Estimation of Parameters of the Weibull Distribution Based on Progressively Censored Data

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Abstract

In this paper, we use the setup proposed by Balakrishnan and Aggarwala (2000) to compute approximate best linear unbiased estimates (ABLUEs) of the location and scale parameters of the Weibull distribution. Further, we derive approximate maximum likelihood estimates (AMLEs) of the location and scale parameters of the Weibull distribution. Finally, we carry out a simulation study to compare between the techniques considered for the estimation.

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1 Introduction

A Type II censored sample is one for which only m smallest observations in a random sample of n items are observed ($1 \leq m \leq n$). Experiments involv-

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ing Type II censoring are often used in life testing. Such tests are time- and cost-effective since it might take a very long time for all items to fail. A generalization of Type II censoring is progressive Type II censoring. In this case, the first failure in the sample is observed and a random sample of size R_1 is immediately drawn from the remaining $n - 1$ unfailed items and removed from the test, leaving $n - 1 - R_1$ items in test. After the second item has failed, R_2 of the still unfailed items are removed, and so on. The experiment terminates after some prefixed series of repetitions of this procedure. Although progressive Type II censored sampling is effective in time and money, it is not very popular in lifetime experiment. It may be due to the complicated calculation of the likelihood function [see Ng, Chan and Balakrishnan (2002)]. Some early works on progressive censoring can be found in Cohen (1963), Mann (1971) and Thomas and Wilson (1972). Viveros and Balakrishnan (1994) proposed a conditional method of inference to derive the exact confidence intervals. Since the publication of the book by Balakrishnan and Aggarwala (2000), considerable amount of research work has been carried out on progressive censoring methodology. Balasooriya and Balakrishnan (2000) and Balasooriya, Saw and Gadag (2000) have studied progressively censored reliability sampling plans for Weibull and lognormal distributions, respectively. Shuo-Jye Wu (2002) has obtained the maximum likelihood estimates of the shape and scale parameters based on concerning a progressively Type-II censored sample from the Weibull distribution. Also, he has constructed an exact confidence interval and an exact confidence region for shape and scale parameters. Ng, Chan and Balakrishnan (2002) have discussed the estimation of the parameters from progressively censored data using the EM Algorithm. Tse Sk, Xiang (2003) have explored the problem of interval estimation for parameters of Weibull-distributed data, which are Type-II progressively censored with random removals. Also, they have considered seven different confidence interval estimation procedures based on a parametric bootstrapping approach and the asymptotic normality method and the likelihood ratio statistic. In addition they, have conducted a Monte Carlo simulation to evaluate the performance of those procedures based on their lengths and their coverage probabilities. Ng, Chan and Balakrishnan (2004) have computed the expected Fisher information and asymptotic variance-covariance matrix of the maximum likelihood estimates based on a progressively Type II censored sample from Weibull distribution. Also, they used these values to determine the optimal progressive censoring plans.

Suppose n independent units are placed on a life-test with the corresponding lifetimes $X_{1:m:n}^{(R_1, R_2, \dots, R_m)}, X_{2:m:n}^{(R_1, R_2, \dots, R_m)}, \dots, X_{m:m:n}^{(R_1, R_2, \dots, R_m)}$, referred to as *progressive Type-II right censored order statistics*. We assume that $X_{i:m:n}^{(R_1, R_2, \dots, R_m)}$, $i = 1, 2, \dots, m$ are i.i.d. with p.d.f. $f(\cdot)$ and c.d.f. $F(\cdot)$. Prior to the experi-

ment, a number $m \leq n$ is determined and the censoring scheme $(R_1, R_2, \dots, R_m)$ with $R_j \geq 0$ and $\sum_{j=1}^m R_j + m = n$ is specified. The special case when $R_1 = R_2 = \dots = R_{m-1} = 0$ so that $R_m = n - m$ is the case of conventional Type-II right censored sampling. Also when $R_1 = R_2 = \dots = R_m = 0$, so that $m = n$, the progressively Type-II right censoring scheme reduces to the case of no censoring (ordinary order statistics). The joint probability density function of the progressively censored failure times $X_{1:m:n}, X_{2:m:n}, \dots, X_{m:m:n}$, is given by [see Balakrishnan and Aggarwala (2000)],

