Nonlinear Incidence Rate of a Pest Management SI Model with Impulsive Infective Transmission¹

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Abstract

In this work, we consider an SI model for pest management, with concerns about impulsive releases of infective pests. we prove that all solutions of

$$\begin{cases} S'(t) = rS(t)(1 - \frac{S(t) + \theta I(t)}{K}) - \frac{\beta S(t)I^2(t)}{1 + aI^2(t)}, t \neq n\tau \\ I'(t) = \frac{\beta S(t)I^2(t)}{1 + aI^2(t)} - wI(t), t \neq n\tau \\ \Delta S(t) = 0, t = n\tau \\ \Delta I(t) = \mu, t = n\tau, n = 1, 2 \cdots \end{cases} \quad (0.1)$$

are uniformly ultimately bounded and there exists globally asymptotic stability periodic solution of pest-extinction when $r\tau < \frac{r\theta\mu}{Kw} + \frac{\beta}{wa} \ln \frac{1-e^{-w\tau}+a\mu}{1-e^{-w\tau}+a\mu e^{-w\tau}}$ is satisfied, and the condition for permanence of system(0.1) is also obtained. it is concluded that the approach of impulsively releasing infective pests provides reliable tactic basis for the practical pest management.

Keywords: Impulsive; Global asymptotic stability; Permanent; Pest-extinction

1 Introduction

According to reports of Food and Agriculture Organization, the warfare between man and pests has sustained for thousands of years. With the development of society and progress of science and technology, man have adopted

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some advanced and modern weapons for instance chemical pesticides, biological pesticides, remote sensing and measure, computers, atomic energy etc., some brilliant achievements have been obtained. however, the warfare will never ne over. Although a great deal of and a large variety of pesticide were used to control pests, the insect pests impairing crops are increasing. especially because of resistance to the pesticide, with pesticide employed, the residual pests breed a large number of pests with resistance to pesticide, so the pesticide is invalid. Moreover, and will continue. on the other hand, the chemical pesticide kills not only pests but also their natural enemies, therefore, insect pests are rampant again, so the effect of chemical control was challenged. Furthermore, the practice proves that long- term adopting chemical control may give rise to disastrous results, for example, environmental contamination and toxicosis of the man and animals and so on.

The use of bacteria, fungi and viruses is potentially one of the most important approaches in pest control. For example *Bacillus thuringiensis* , which is available in commercial preparations, is used in in the control of a large number of pests[1, 3, 9]. an advantage of using insect pathogens is that they are safe to man and are usually safe to beneficial insects.

There is a vast amount of literatures on the applications of microbial disease to suppress pests[3, 4, 7, 9, 10, 18], but there are fewer papers on mathematical model of the dynamical of microbial disease in pest control [5, 6, 11]. in this paper, insect pathogens are used like a nonresidual chemical insecticide, we shall examine the use of pathogens in a more flexible manner.

2 Model formulation

There are many works concerning *SI* or *SIR* infective disease models, and many good results are obtained [15 ~ 17, 19 ~ 24]. but there are fewer which applied *SI* model pest management[12]. The basic *SI* model is

$$\begin{cases} S'(t) = -\beta S(t)I(t) \\ I'(t) = \beta S(t)I(t) - wI(t) \end{cases} \quad (2.1)$$

where $\beta > 0$ is called the transmission coefficient, $w > 0$ is called the death coefficient of $I(t)$, $S(t)$ denotes the number of susceptible insects and $I(t)$ denotes the number of infective insects, in our models we assume that all newborns are susceptible, and the basic model considered in [12] is following

model of the epidemic under a control variable

$$\begin{cases} S'(t) = -\beta S(t)I(t) \\ I'(t) = \beta S(t)I(t) - wI(t) + u(t) \end{cases} \quad (2.2)$$

