Ranking of Extreme and Non-Extreme Efficient DMUs

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Abstract

There are several methods for ranking of the extreme efficient DMUs. The non-extreme efficient DMUs cannot rank by these methods. In this paper, we propose a method for ranking all efficient DMUs (extreme and non-extreme efficient DMUs) by using reference DMUs.

Keywords: Data Envelopment Analysis, Ranking, Efficiency, and reference set

1. Introduction

Data Envelopment Analysis (DEA), Charnes et. al [3], is a mathematical programming technique for identifying efficient frontiers for peer Decision Making Units (DMUs). In most models of DEA, the best performers have efficiency score unity, and, from experience, we know that usually there are plural DMUs, which have this "efficient status". To discriminate between these efficient DMUs is an interesting research subject. Several authors have proposed methods for ranking the best performers. See Andersen and Petersen (AP)[1], Seiford and Zhu [6], Zhu [7] and Mehrabian, Alirezai and Jahanshahloo (MAJ)[5] and Jahanshahloo et al. [4] among others.

All these ranking methods have been applied for ranking only extreme efficient DMUs. In this paper, we use the convex combination of extreme efficient DMUs rank, which are reference DMUs, for ranking non-extreme

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efficient DMUs. We assume that there is n DMUs to be evaluated, indexed by $j = 1, \dots, n$ and $Y_j = (y_{1j}, \dots, y_{sj})$ each DMU is assumed to produce s different outputs from m different inputs. Let the observed input and output vectors of DMU _{j} be $X_j = (x_{1j}, \dots, x_{mj})$ and respectively, that all components of vectors X_j and Y_j for all DMUs are non-negative and each DMU has at least one strictly positive input and output. To obtain efficiency of DMUs we use the CCR [5], which is as follows:

$$\begin{aligned} \theta_{CCR}^* = \quad & \text{Min} \quad \theta_{CCR} \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} \leq \theta_o x_{io}, \quad i = 1, \dots, m \\ & \sum_{j=1}^n \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \quad (1)$$

Assume that the DMU _{o} is extreme efficient. To obtain the ranking score of DMU _{o} , consider the model AP,[2], as follows:

$$\begin{aligned} \theta_{AP}^* = \quad & \text{Min} \quad \theta_{AP} \\ \text{s.t.} \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij} \leq \theta_o x_{io}, \quad i = 1, \dots, m \\ & \sum_{j=1, j \neq o}^n \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j = 1, \dots, n, j \neq o. \end{aligned} \quad (2)$$

The structure of the paper is as follows. In section 2 we propose a method for ranking non-efficient DMUs. In section 3, a numerical example is considered which illustrates the presented method. Section 4 gives our conclusive remarks.

2. Ranking of Non-Extreme Efficient DMUs

The previous ranking methods are applied for ranking only extreme efficient DMUs. In this paper, we use the convex combination of extreme efficient DMUs rank, which are references DMUs, for ranking non-extreme efficient DMUs. We know each non-extreme efficient DMU has at least two extreme efficient references. So, suppose that the DMU _{o} is non-extreme efficient and $\Lambda_o = o_1, \dots, o_p$ be the set of index DMUs which are references DMUs for DMU _{o} .

Theorem 1: *Each non-extreme efficient DMU can be represented by the convex combination of its extreme efficient reference set DMUs.*

Proof: The proof is similar to representation theorem in [2].

Since DMU_o and its reference set DMUs are almost similar. So their performance and rank are nearly similar. Therefore from theorem 1 and above discussion, we define the rank of non-extreme DMUs based on the density of reference set DMUs in CCR model as definition 1.

Definition 1: Assume that DMU_o is non-extreme efficient, then its rank (R_o) is defined as, $R_o = \sum_{o_j \in \Lambda} \lambda_{o_j}^* R_{o_j}^*$, where for $o_j \in \Lambda_o$, R_{o_j} is the rank of o_j -th DMU and $\lambda_{o_j}^*$ is its intensity in CCR model and can be obtained from optimal solution of model (1) when DMU_o is evaluated.

3. Example

Consider 11 DMUs with 2 inputs and 1 output. The data of these DMUs have been reported in Table 1.

DMUs	A	B	C	D	E	F	G	H	I	L	M
Input1	1	2	4	1.5	1.8	1.25	3	4	5	3	5
Input2	4	2	1	3	2.4	3.5	1.5	2	3	4	5
Output	1	1	1	1	1	1	1	1	1	1	1

Table 1: The values of inputs and outputs

By CCR model DMUs D, E, F and G are non-extreme efficient and DMUs A, B and C are extreme efficient. By using formula $R_o = \sum_{o_j} \lambda_{j_o}^* R_{j_o}$ the rank of on-extreme efficient DMUs have been reported in row 5 of table 2.

DMU_o	A	B	C	D	E	F	G
θ_{CCR}^*	1	1	1	1	1	1	1
θ_{AP}^*	1.25	1.0714	1.4999	1	1	1	1
Λ_o	{A}	{B}	{C}	{A,B}	{A,B}	{A,B}	{B,C}
R_o	1.25	1.0714	1.4999	1.1607	1.1071	1.2053	1.2857
$\lambda_{j_o}^*, o_j \in \Lambda$	$\lambda_A^* = 1$	$\lambda_B^* = 1$	$\lambda_C^* = 1$	$\lambda_A^* = 0.5,$ $\lambda_B^* = 0.5$	$\lambda_A^* = 0.2,$ $\lambda_B^* = 0.8$	$\lambda_A^* = 0.75,$ $\lambda_B^* = 0.25$	$\lambda_B^* = 0.5,$ $\lambda_C^* = 0.5$

Table 2: CCR efficiency, reference set, and the ranks

In Table 2, the numbers of the rows 2 and 3 show the efficiency and rank of efficient DMUs, respectively. In this Table rows 4 and 6 show reference set and their density, respectively.

4. Conclusion

In this paper, we proposed a ranking method for ranking extreme and non-extreme efficient DMUs based on the density of reference set DMUs in CCR model. So, this method is able to rank all efficient DMUs with constraint and variable returns to scale. It seems that ranking by this approach is more precise than other methods.

References

- [1] Andersen, P., Petersen, N.C, 1993, A procedure for ranking efficient units in data envelopment analysis, *Management Science* 39(10), 1261-1294.
- [2] Charnes, A., W.W. Cooper, E. Rhodes, 1978, Measuring the efficiency of decision making units, *European Journal of Operational Research* 2, 429-444.
- [3] Bazaraa, M.S., Hanif D. Sherali, C.M. Shertty, 1993, *linear programming, and network flow*, second Edition.
- [4] Jahanshahloo, G.R., F. Hosseinzadeh Lotfi, N. Shoja, G. Tohidi, S. Razavyan, 2004, Ranking using l1-norm in data envelopment analysis, *Applied Mathematics and Computation*, 153, 215-224.
- [5] Mehrabian, S., M. R. Alirezaee, G. R. Jahanshahloo 1999, A complete efficiency ranking of decision making units in data envelopment analysis, *Computational Optimization and Applications*, 14, 261-266.
- [6] Seiford, L.M. and J. Zhu 1999, Infeasibility of super efficiency data envelopment analysis models, *INFOR* 37(2), 174-187.
- [7] Zhu, J., 2001, "Super-efficiency and DEA sensitivity analysis", *European Journal of Operational Research*, 129, 443-455.