$$f_{X_{1:m:n}, X_{2:m:n}, \dots, X_{m:m:n}}(x_1, x_2, \dots, x_m) = A_{n;R_1, \dots, R_{m-1}} \prod_{i=1}^m f(x_i) \{1 - F(x_i)\}^{R_i},$$

$$-\infty < x_1 < x_2 < \dots < x_m < \infty, (1.1)$$

where

$$A_{n;R_1, \dots, R_{m-1}} = n(n - R_1 - 1) \dots (n - R_1 - R_2 - \dots - R_{m-1} - m + 1). \quad (1.2)$$

For simplicity, we write $A_{n;R_1, \dots, R_{m-1}} = A_{n; \tilde{R}_{m-1}}$; $1 \leq m \leq n$ and $A_{n; \tilde{R}_0} = n$.

In this paper, we are concerned with progressive Type-II censored data from the Weibull distribution which is commonly used to model failure-time distributions, due to desirable properties such as a positive, increasing hazard rate. The three-parameter Weibull distribution has its density as

$$f(x) = \frac{\delta}{\sigma} \left(\frac{x - \theta}{\sigma} \right)^{\delta-1} \exp \left\{ - \left(\frac{x - \theta}{\sigma} \right)^{\delta} \right\}, \quad x \geq \theta, \delta > 0, \sigma > 0, \quad (1.3)$$

and

$$F(x) = 1 - \exp \left\{ - \left(\frac{x - \theta}{\sigma} \right)^{\delta} \right\}, \quad x \geq \theta, \delta > 0, \sigma > 0. \quad (1.4)$$

In Section 2 of this paper, we use the setup proposed by Balakrishnan and Aggarwala (2000) to compute approximate best linear unbiased estimates (ABLUEs) of the location and scale parameters of the Weibull distribution. Next, we derive approximate maximum likelihood estimates (AMLEs) of location and scale parameters of Weibull distribution. Finally, we carry out a simulation study to compare between the techniques considered for the estimation. Also, we draw some comparisons and conclusions.

2 Estimation

In this section, we apply two different methods to estimate the location and scale parameters of the Weibull distribution.

2.1 Approximate best linear unbiased estimates

Following the technique proposed by Balakrishnan and Aggarwala(2000), we calculate the approximate best linear unbiased estimates (ABLUEs) for the location and scale parameter θ^* and σ^* , as follows

$$\theta^* = \sum_{i=1}^n a_i X_{i:m:n}^{(R_1, \dots, R_m)}, \quad (2.1)$$

$$\sigma^* = \sum_{i=1}^n b_i X_{i:m:n}^{(R_1, \dots, R_m)}, \quad (2.2)$$

where the a_i and b_i are the coefficients of the ABLUEs given in Balakrishnan and Aggarwala(2000). Table 1 gives the coefficients a_i and b_i for different sample sizes and different censoring schemes

n	m	censoring scheme
8	3	R1 = (1, 1, 3)
10	4	R2 = (2, 0, 2, 2)
15	5	R3 = (4, 0, 2, 0, 4)
20	6	R3 = (6, 0, 2, 0, 0, 6)
30	7	R4 = (10, 0, 0, 3, 0, 0, 10)
35	8	R5 = (10, 0, 3, 2, 0, 2, 0, 10)

2.2 Approximate maximum likelihood estimates

In this section, we use the approximate maximum likelihood estimation method AMLEs developed by Balakrishnan(1989 a,b, 1990 a,b,c) to estimate the scale and location parameters σ and θ (we shall denote them by $\tilde{\sigma}$ and $\tilde{\theta}$ [for more details, see Balakrishnan and Varadan(1991), Balakrishnan and Wong(1991) and Chan(1989)]. The likelihood function based on progressive Type-II right censored sample