where $u(t)$, which is a control variable, denotes the rate of pests infected in the laboratory, and there are some other conditions for the above system, but we consider that the variable $u(t)$ is difficult in practice. and the susceptible pests $S(t)$ will not go extinction. Bilinear and standard incidence rates have been frequently used in classical epidemic models (Hethcote [25]). Simple dynamics of these models seem related to such functions. Several different incidence rates have been proposed by researchers. With regard to the human beings and some mass residing animals, Anderson, May etc. pointed out standard incidence is more suitable than bilinear incidence [29, 30, 31]. Levin etc. have adopted incidence form like $\beta S^q I^p$ or $\frac{\beta S^q I^p}{N}$ which depending different infective disease and environments [29, 30]. Chen [27] sets forth transmission effect like saturation effect $\frac{\beta S(t)}{1+aS(t)}$ and crowded effect $\frac{\beta S(t)}{1+aS(t)+bS^2(t)}$ as incidence rate. But V. Capasso, G. Serio [28] introduced a saturated incidence rate $f(I)S$ into epidemic models. This is very important because the number of effective contacts between infective individuals and susceptible individuals may saturate at high infective levels due to crowding of infective individuals. Ruan and Wang [29] has also put forward such saturation effect $\frac{\beta I^h(t)}{1+\alpha I^l(t)}$ as incidence rate. We will focus on the case when $h = l = 1$. So we develop (2.2) by introducing a constant periodic releasing for the infective pests at fixed moment. Then we consider the following impulsive differential equation with a fixed moment

$$\begin{cases} S'(t) = rS(t)\left(1 - \frac{S(t) + \theta I(t)}{K}\right) - \frac{\beta S(t)I^2(t)}{1 + aI^2(t)}, t \neq n\tau \\ I'(t) = \frac{\beta S(t)I^2(t)}{1 + aI^2(t)} - wI(t), t \neq n\tau \\ \Delta S(t) = 0, t = n\tau \\ \Delta I(t) = \mu, t = n\tau, n = 1, 2, \dots \end{cases} \quad (2.3)$$

where $\frac{\beta I^2(t)}{1+aI^2(t)}$ is the saturation effect of incidence rate, $r > 0$ is the intrinsic growth rate of pests, $K > 0$ is the pests capacity of environment, $\Delta I(t) = I(t^+) - I(t)$, $0 < \theta < 1$, $\mu \geq 0$ is the releasing amount of infective pests at $t = n\tau, n \in Z_+$, $1 > \mu_1 \geq 0$, and $Z_+ = \{1, 2, \dots\}$, $a > 0$ and τ is the period of the impulsive effect, that is, we can use a biological tactics to eradicate pests or keep the pest population below the damage level.

3 Qualitative analysis for model (2.3)

The solution of (2.3), denote by $x(t) = (S(t), I(t))^T$, is a piecewise continuous function $x: R_+ \rightarrow R_+^2$, $x(t)$ is continuous on $(n\tau, (n+1)\tau]$, $n \in Z_+$ and $x(n\tau^+) = \lim_{t \rightarrow n\tau^+} x(t)$ exists. Obviously the global existence and uniqueness of solutions of (2.3) is guaranteed by the smoothness properties of f , which denotes the mapping defined by right-side of system (2.3) (see Lakshmikantham, [13]). Before we have the the main results. we need give some lemmas which will be used in the next. Since $(S'(t) = 0$ whenever $S(t) = 0$, $I'(t) = 0$ whenever $I(t) = 0, t \neq n\tau$, and $I(n\tau^+) = I(n\tau) + \mu, \mu \geq 0$. So we have

Lemma 1. Suppose $x(t)$ is a solution of (2.3) with $x(0^+) \geq 0$, then $x(t) \geq 0$ for $t \geq 0$. and further $x(t) > 0$ $t \geq 0$ for $x(0^+) > 0$.

