy left h -ideal of R . By Theorem 3.16, ν^+ is a maximal anti fuzzy left h -ideal of R . Note that

$$\begin{aligned}\nu^+(x) &= \nu(x) + 1 - \nu(0) = \frac{1}{2}(\mu(x) + \mu(x_0)) + 1 - \frac{1}{2}(1 + \mu(x_0)) \\ &= \frac{1}{2}(\mu(x_0) + 1) = \nu(x_0),\end{aligned}$$

and $\nu^+(x_0) < 1 = \nu^+(0)$. Hence ν^+ is a non-constant, and μ is not a maximal element of $\mathcal{N}(R)$. This is a contradiction. This ends the proof. $\square$

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