$X_{1:m:n}, X_{2:m:n}, \dots, X_{m:m:n}$ can be written as

$$L(\theta, \sigma) = (Constant) \left(\frac{1}{\sigma}\right)^m \prod_{i=1}^m f(Z_{i:m:n}) (1 - F(Z_{i:m:n}))^{R_i}. \quad (2.3)$$

Upon partial differentiation of the logarithm of the likelihood function with respect to θ and σ , the score equations to be solved for θ and σ in this case are given by

$$\frac{\partial \ln L}{\partial \theta} = \frac{-1}{\sigma} \left\{ \sum_{i=1}^m \frac{f'(Z_{i:m:n})}{f(Z_{i:m:n})} - \sum_{i=1}^m \frac{R_i f(Z_{i:m:n})}{(1 - F(Z_{i:m:n}))} \right\} = 0, \quad (2.4)$$

and

$$\frac{\partial \ln L}{\partial \sigma} = \frac{-m}{\sigma} - \sum_{i=1}^m \frac{f'(Z_{i:m:n})}{f(Z_{i:m:n})} \frac{(x_i - \theta)}{\sigma^2} + \sum_{i=1}^m \frac{(x_i - \theta)}{\sigma^2} \frac{R_i f(Z_{i:m:n})}{(1 - F(Z_{i:m:n}))} = 0, \quad (2.5)$$

where $Z_{i:m:n} = (X_{i:m:n} - \theta)/\sigma$, $F(z) = 1 - e^{-z^\delta}$, and $f(z) = \frac{\delta}{\sigma} z^{\delta-1} e^{-z^\delta}$. The likelihood equations (2.4) and (2.5) do not admit explicit solutions. However, by expanding the functions $\dot{f}(Z_{i:m:n})/f(Z_{i:m:n})$ and $f(Z_{i:m:n})/(1 - F(Z_{i:m:n}))$ in a Taylor series around the point ξ_i , where

$$\xi_i = F^{-1}(p_i) = (-\ln(1 - p_i))^{\frac{1}{\delta}}, \quad (2.6)$$

and $p_i = 1 - q_i = 1 - \prod_{j=m-i+1}^m \alpha_j$. Balakrishnan and Aggarwala (2000) deduced that: if $U_{i:m:n}$, $i = 1, \dots, m$ denote a progressive Type-II right censored sample from the uniform(0, 1) distribution obtained from a sample of size n with the censoring scheme $(R_1, R_2, \dots, R_m)$, then, $V_i, i = 1, \dots, m$ are all statistically independent random variables with $V_i = \text{Beta}(i + \sum_{j=m-i+1}^m R_j, 1)$, $i = 1, \dots, m$ such that

$$U_{i:m:n} = 1 - \prod_{j=m-i+1}^m V_j, \quad i = 1, \dots, m, \quad (2.7)$$

and

$$E(U_{i:m:n}) = 1 - \prod_{j=m-i+1}^m \alpha_j, \quad i = 1, 2, \dots, m, \quad (2.8)$$

where

$$\alpha_j = \frac{j + \sum_{i=m-j+1}^m R_i}{1 + j + \sum_{i=m-j+1}^m R_i}, \quad j = 1, \dots, m. \quad (2.9)$$

We may then consider the following approximations

$$\frac{\dot{f}(Z_{i:m:n})}{f(Z_{i:m:n})} \simeq \gamma_i + \beta_i Z_{i:m:n}, \quad (2.10)$$

and

$$\frac{f(Z_{i:m:n})}{(1 - F(Z_{i:m:n}))} \simeq \lambda_i + \mu_i Z_{i:m:n}, \quad (2.11)$$

where

$$\gamma_i = (\delta^2 - 2\delta)\xi_i^{\delta-1} + 2(\delta - 1)\xi_i^{-1}, \quad (2.12)$$