Lemma 2. (see[13],Page 23 Lemma 2.2) Let the function $m \in PC'[R^+, R]$ satisfies the inequalities

$$\begin{cases} m'(t) \leq p(t)m(t) + q(t), \\ t \geq t_0, t \neq t_k, k = 1, 2, \dots, \\ m(t_k^+) \leq d_k m(t_k) + b_k, t = t_k, \end{cases} \quad (3.1)$$

where $p, q \in PC[R^+, R]$ and $d_k \geq 0, b_k$ are constants ,then

$$\begin{aligned} m(t) &\leq m(t_0) \prod_{t_0 < t_k < t} d_k \exp\left(\int_{t_0}^t p(s) ds\right) \\ &+ \sum_{t_0 < t_k < t} \left(\prod_{t_k < t_j < t} d_j \exp\left(\int_{t_0}^t p(s) ds\right) \right) b_k \\ &+ \int_{t_0}^t \prod_{s < t_k < t} d_k \exp\left(\int_s^t p(\sigma) d\sigma\right) q(s) ds, t \geq t_0 \end{aligned}$$

Now, we show that all solutions of (2.3) are uniformly ultimately bounded.

Lemma 3. There exists a constant $M > 0$ such that $S(t) \leq M, I(t) \leq M$ for each solution $(S(t), I(t))$ of (2.3) with all t large enough.

Proof. Define $V(t) = S(t) + I(t)$. then $t \neq n\tau$ we have

$$\begin{aligned} D^+V(t) + wV(t) &= (r + w)S(t) - \frac{rS^2(t)}{K} - \frac{r\theta S(t)I(t)}{K} \\ &\leq (r + w)S(t) - \frac{rS^2(t)}{K} \leq M_0 \end{aligned}$$

where $M_0 = \frac{K(r+w)^2}{4r}$, when $t = n\tau, V(n\tau^+) = V(n\tau) + \mu$. By lemma 2., for $t \in (n\tau, (n+1)\tau]$ we have

$$V(t) \leq V(0) \exp(-wt) + \int_0^t M_0 \exp(-w(t-s)) ds + \sum_{0 < n\tau < t} \mu \exp(-w(t-n\tau))$$

$$\begin{aligned}
&= V(0) \exp(-wt) + \frac{M_0}{w} (1 - \exp(-wt)) + \mu \frac{\exp(-w(t - \tau)) - \exp(-w(t - (n + 1)\tau))}{1 - \exp(w\tau)} \\
&< V(0) \exp(-wt) + \frac{M_0}{w} (1 - \exp(-wt)) + \frac{\mu \exp(-w(t - \tau))}{1 - \exp(w\tau)} + \frac{\mu \exp(w\tau)}{\exp(w\tau) - 1} \\
&\rightarrow \frac{M_0}{w} + \frac{\mu \exp(w\tau)}{\exp(w\tau) - 1}, \quad \text{as } t \rightarrow \infty.
\end{aligned}$$

So $V(t)$ is uniformly ultimately bounded. hence, by the definition of $V(t)$, we have there exists a constant $M > 0$ such that $S(t) \leq M, I(t) \leq M$ for t large enough. the proof is complete.

If $S(t) = 0$, we have the following subsystem of (2.3)

$$\begin{cases} I'(t) = -wI(t), t \neq n\tau \\ I(n\tau^+) = I(n\tau) + \mu, t = n\tau, n = 1, 2, \dots \end{cases} \quad (3.2)$$

Obviously $\widetilde{I}(t) = \frac{\mu \exp(-w(t - n\tau))}{1 - \exp(-w\tau)}, t \in (n\tau, (n + 1)\tau], n \in Z_+, \widetilde{I}(0^+) = \frac{\mu}{1 - \exp(-w\tau)}$ is a positive periodic solution of (3.2). Since the solution of (3.2) is $I(t) = (I(0^+) - \frac{\mu}{1 - \exp(-w\tau)}) \exp(-wt) + \widetilde{I}(t), t \in (n\tau, (n + 1)\tau], n \in Z_+$, so we derive

Lemma 4. System (3.2) has a positive periodic solution $\widetilde{I}(t)$ and for every solution $I(t)$ of (3.2) we have $I(t) \rightarrow \widetilde{I}(t)$ as $t \rightarrow \infty$.

From above discuss we know that (2.3) has a pest-extinction periodic solution

$$(0, \widetilde{I}(t)) = (0, \frac{\mu \exp(-w(t - n\tau))}{1 - \exp(-w\tau)}) \quad (3.3)$$

for $n\tau < t \leq (n + 1)\tau$, and $I(n\tau^+) = \widetilde{I}(0^+) = \frac{\mu}{1 - \exp(-w\tau)}, n \in Z_+$.

Theorem 1. Let $(S(t), I(t))$ be any solution of (2.3), If

$$r\tau < \frac{r\theta\mu}{Kw} + \frac{\beta}{2wa} \ln \frac{(1 - e^{-w\tau})^2 + a\mu^2}{(1 - e^{-w\tau})^2 + a\mu^2 e^{-2w\tau}}$$

holds, then pest-extinction solution $(0, \widetilde{I}(t))$ of (2.3) is globally asymptotically stable.

Proof. First, we prove the local stability. Defining $s(t) = S(t), i(t) = I(t) - \widetilde{I}(t)$, then we have the following linearly similar system for (2.3) which is concerning one periodic solution $(0, \widetilde{I}(t))$

$$\begin{pmatrix} \frac{ds(t)}{dt} \\ \frac{di(t)}{dt} \end{pmatrix} = \begin{pmatrix} r - \frac{r\theta}{K}\widetilde{I(t)} - \frac{\beta\widetilde{I(t)}^2}{1+a\widetilde{I(t)}^2} & 0 \\ \frac{\beta\widetilde{I(t)}^2}{1+a\widetilde{I(t)}^2} & -w \end{pmatrix} \begin{pmatrix} s(t) \\ i(t) \end{pmatrix}$$

It is easy to obtain the fundamental solution matrix

$$\Phi(t) = \begin{pmatrix} \exp(\int_0^t (r - \frac{r\theta}{K}\widetilde{I(s)} - \frac{\beta\widetilde{I(s)}^2}{1+a\widetilde{I(s)}^2})ds) & 0 \\ * & \exp(-wt) \end{pmatrix}$$

There is no need to calculate the exact form of $*$ as it is not required in the analysis that follows. The linearization of the third and fourth equations of (2.3) is

$$\begin{pmatrix} s(n\tau^+) \\ i(n\tau^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} s(n\tau) \\ i(n\tau) \end{pmatrix}$$

The stability of the periodic solution $(0, \widetilde{I(t)})$ is determined by the eigenvalues of

$$M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \phi(\tau)$$

where are

$$\lambda_1 = \exp(-w\tau) < 1, \lambda_2 = \exp(\int_0^\tau (r - \frac{r\theta}{K}\widetilde{I(t)} - \frac{\beta\widetilde{I(t)}^2}{1+a\widetilde{I(t)}^2})ds)$$

according to the Floquet theory [14], if $|\lambda_2| < 1$, we have

$$r\tau < \frac{r\theta\mu}{Kw} + \frac{\beta}{2wa} \ln \frac{(1 - e^{-w\tau})^2 + a\mu^2}{(1 - e^{-w\tau})^2 + a\mu^2 e^{-2w\tau}} \quad (3.4)$$

holds, then $(0, \widetilde{I(t)})$ is locally stable.