$$\beta_i = (1 - \delta)(\xi_i^{-2} + \delta\xi_i^{\delta-2}), \quad (2.13)$$

$$\lambda_i = \frac{\delta(2 - \delta)}{\sigma} \xi_i^{\delta-1}, \quad (2.14)$$

$$\mu_i = \frac{\delta(\delta - 1)}{\sigma} \xi_i^{\delta-2}. \quad (2.15)$$

From (2.4) and (2.5) by using the above relations, the approximate likelihood equations for θ and σ can be written as

$$\frac{\partial \ln L}{\partial \theta} \simeq \frac{-1}{\sigma} \left\{ \sum_{i=1}^m (\gamma_i - R_i \lambda_i) + \sum_{i=1}^m (\beta_i - R_i \mu_i) Z_{i:m:n} \right\} = 0, \quad (2.16)$$

and

$$\frac{\partial \ln L}{\partial \sigma} \simeq \frac{-1}{\sigma} \left\{ m + \sum_{i=1}^m Z_{i:m:n} (\gamma_i + \beta_i Z_{i:m:n}) - \sum_{i=1}^m R_i Z_{i:m:n} (\lambda_i + \mu_i Z_{i:m:n}) \right\} = 0 \quad (2.17)$$

By solving (2.16) and (2.17), we obtain the approximate MLEs of θ and σ as

$$\tilde{\theta} = W + U \tilde{\sigma}, \quad (2.18)$$

and

$$\tilde{\sigma} = \{-B + (B^2 + 4AC)^{1/2}\}/2A, \quad (2.19)$$

where

$$\begin{aligned} U &= \frac{\sum_{i=1}^m (\gamma_i - R_i \lambda_i)}{\sum_{i=1}^m (\beta_i - R_i \mu_i)}, \\ W &= \frac{\sum_{i=1}^m (\beta_i - R_i \mu_i) X_{i:m:n}}{\sum_{i=1}^m (\beta_i - R_i \mu_i)}, \\ A &= m - \sum_{i=1}^m (R_i \mu_i - \beta_i) U^2 + \sum_{i=1}^m (R_i \lambda_i - \gamma_i) U, \\ B &= -\sum_{i=1}^m (R_i \lambda_i - \gamma_i) (X_{i:m:n} - W) + 2 \sum_{i=1}^m (R_i \mu_i - \beta_i) (X_{i:m:n} - W) U \\ C &= \sum_{i=1}^m (R_i \mu_i - \beta_i) (X_{i:m:n} - W)^2, \end{aligned}$$

and γ_i , β_i , λ_i and μ_i are given in (2.12-2.15). By solving (2.17) for σ , we obtain a quadratic equation in σ that has two roots; however, one of them drops out, under the condition that $C > 0$.

3 Simulation Study

Using the algorithm given in Balakrishnan and Aggarwala (2000), progressively Type-II right censored samples the from Weibull distribution with location parameter $\theta = 0.0$, scale parameter $\sigma = 1.0$ and shape parameters $\delta = 2, 3, 4, 5$ was generated based on 10000 Monte Carlo runs. Next, we calculate the MSE and Bias of the ABLUEs and AMLEs based on different censoring schemes and the numerical results are parented in Tables 2 and 3.

Illustrative example:

A progressively Type-II right censored sample of size $m = 6$ from a sample of size $n = 20$ from the Weibull distribution with $\theta = 0.0$, $\sigma = 1.0$, $\delta = 3.0$ and censoring scheme $R = (6, 0, 2, 0, 0, 6)$, was simulated by using an algorithm given in Balakrishnan and Aggarwala(2000). The simulated progressively Type-II right censored sample is

$$0.5381790, 0.5527108, 0.5998031, 0.6260462, 0.7449507, 0.7478878.$$

By making use of (2.1) and (2.2) and the coefficients a_i and b_i for $n = 20$, $m = 6$ and $\delta = 3.0$ given in Table 1, we determine the ABLUEs of θ and σ as follows