In the following, we will prove the global attraction, choose a $\varepsilon > 0$ such that

$$\rho = \exp(\int_0^\tau (r - \frac{r\theta}{K}(\widetilde{I(t)} - \varepsilon) - \frac{\beta(\widetilde{I(t)} - \varepsilon)}{1+a(\widetilde{I(t)} - \varepsilon)}dt) < 1$$

From the second equation of (2.3) we Notice that $\frac{dI(t)}{dt} \geq -wI(t)$, so we consider

following impulsive differential equation

$$\begin{cases} \frac{dy(t)}{dt} = -wy(t), t \neq n\tau \\ y(t^+) = y(t) + \mu, t = n\tau \\ y(0^+) = I(0^+) \end{cases} \quad (3.5)$$

From lemma 2 and comparison theorem of impulsive equation (see theorem 3.1.1 in [13]), we have $I(t) \geq y(t)$ and $y(t) \rightarrow \widetilde{I(t)}$ as $t \rightarrow \infty$. Then

$$I(t) \geq y(t) \geq \widetilde{I(t)} - \varepsilon \quad (3.6)$$

for all t large enough, for convenience we may assume (3.5) hold for all $t \geq 0$, From (2.3), (3.5) and the function $y = \frac{x}{1+ax}$ is increasing on $(-\infty, +\infty)$. We get

$$\frac{dS(t)}{dt} \leq rS(t) \left(1 - \frac{r\theta}{K} (\widetilde{I(t)} - \varepsilon) - \frac{\beta(\widetilde{I(t)} - \varepsilon)^2}{1 + a(\widetilde{I(t)} - \varepsilon)^2} \right) \quad (3.7)$$

So $S(t) \leq S(0^+) \exp(\int_0^s (r - \frac{r\theta}{K} (\widetilde{I(s)} - \varepsilon) - \frac{\beta(\widetilde{I(s)} - \varepsilon)^2}{1 + a(\widetilde{I(s)} - \varepsilon)^2}) ds)$, thus

$$S((n+1)\tau) \leq S(n\tau^+) \exp(\int_{n\tau}^{(n+1)\tau} (r - \frac{r\theta}{K} (\widetilde{I(s)} - \varepsilon) - \frac{\beta(\widetilde{I(s)} - \varepsilon)^2}{1 + a(\widetilde{I(s)} - \varepsilon)^2}) ds)$$

Hence $S(n\tau) \leq S(0^+) \rho^n$ and $S(n\tau) \rightarrow 0$ as $n \rightarrow \infty$, therefore $S(t) \rightarrow 0$ as $t \rightarrow \infty$.

Next we prove that $I(t) \rightarrow \widetilde{I(t)}$ as $t \rightarrow \infty$, For $0 < \beta\varepsilon \leq w$, there must exist a $t_0 > 0$ such that $0 < S(t) < \varepsilon$ for all $t \geq t_0$, without loss of generality, we may assume that $0 < S(t) < \varepsilon$ for all $t \geq 0$, then for system (2.3) we have

$$-wI(t) \leq \frac{dI(t)}{dt} \leq (-w + \beta\varepsilon)I(t) \quad (3.8)$$

then we have $z_1(t) \leq I(t) \leq z_2(t)$ and $z_1(t) \rightarrow \widetilde{I(t)}, z_2(t) \rightarrow \widetilde{I(t)}$ as $t \rightarrow \infty$. while $z_1(t)$ and $z_2(t)$ are the solutions of

$$\begin{cases} \frac{dz_1(t)}{dt} = -wz_1(t), t \neq n\tau \\ z_1(t^+) = z_1(t) + \mu, t = n\tau \\ z_1(0^+) = I(0^+) \end{cases} \quad (3.9)$$

and

$$\begin{cases} \frac{dz_2(t)}{dt} = (-w + \beta\varepsilon)z_2(t), t \neq n\tau \\ z_2(t^+) = z_2(t) + \mu, t = n\tau \\ z_2(0^+) = I(0^+) \end{cases} \quad (3.10)$$

respectively, $\widetilde{z_2}(t) = \frac{\mu \exp((-w+\beta\varepsilon)(t-n\tau))}{1-\exp((-w+\beta\varepsilon)\tau)}$ for $n\tau < t \leq (n+1)\tau$. therefore, for any $\varepsilon_1 > 0$. there exists a $t_1, t > t_1$ such that

$$\widetilde{I}(t) - \varepsilon_1 < I(t) < \widetilde{z_2}(t) + \varepsilon_1$$

Let $\varepsilon \rightarrow 0$, so we have

$$\widetilde{I}(t) - \varepsilon_1 < I(t) < \widetilde{I}(t) + \varepsilon_1$$

for t large enough, which implies $I(t) \rightarrow \widetilde{I}(t)$ as $t \rightarrow \infty$. this completes the proof.

The next work is the investigation of permanence of the system(2.3). Before starting our theorem, we give the following definition.

Definition 3.1. System (2.3) is said to be permanent if there are constants $m, M > 0$ (independent of initial value) and a finite time T_0 such that for all solutions $(S(t), I(t))$ with all initial values $S(0^+) > 0, I(0^+) > 0, m \leq S(t) \leq M, m \leq I(t) \leq M$ holds for all $t \geq T_0$. Here T_0 may depend on the initial values $(S(0^+), (I(0^+))$.

Theorem 2. Let $(S(t), I(t))$ be any solution of (2.3), If

$$r\tau > \frac{r\theta\mu}{Kw} + \frac{\beta}{2wa} \ln \frac{(1 - e^{-w\tau})^2 + a\mu^2}{(1 - e^{-w\tau})^2 + a\mu^2 e^{-2w\tau}}$$

holds, then the system (2.3) is permanent.

Proof. Suppose $(S(t), I(t))$ is a solution of (2.3) with $S(0) > 0, I(0) > 0$. By lemma 3, we have proved there exists a constant $M > 0$ such that $S(t) \leq M, I(t) \leq M$ for t large enough, we may assume $S(t) \leq M, I(t) \leq M$ and $M > \frac{r}{\beta}$ for $t \geq 0$. From (3.5) we know $I(t) > \widetilde{I}(t) - \varepsilon_2$ for all t large enough and $\varepsilon_2 > 0$, so $I(t) \geq \frac{\mu \exp(-w\tau)}{1-\exp(-w\tau)} - \varepsilon_2 = m_2$ for t large enough. Thus, we only need to find $m_1 > 0$ such that $S(t) \geq m_1$ for t large enough, we will do it in the following two steps.

1° Since $r\tau < \frac{r\theta\mu}{Kw} + \frac{\beta}{wa} \ln \frac{1-e^{-w\tau}+a\mu}{1-e^{-w\tau}+a\mu e^{-w\tau}}$, we can select $m_3 > 0, \varepsilon_1 > 0$ small enough such that $m_3 < \frac{w}{\beta}, \delta = \beta m_3 < w$ and

$$\sigma = r\tau - \frac{r}{K}m_3\tau - \frac{r\theta\varepsilon}{K}\tau - \frac{r\mu\theta}{K(w-\delta)}$$

$$\begin{aligned}
& -\frac{\beta\mu}{a(w-\delta)(a\varepsilon_1^2+1)} \ln \frac{a(\mu+b\varepsilon_1)^2+b^2}{a(\mu e^{(-w+\delta)\tau}+b\varepsilon_1)^2+b^2}) \\
& -\frac{\beta\mu\varepsilon_1}{\sqrt{a}(w-\delta)(a\varepsilon_1^2+1)} (\arctan \frac{\sqrt{a}\mu}{b} - \arctan \frac{\sqrt{a}\mu}{b} e^{(-w+\delta)\tau}) \\
& -\frac{\beta\varepsilon_1^2}{(w-\delta)(a\varepsilon_1^2+1)} \ln \frac{\mu-b\varepsilon_1}{\mu e^{(-w+\delta)\tau}-b\varepsilon_1} > 0
\end{aligned}$$

where $c = 1 - a\mu e^{(-w+\delta)\tau}$, $b = (1 + a\varepsilon_1)(1 - a\mu e^{(-w+\delta)\tau})$. We will prove $S(t) < m_3$ can not hold for $t \geq 0$. otherwise,