$$\begin{aligned}\theta^* &= (1.3708 \times 0.5381790) + (0.40274 \times 0.5527108) + (0.17174 \times 0.5998031) \\ &\quad + (0.13216 \times 0.6260462) + (0.04842 \times 0.7449507) + (-1.1258 \times 0.7478878) \\ &= 0.34\end{aligned}$$

and

$$\begin{aligned}\sigma^* &= (-1.7460 \times 0.5381790) + (-0.51487 \times 0.5527108) + (-0.07314 \times 0.5998031) \\ &\quad + (-0.10533 \times 0.6260462) + (0.02891 \times 0.7449507) + (2.4104 \times 0.7478878) \\ &= 0.49.\end{aligned}$$

The standard errors of the estimates θ^* and σ^* are:

$$\begin{aligned}SE(\theta^*) &= \sigma^*(Var(\theta^*))^{1/2} = 0.49 \times (0.042)^{1/2} = 0.10, \\ SE(\sigma^*) &= \sigma^*(Var(\sigma^*))^{1/2} = 0.49 \times (0.098)^{1/2} = 0.15.\end{aligned}$$

Table 1: Coefficients of the ABLUEs of θ and σ when $\theta = 0.0$ and $\sigma = 1.0$

n	m	$\delta = 2.0$		$\delta = 3.0$		$\delta = 4.0$		$\delta = 5.0$	
		a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
8	3	1.8793	-2.8220	2.4202	-3.1628	2.9750	-3.6315	3.5360	-4.1449
		.14418	.09394	.43875	-.33742	.73841	-.69641	1.039053	-1.0299
		-1.0235	2.7281	-1.8589	3.5002	-2.7134	4.3279	-3.5751	5.1749
10	4	1.4914	-2.0348	1.8075	-2.2077	2.1462	-2.4869	2.4951	-2.8035
		.24712	-.25343	.51654	-.57889	.77499	-.85833	1.0278	-1.1217
		-.13068	.62229	-.11655	.51413	-.08541	.43833	-.04759	.37446
		-.60781	1.6659	-1.2075	2.2725	-1.8358	2.9069	-2.4753	3.5507
15	5	1.3129	-1.8723	1.5306	-1.9206	1.7771	-2.0993	2.0364	-2.3227
		.23244	-.29349	.45213	-.55269	.65597	-.76833	.85260	-.96984
		.00778	.27666	.10644	.08785	.21375	-.05928	.32347	-.19121
		.01079	.11577	.08576	-.00081	.17718	-.11404	.27555	-.22574
		-.56399	1.7733	-1.1749	2.3862	-1.8240	3.0409	-2.4880	3.7095
20	6	1.2095	-1.7677	1.3708	-1.7460	1.5656	-1.8694	1.7752	-2.0418
		.21634	-.30119	.40274	-.51487	.57129	-.68915	.73223	-.85146
		.05537	.12185	.17174	-.07314	.29101	-.22508	.41006	-.36169
		.04072	.02616	.13216	-.10533	.23452	-.22715	.34076	-.34479
		.00104	.10865	.04842	.02891	.11032	-.05056	.17868	-.12991
		-.52299	1.8123	-1.1258	2.4104	-1.7727	3.0614	-2.4369	3.7296
30	7	1.1472	-1.8649	1.2769	-1.7467	1.4429	-1.8179	1.6249	-1.9510
		.20287	-.33647	.36658	-.51817	.51198	-.66375	.64987	-.79957
		.10308	-.13895	.22925	-.30304	.35314	-.44085	.47481	-.56927
		.00374	.23501	.06528	.08716	.13541	-.02058	.20822	-.11424
		.02935	.02027	.10038	-.08917	.18290	-1.91092	.26994	-.28973
		.00751	.07135	.05242	-.00638	.11073	-.08276	.17515	-.15853
		-.49375	2.0137	-1.0908	2.5763	-1.7371	3.2169	-2.4029	3.8823
35	8	1.0797	-1.8018	1.1640	-1.6162	1.2909	-1.6430	1.4361	-1.7376
		.19969	-.34153	.34475	-.49388	.46949	-.61378	.58650	-.72597
		.08948	-.00691	.20714	-.19777	.31958	-.33628	.42855	-.45789
		.04059	.07030	.12255	-.07484	.20845	-.18697	.29479	-.28734
		.03717	-.01023	.11001	-.11798	.19078	-.21537	.27414	-.30827
		-.02854	.24769	-.02216	.19093	-.00577	.15004	.01502	.11473
		.00244	.07243	.03488	.01312	.07910	-.04624	.12882	-.10560
		-.42051	1.7700	-.96121	2.2966	-1.5526	2.8916	-2.1639	3.5079

Table 2: Bias and MSE for the ABLUEs of the location and scale parameters

δ	n	m	scheme	BIAS(θ^*)	BIAS(σ^*)	MSE(θ^*)	MSE(σ^*)
2	8	3	R1	-.05526	.07561	.09088	.30862
	10	4	R2	-.05437	.07700	.05839	.18724
	15	5	R3	-.03811	.05302	.03439	.13344
	20	6	R4	-.03069	.04114	.02366	.10326
	30	7	R5	-.02564	.03434	.01507	.08512
	35	8	R6	-.02408	.03308	.01227	.07196
3	8	3	R1	-.09603	.10968	.15232	.31283
	10	4	R2	-.08996	.10297	.09825	.18784
	15	5	R3	-.06791	.07761	.062663	.13250
	20	6	R4	-.05685	.06449	.04563	.10223
	30	7	R5	-.05028	.05951	.03247	.08494
	35	8	R6	-.04679	.05609	.02702	.07175
4	8	3	R1	-.12460	.13604	.19713	.32653
	10	4	R2	-.11380	.12337	.12672	.19664
	15	5	R3	-.08846	.09630	.08379	.13904
	20	6	R4	-.07519	.08183	.06256	.10771
	30	7	R5	-.06822	.07742	.04696	.09045
	35	8	R6	-.06314	.07220	.03936	.07646
5	8	3	R1	-.14521	.15532	.23018	.33906
	10	4	R2	-.13049	.13823	.14735	.20505
	15	5	R3	-.10304	.10978	.09948	.14548
	20	6	R4	-.08827	.09422	.07531	.11318
	30	7	R5	-.08128	.09002	.05834	.09575
	35	8	R6	-.07491	.08340	.04904	.08095

Table 3: Bias and MSE for the AMLEs of the location and scale parameters

δ	n	m	scheme	BIAS(θ^*)	BIAS(σ^*)	MSE(θ^*)	MSE(σ^*)
2	8	3	R1	.23617	-.38399	.16871	.18133
	10	4	R2	.18706	-.28365	.12692	.10752
	15	5	R3	.14479	-.23231	.09586	.07194
	20	6	R4	.11982	-.19873	.07808	.05269
	30	7	R5	.09323	-.17349	.06396	.03885
	35	8	R6	.08289	-.15457	.05463	.03124
3	8	3	R1	.20039	-.35712	.15799	.19436
	10	4	R2	.11962	-.23189	.11289	.11008
	15	5	R3	.05496	-.16401	.08574	.07066
	20	6	R4	.01820	-.12312	.07102	.05047
	30	7	R5	-.01609	-.10626	.06076	.03876
	35	8	R6	-.02729	-.09154	.05214	.03158
4	8	3	R1	.19332	-.33314	.16761	.20412
	10	4	R2	.09481	-.19754	.11888	.11854
	15	5	R3	.01909	-.12078	.09469	.08009
	20	6	R4	-.02271	-.07679	.08162	.06041
	30	7	R5	-.05885	-.06060	.07364	.04848
	35	8	R6	-.07252	-.04461	.06483	.03999
5	8	3	R1	.19169	-.31493	.17896	.21269
	10	4	R2	.08184	-.17350	.12721	.12742
	15	5	R3	-.00058	-.09105	.10517	.09099
	20	6	R4	-.04513	-.04526	.09292	.07196
	30	7	R5	-.08196	-.02813	.08621	.05995
	35	8	R6	-.09776	-.01058	.07709	.05039