$$\frac{dI(t)}{dt} \leq (-w + \delta)I(t)$$

By lemmas 3 and 4, we have $I(t) \leq u(t)$ and $u(t) \rightarrow \overline{u(t)}$, $t \rightarrow \infty$, where $u(t)$ is the solution of

$$\begin{cases} \frac{du(t)}{dt} = (-w + \delta)u(t), t \neq n\tau \\ u(t^+) = u(t) + \mu, t = n\tau \\ u(0^+) = I(0^+) \end{cases} \quad (3.11)$$

and $\overline{u(t)} = \frac{\mu \exp((-w+\delta)(t-n\tau))}{1 - \exp((-w+\delta)\tau)}$, $t \in (n\tau, (n+1)\tau]$. Therefore, exists a $T_1 > 0$ such that

$$I(t) \leq u(t) \leq \overline{u(t)} + \varepsilon_1$$

and

$$\frac{dS(t)}{dt} \geq S(t) \left(r - \frac{r}{K} m_3 - \frac{r\theta}{K} (\overline{u(t)} + \varepsilon_1) - \frac{\beta(\overline{u(t)} + \varepsilon_1)}{1 + a(\overline{u(t)} + \varepsilon_1)} \right). \quad (3.12)$$

for $t \geq T_1$, Let $N_1 \in N$ and $N_1\tau > T_1$, Integrating (3.11) on $(n\tau, (n+1)\tau)$, $n \geq N_1$, we have

$$\begin{aligned}
S((n+1)\tau) & \geq S(n\tau^+) \exp \left(\int_{n\tau}^{(n+1)\tau} \left(r - \frac{r}{K} m_3 - \frac{r\theta}{K} (\overline{u(t)} + \varepsilon_1) - \frac{\beta(\overline{u(t)} + \varepsilon_1)}{1 + a(\overline{u(t)} + \varepsilon_1)} \right) dt \right) \\
& = S(n\tau) e^\sigma
\end{aligned}$$

then $S((N_1 + k)\tau) \geq S(N_1\tau^+) e^{k\sigma} \rightarrow \infty$, as $k \rightarrow \infty$, which is a contradiction, Hence exists a $t_1 > 0$ such that $S(t) \geq m_3$.

2° If $S(t) \geq m_3$ for $t \geq t_1$, then our aim is obtained, Hence, we need only to consider those solutions which leave region $R = \{(S(t), I(t)) \in R_+^2 : S(t) < m_3\}$ and reenter it again. Let $t^* = \inf_{t \geq t_1} \{S(t) < m_3\}$, then $S(t) \geq m_3$ for

$t \in [t_1, t^*)$ and $S(t^*) = m_3$. Since $S(t)$ is continuous, suppose $t^* \in [n\tau, (n+1)\tau)$, $n_1 \in N$. select $n_2, n_3 \in N$, such that

$$n_2\tau > T_2 = \frac{\ln(\frac{\varepsilon_1}{M+\mu})}{-w+\delta}, \quad e^{(n_2+1)\sigma_1\tau} e^{n_3\sigma} > 1$$

where $\sigma_1 = r - \frac{rm_3}{K} - \frac{r\theta}{K}M - \frac{\beta M^2}{1+aM^2} < 0$, Let $T = n_1\tau + n_2\tau$, we claim that there must be a $t_2 \in [(n_1+1)\tau, (n_1+1)\tau + T]$, consider (3.11) with $u((n_1+1)\tau^+) = I((n_1+1)\tau^+)$. We have

$$u(t) = (u((n_1+1)\tau^+) - \frac{\mu}{1-e^{(-w+\delta)\tau}})e^{(-w+\delta)(t-(n_1+1)\tau)} + \overline{u(t)}$$

$t \in (n\tau, (n+1)\tau]$, $n_1+1 \leq n \leq n_1+1+n_2+n_3$, then

$$|u(t) - \overline{u(t)}| < (\mu + M)e^{(-w+\delta)(t-(n_1+1)\tau)}$$

and $I(t) \leq u(t) \leq \overline{u(t)} + \varepsilon_1$, $(n_1+1+n_2)\tau \leq t \leq (n_1+1+T)\tau$, which implies (3.12) holds for $(n_1+1+n_2)\tau \leq t \leq (n_1+1)\tau + T$. As in step1, we have

$$S((n_1+1+n_2+n_3)\tau) \geq S((n_1+1+n_2)\tau)e^{n_3\sigma}$$

The first equation of (2.3) gives

$$\frac{dS(t)}{dt} \geq S(t)(r - \frac{r}{K}m_3 - \frac{r\theta}{K}M - \frac{\beta M^2}{1+aM^2}) = \sigma_1 S(t)$$

Integrating it on $[t^*, (n_1+1+n_2)\tau]$, we have

$$S((n_1+1+n_2)\tau) \geq m_3 e^{\sigma_1(n_2+1)\tau}$$

Then

$$S((n_1+1+n_2+n_3)\tau) \geq m_3 e^{\sigma_1(n_2+1)\tau} e^{n_3\sigma} > m_3$$

this is a contradiction. Let $\bar{t} = \inf_{t \geq t^*} \{S(t) \geq m_3\}$, thus $S(\bar{t}) \geq m_3$ for $t \in [t^*, \bar{t}]$, we have $S(t) \geq S(t^*)e^{\sigma_1(t-t^*)} \geq m_3 e^{\sigma_1(1+n_2+n_3)} = m_1$ for $t \geq \bar{t}$. The same arguments can be continued since $S(\bar{t}) \geq m_3$. Hence $S(t) \geq m_1$ for all $t \geq t_1$, the proof is complete.

Remark 3.1. Let $f(\tau) = r\tau - \frac{r\theta\mu}{Kw} + \frac{\beta}{2wa} \ln \frac{(1-e^{-w\tau})^2 + a\mu^2}{(1-e^{-w\tau})^2 + a\mu^2 e^{-2w\tau}}$, we easily know $f(0) = -\ln \frac{1}{1-\mu_1} < 0$, $f(\tau) \rightarrow \infty$ as $\tau \rightarrow \infty$, and

$$f''(\tau) > 0$$

So $f(\tau) = 0$ has a unique positive root, denoted by τ_{max} . From theorem 1 and theorem 2 we know τ_{max} is a threshold, If $\tau < \tau_{max}$, then the pest-extinction

solution $(0, \widetilde{I(t)})$ of system (2.3) is globally asymptotically stable. If $\tau > \tau_{max}$ the system(2.3) is permanent.

Remark 3.2. Let $f(\mu) = r\tau - \frac{r\theta\mu}{Kw} + \frac{\beta}{2wa} \ln \frac{(1-e^{-w\tau})^2 + a\mu^2}{(1-e^{-w\tau})^2 + a\mu^2 e^{-2w\tau}}$, we easily know $f(0) > 0, f(\tau) \rightarrow -\infty$ as $\tau \rightarrow \infty$, and

$$f''(\mu) < 0$$

So $f(\mu) = 0$ has a unique positive root, denoted by μ_{max} . From theorem 1 and theorem 2 we know μ_{max} is a threshold, If $\mu > \mu_{max}$, then the pest-extinction solution $(0, \widetilde{I(t)})$ of system (2.3) is globally asymptotically stable. If $\mu < \mu_{max}$ the system(2.3) is permanent.

4 Discussion

In this paper, according to the fact, an SI model with impulsive infective pest transmission at fixed moment is proposed and investigated, we analyzed the pest-extinction periodic solution of such system is global asymptotic stability, and the condition for the permanent of system (2.3). there are some interesting problems: how to optimize the period and the amount of releasing infective pest? How to do with the determining time of impulsive releasing infective pests depending on the state of insect pest? and how this impulsive releasing infective pest affect the system? In the real world, For the insect pest is with seasonal damages, Should impulsive releasing infective be considered on finite interval? We will continue to study these problems in the future.

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