3.1 Comparisons and Conclusions

By using the different censoring schemes given above in Section 2 and the algorithm given in Balakrishnan and Aggarwala (2000), we generate a progressively Type-II right censored samples from the Weibull distribution with location parameter $\theta = 0.0$ and scale parameter $\sigma = 1.0$ based on 10000 Monte Carlo runs. The coefficients of the ABLUEs are presented in Table 1. The Bias and the MSE of ABLUEs and AMLEs of the location and scale parameters from Weibull distribution are presented in Tables 2 and 3. From the numerical results presented in Tables 1, 2 and 3, we notice the following:

1. As a check of the entries of Table 1, for the coefficients of the ABLUEs, we see that $\sum_{i=1}^n a_i \simeq 1$ and $\sum_{i=1}^n b_i \simeq 0$.
2. From Table 2, we see that when n and δ increase, the mean square errors

$MSE(\theta^*)$ and $MSE(\sigma^*)$ decrease for all censoring schemes.

3. From Table 3, we see that when n and δ increase, the mean square errors $MSE(\tilde{\theta})$ and $MSE(\tilde{\sigma})$ decrease for all censoring schemes.

In order to compare the performance of the ABLUEs and AMLEs, Balakrishnan and Lee (1998) have defined the relative efficiency between the two methods of estimation as follows

$$\text{Eff}(\theta) = \frac{MSE(\tilde{\theta})}{\text{Var}(\theta^*)} \times 100, \quad (3.1)$$

and

$$\text{Eff}(\sigma) = \frac{MSE(\tilde{\sigma})}{\text{Var}(\sigma^*)} \times 100. \quad (3.2)$$

The values of the relative efficiency in (3.1) and (3.2) can be interpreted as follows:

- if $\text{Eff}(\theta) > 100$, we conclude that the estimation of θ based on the ABLUEs is more efficient than that based on the AMLEs.
- if $\text{Eff}(\theta) < 100$, we conclude that the estimation of θ based on the AMLEs is more efficient than that based on the ABLUEs.

By applying this technique to our results in given in Tables 2 and 3, we have the relative efficiency for our ABLUEs and AMLEs as given in Table 4 below:

Table 4: Relative efficiency between the ABLUEs and the AMLEs

n		$\delta = 2.0$	$\delta = 3.0$	$\delta = 4.0$	$\delta = 5.0$
8	$\text{Eff}(\theta)$	144.51	110.40	92.29	85.59
	$\text{Eff}(\sigma)$	35.49	64.62	66.27	67.54
10	$\text{Eff}(\theta)$	214.42	125.21	111.56	97.61
	$\text{Eff}(\sigma)$	65.38	62.11	59.26	68.53
15	$\text{Eff}(\theta)$	291.04	147.69	124.65	118.36
	$\text{Eff}(\sigma)$	55.07	55.86	61.72	68.19
20	$\text{Eff}(\theta)$	343.65	167.51	143.41	137.62
	$\text{Eff}(\sigma)$	51.88	51.46	59.80	68.99
30	$\text{Eff}(\theta)$	443.75	202.97	174.08	166.66
	$\text{Eff}(\sigma)$	46.28	47.62	57.40	68.40
35	$\text{Eff}(\theta)$	726.18	261.14	154.45	177.54
	$\text{Eff}(\sigma)$	44.08	58.28	43.84	68.10

Form Table 4, we see that the ABLUE of θ is more efficient than the AMLE for $n \geq 15$ and $\delta = 2, 3, 4, 5$. The AMLE of σ is more efficient than the ABLUE for $n \geq 8$ and $\delta = 2, 3, 4, 5$.

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